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## Price elasticity of demand and capacity expansion features in an enhanced ABC product-mix decision model

Wen-Hsien Tsai<sup>a\*</sup>, Lopin Kuo<sup>b</sup>, Thomas W. Lin<sup>c</sup>,  
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In recent years, activity-based costing (ABC) has become a popular cost and operations management technique to improve the accuracy of firms' product or service costs in order to help the firms stay competitive. Since the product-mix decision is an important ABC application, most studies in the ABC literature were generally focused on the effect of ABC analysis on the product-mix decision or product cost calculation. However, these studies usually ignored some important factors, such as: capacity expansions, management's degree of control over resources, purchase discount, and change of product's price. Hence, in this paper, we consider these factors to propose a more general model. This model can help managers to make a product-mix decision and identify excess resources so that managers can redeploy them to optimise resource usage. Furthermore, since previous studies did not consider the impact of price changes on product-mix decisions, this paper also examines the impact of reducing product price with different price elasticity of demand ( $\epsilon^D$ ) on the simulated company's profit.

**Keywords:** activity-based costing (ABC); price elasticity of demand ( $\epsilon^D$ ); capacity expansion; theory of constraints (TOC); management's control over resources ability

### 1. Introduction

Traditional cost accounting (TCA), which mainly uses direct labour hours or cost to allocate the overhead costs, systematically distorts product costs in advanced manufacturing environments in which overhead costs are a significant portion of product costs. Since incorrect product cost information can lead to poor decisions, activity-based costing (ABC) was developed by General Electric and other firms to improve the usefulness of accounting information (Johnson 1992). Experts believe that ABC can provide more accurate product costing information than TCA when products are diverse in size, complexity, material requirements, and/or setup procedures (Cooper and Kaplan 1988a). ABC uses a two-stage procedure of cost assignment to achieve the accurate costs.

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First, resource costs are traced to various activities by using resource drivers according to the consumption of resources by activities. Then, activity costs are traced to various products or services by using activity drivers according to the consumption of activities by products or services (Cooper 1989, Turney 1991). The main features of the ABC model are the identification of activities and the use of both volume and non-volume drivers to assign resource costs to products or services. In ABC models, the hierarchy of company activities is composed of the following categories: unit-level activities (performed one time for one unit of product or service, e.g., machining, finishing); batch-level activities (performed one time for a batch of products or services, e.g., setup, scheduling); product-level activities (performed to benefit all units of a particular product or service, e.g., product design); and facility-level activities (performed to sustain the manufacturing or service facility, e.g., plant guard and management) (Cooper 1990, Cooper and Kaplan 1991). ABC uses these four categories of activities to facilitate the identification of costs and drivers. Furthermore, appropriate activity drivers should be chosen for different kinds of activity costs. For example, machine hour is used as the activity driver for the activity machining; setup hours or the number of setups for machine setup; and the number of drawings or the number of design hours for product design (Tsai 1996a, 1996b).

Since 1988, ABC has rapidly evolved from the concept stage to implementation. The applications of ABC have been extended from manufacturing industries (Zhuang and Burns 1992, Dhavale 1993, Brewer *et al.* 2003, Needy *et al.* 2003) to service industries (Carlson and Young 1993, Chan 1993, Tsai and Kuo 2004, Tsai and Hsu 2008), non-profit organisations (Antos 1992), and governmental agencies (Harr 1990). In addition, the information obtained from the ABC approach has also been applied in various fields such as product mix decision, joint products decision (Tsai 1996b, Tsai and Lai 2007, Tsai *et al.* 2008), outsourcing (Tsai and Lai 2007, Tsai *et al.* 2007), quality improvement (Tsai 1998), environmental management (Tsai *et al.* 2007, Tsai and Hung 2009a, 2009b), project management (Raz and Elnathan 1999), software development (Fichman and Kemerer 2002), and so on.

Among the above ABC applications, the product-mix decision is important both in management practice and academic research. Product-mix decision problems exist at all times, and it has been applied to mathematical programming on semiconductor foundry manufacturing (Chou and Hong 2000), PCB manufacturing (Bhatnagar *et al.* 1999), and commercial banks (Wheelock and Wilson 2001, Young and Roland 2001). Previous research (Cooper *et al.* 1992, Swenson and Flesher 1996, Foster and Swenson 1997), indicated that the product-mix decision is a dominant application of ABC in their examined firms. In addition, the longitudinal study of Innes *et al.* (2000) demonstrated that ABC users perceive its use as more important and successful for making pricing and product-mix decisions over time.

However, ABC has been criticised for its failure to identify and remove constraints in the production process (Johnson 1992). Therefore, Kee (1995) integrated ABC with the capacity constraints to propose a liner programming model, and adopted the concept of bottleneck recognition and relieve coming from the theory of constraints (TOC) to conduct a sensitivity analysis. After Kee (1995), there were many other factors, such as capacity expansions, management's degree of control over resources, purchase discount, and change of product's price, considered by subsequent researchers to enhance this model (Holmen 1995, Campbell *et al.* 1997, Kee and Schmidt 2000, Lea and Fredendall 2002).

Kaplan (2005) indicated that 'when the capacity of existing resources is exceeded, the pain is obvious through shortages, increased pace of activity, delays, or

poor-quality work'. The ABC approach helps us make clear that such shortages not only occur on machine, but also occur for designing, scheduling, maintaining, and other people resources performing support activities (Kaplan 2005). Facing such shortages, companies can increase the supply of resources by spending more cost to relieve the bottleneck, where Banker and Hughes (1994) and subsequent researchers referred to the ability to acquire additional resources as a soft constraint. Capacity expansion has been applied in power system planning (Wang and Sparrow 1999). However, as what Kee (2008) indicated, the cost of capacity expansion associated with a product-mix decision creates costs that are not proportional to the quantity of a product produced. Therefore, the factor of capacity expansion should be incorporated into the product-mix decision model.

Besides signalling those resources that are at capacity constraints, ABC also signals where unused capacity exists (Kaplan 2005). To obtain the benefit from unused resources, managers may reduce the supply of these resources or redeploy them to other profitable activities. On the contrary, if managers do nothing about the unused capacity, the reduced cost of resources used will only be offset by an equivalent increase in the cost of unused capacity (Kaplan 2005). Unused capacity is an important topic and has been applied in the real world such as the flexible manufacturing system (Sebestyen and Juhasz 2003). Managers have to exploit the unused capacity according to their discretionary power over it. Therefore, identifying management's degree of control over resources is significant for making a product-mix decision.

Pricing is another dominant application of ABC. Since ABC offers more accurate information of product costing, some companies use their ABC information to reprice their products, services, or customers so that the revenues (resources) received exceed the costs of resources used to produce products for individual customers (Cooper and Kaplan 1992). For maximising the net income, product-mix and prices of the products should be considered together. 'A product whose revenue is less than its activity-based cost may be beneficial for the firm to produce when the firm has excess resources that cannot be terminated or deployed elsewhere in the firms operations', as indicated in Kee and Schmidt (2000). On the other hand, re-pricing a product not only affects the revenue of this product directly, price elasticity is also an important factor to consider. Price elasticity consideration has been applied in airport pricing (Zhang and Zhang 2003); and evidence on the cyclical behaviour for food manufacturing industries (Field and Pagoulatos 1997) indicates that the demand of a product varies with its price as well. Furthermore, managers usually determine the amount of resources acquired according to the demand of products, and that may change the cost of resources. For example, the vendor of a material may allow a purchase discount for purchases that qualify a specific threshold.

All factors mentioned above are important, and there are interactions not only among them but also between each of them and the product-mix decision. Therefore, these factors should be considered and evaluated simultaneously within a product-mix decision model. However, most of the related works only consider one or two factors of them. In this paper, we develop a more general model which integrates ABC with the TOC principles to reflect different degree of management's control over resources. In addition, our model incorporates the factors of capacity expansions and purchase discount. We also examine the impact of reducing product price with different price elasticity of demand ( $\epsilon^D$ ) on the simulated company's profit with our model.

In the next section, we review the literature on product-mix decision studies of ABC. In the third section, we develop an enhanced general model. A numerical example is used to illustrate the application of this model in Section 4, and an examination of the impact

of reducing product price with different price elasticity of demand is analysed in Section 5. In the last section, we offer concluding remarks.

## 2. Literature review

The product-mix decision is one of the most important ABC applications (Cooper and Kaplan 1988b, Roth and Borthick 1989, Bakke and Hellberg 1991, Smith and Leksan 1991, Kee 1995, Lea 1998, Kee and Schmidt 2000, Lea and Fredendall 2002). Cooper and Kaplan (1988b), Roth and Borthick, (1989), and Smith and Leksan (1991) focused their analyses on product cost calculation or the impact of the product-mix decision on profits or product costs under ABC. Additionally, some researchers utilised several kinds of mathematical programming to improve product-mix decision models (Charnes *et al.* 1963, Sheshai *et al.* 1977, Tsai and Lin 1990, Tsai 1994, Tsai and Lin 2004).

Kee (1995) provided a numerical example that integrated ABC with the TOC principles to illustrate the economic consequences of production-related decisions. ABC and the theory of constraints (TOC) represent alternative paradigms to traditional cost-based accounting systems. Both paradigms are designed to overcome limitations of traditional cost-based systems. TOC, developed by Goldratt (1990), is a systems-management philosophy and a process of ongoing improvement. TOC is composed of a series of focusing procedures for identifying the bottlenecks and managing the production system concerned with these constraints. After removing a bottleneck, the firm moves to a higher level of goal attainment. The cycle of managing the firm by being concerned with new bottlenecks is repeated, leading to on-going, effective improvements in the firm's operation and performance. In the literature, some researchers (Bakke and Hellberg 1991, MacArthur 1993, Holmen 1995) examined the complementary nature of ABC and TOC, and addressed that the strengths of ABC and TOC could make up for each other's limitations. Under the TOC, direct material is treated as a variable cost, while direct labour and all other costs are treated as fixed. The objective of the TOC is to maximise throughput subject to the capacity of the individual production activities of the firm. TOC assumes that production capacity is constrained while ABC assumes that production capacity is unconstrained. The TOC assumes that there is always a bottleneck. Under TOC, only the products with the highest contribution per unit of the bottleneck should be produced. TOC using throughput maximisation as a decision criterion may lead to suboptimal decisions in some circumstances. For intermediate and longer-run decisions in which management has discretionary power over direct labour or overhead cost items, the global operational measures of TOC ignore factors relevant to decision process. Alternatively, ABC provides a comprehensive framework for modelling the economic attributes of the production process. Thereby, integrating ABC with TOC means to incorporate the opportunity cost of a production constraint, i.e., it can identify those products with the highest potential profit under the firm's limited production opportunities. Simultaneously, it selects products where the marginal revenue from the last unit produced is equal to the marginal cost of the resources used in the manufacturing. On the application of integrating ABC and TOC, Campbell *et al.* (1997) distinguished resources into people-intensive and machine-intensive, and then showed how a hybrid ABC and TOC approach leads to better profitability estimation.

Many researchers suggested that TOC is appropriate for decisions in the short-run, while ABC is appropriate for decisions in the long-run. Kee and Schmidt (2000) argued

that management's discretionary power over labour and overhead determines when the TOC and ABC lead to an optimal product mix. Therefore, Kee and Schmidt (2000) indicated that managers should focus on the discretionary power they have over the resources over a given time horizon rather than focusing on time alone. The assumptions of TOC and ABC were that a firm's management has either no control or has complete control over its labour and overhead resources, respectively. When the respective assumptions are met, the TOC and ABC can lead to optimal product-mix decisions. However, when a firm has varying degrees of control over labour and overhead resources, neither the TOC nor ABC may lead to an optimal product-mix. Kee and Schmidt (2000) developed a more general product-mix decision model that overcame the stringent requirements of the TOC and ABC, and demonstrated that the TOC and ABC were special cases of their model. Lea and Fredendall (2002) found in an extension study of Kee and Schmidt's (2000) model that the performance of management accounting systems or product-mix algorithms or product structures was not different between the short and the long term.

Kee (1995), and Kee and Schmidt's (2000) ABC product-mix models neglected the mechanism of capacity expansion in production. The feasibility or benefits of capacity expansions have usually been evaluated by using post-optimal (sensitivity) analysis. As argued by Tsai and Lin (2004), it is difficult to simultaneously consider two or more kinds of capacity expansions by using this kind of product-mix decision models. Tsai and Lin (2004) utilised mixed-integer programming to develop a product-mix model with capacity expansion features and illustrated how to quickly acquire the product-mix information. However, the basis for making these decisions in Tsai and Lin's (2004) ABC model is to assume that a firm's management has complete control over its labour and most of the overhead resources.

Previous studies usually regarded price as fixed in their models, while the higher a product's price, the lower the quantity of the product that will be produced and sold (Kee, 2008). Kee (2008) considered the different price and its corresponding demand of a product. Kee (2008) also considered the purchase discount for further improvement in the product-mix model. However, in Kee (2008), price of a product was only divided into high, medium, and low levels, and the different price elasticity of the product was ignored. Because the demand curve is downward sloping, when the price is lower, more customers will demand or purchase. When the price is lower, then customers will increase demand or purchases, because the demand curve is downward sloping. Companies in order to satisfy the customers' demand, will adopt the capacity expansion strategy. The greater  $\epsilon^D$  value in the product the more customers will demand or purchase by a lower price. Hence, the factors of capacity expansion and price elasticity should be considered. Both of them have been applied in water supply in a planning support system (Kim and Hopkins 1996) and manufacturing industry (Chomiakow 2007). On the other hand, Kee (2008) also considered the purchase discount for further improvement in the product-mix model. However, in Kee (2008), price of a product was only divided into high, medium, and low levels, and the different price elasticity of the product was ignored.

In this paper, we integrate the ideas of Kee and Schmidt (2000) and Tsai and Lin (2004), and develop a more general model which considers both management's degree of control over resources and capacity expansion. In addition, our study also considers the impact of price elasticity of demand to help managers find an optimal product-mix solution.

### 3. Model formulation

In this section, we propose an enhanced general model which incorporates four factors: capacity constraint, management's degree of control over resource, capacity expansions, and purchase discount to determine an optimal product-mix decision.

#### 3.1 Assumptions

Suppose a company plans to add a new product into the existing product line. The following assumptions are incorporated in our enhanced general product-mix decision model. First, the overhead activities in a multi-product manufacturing company have been classified as unit-level, batch-level, product-level, and facility-level activities, and resource drivers and activity drivers have been chosen by the company's ABC task force through an ABC study. Second, the data on running actual activity cost (Tyson *et al.* 1989) per activity driver for each activity has been collected and used in the product-mix decision model. Third, unit prices and direct material costs for existing products remain fixed within a certain relevant range in the short term. For some materials, a purchase discount will be offered if the quantity in an order satisfies a specific threshold. Fourth, this model does not consider the substitution of resources in the product-mix decision; for example, this model does not consider the replacement of labour hours by machine hours. Fifth, under ABC, facility-level activity costs will, however, be smaller in amount than traditional fixed costs because they represent only that portion of the traditional fixed costs that do not vary with batch- or product-level activities (Lere 2000). They include costs such as depreciation, property taxes and insurance on the factory building. Floor space occupied is often referred to as the facility-level driver for assigning facility-level costs. However, this stretches the idea of a driver, because it is rare that the total floor space devoted to each product or unit can be identified (Carter and Usry 2002). For the purpose of planning, we assume, in this paper, that the cost function of facility-level activity is a stepwise function that varies with the machine-hours range (Tsai and Lin 2004). Sixth, the company has established good relationships and contracts with some vendors, so renting additional machines from vendors can expand the machine hour resource in the short term. Seventh, in conforming to government's policy, the direct labour resource can be expanded by using overtime work or additional night shifts with a higher wage rate in the short term. Eighth, management can acquire accurate cost information from the company's financial department, so the company's production policy will be based on a maximising profit rule instead of only considering customers' demand. This study implies that the company produces both the old and new products based on aggregate market demand and related cost information of products. Ninth, the price of potential competitors' products is not sensitive to temporary price changes in the company's products in the short term. The main reason for this is because the products of each company have their own customer segments and price orientation.

#### 3.2 Notation

The following notation is used in this paper:

- $\pi$  the company's profit;
- $p_i$  the unit selling price of product  $i$ ;

- $Y_i$  the production quantity of old product  $i$ ;  
 $q_i$  the production quantity of new product  $i$ ;  
 $l_r$  the unit cost of the  $r$ th material without a purchase discount used;  
 $lD_r$  the unit cost of the  $r$ th material with a purchase discount used ( $r \in D$ );  
 $A_r$  the quantities of the  $r$ th material without a purchase discount used;  
 $AD_r$  the quantities of the  $r$ th material with a purchase discount used ( $r \in D$ );  
 $TD_r$  the quantities of the  $r$ th material that an order has to satisfy for receiving a purchase discount;  
 $SD_r$  a 0–1 variable.  $SD_r = 1$  means that the quantities of the  $r$ th material satisfy the threshold of discount ( $r \in D$ ), otherwise,  $SD_r = 0$ ;  
 $ND_r$  a 0–1 variable.  $ND_r = 1$  means that the quantities of the  $r$ th material dissatisfy the threshold of discount ( $r \in D$ ), otherwise,  $ND_r = 0$ ;  
 $b_{ir}$  the requirement of the  $r$ th material for one unit of product  $i$ ;  
 $W_r$  the available quantity of the  $r$ th material;  
 $G_1$  the available normal direct labour hours;  
 $G_2$  total direct labour hours under overtime work or additional night shift situation;  
 $C_1$  total direct labour cost in  $G_1$ ;  
 $C_2$  total direct labour cost in  $G_2$ ;  
 $\eta_1, \eta_2$  an SOS1 (special ordered set of type 1) set of 0–1 variables within which exactly one variable must be non-zero (Williams 1985);  
 $\alpha_0, \alpha_1, \alpha_2$  an SOS2 (special ordered set of type 2) set of non-negative variables within which at most two adjacent variables, in the ordering given to the set, can be non-zero (Williams 1985);  
 $a_j$  the actual running activity cost per activity driver for activity  $j$ ;  
 $a_h$  the cost per direct machine hour;  
 $\phi_{ij}$  the requirement of the activity driver of unit-level activity  $j$  ( $j \in U$ ) for one unit of product  $i$ ;  
 $N_j$  the capacity limit of the activity driver of unit-level activity  $j$  ( $j \in U$ );  
 $\mu_{ij}$  the requirement of the activity driver of batch-level activity  $j$  ( $j \in B$ ) for product  $i$ ;  
 $B_{ij}$  the number of the batches of batch-level activity  $j$  ( $j \in B$ ) for product  $i$ ;  
 $\sigma_{ij}$  the number of units per batch of batch-level activity  $j$  ( $j \in B$ ) for product  $i$ ;  
 $T_j$  the capacity limit of the activity driver of batch-level activity  $j$  ( $j \in B$ );  
 $\rho_{ij}$  the requirement of the activity driver of product-level activity  $j$  ( $j \in P$ ) for product  $i$ ;  
 $Z_i$  the indicator for producing product  $i$  ( $Z_i = 1$ ) or not producing product  $i$  ( $Z_i = 0$ );  
 $Q_j$  the capacity limit of the activity driver of product-level activity  $j$  ( $j \in P$ );  
 $D_i$  the maximal (aggregate market) demand of product  $i$ ;  
 $M_k$  the machine hours of  $k$ th level capacity;  
 $F_k$  the facility-level activity cost in machine hours of the  $k$ th level capacity;  
 $\varphi_{ih}$  the requirement of the machine hours for one unit of product  $i$ ;  
 $\theta_k$  an SOS1 set of 0–1 variables within which exactly one variable must be non-zero,  $\theta_k = 1$  ( $k \neq 0$ ) means that the capacity needs to be expanded to the  $k$ th level, i.e.,  $M_k$  machine hours (Williams 1985);  
 TL total direct labour hours;



$\varepsilon^D$  the price elasticity of demand (using the measurement of arc elasticity in this paper).

**3.3 The enhanced ABC product-mix decision model**

In the remainder of this section, we first introduce two product-mix decision models under ABC and TOC, respectively, and then, we propose our enhanced general model, where both TOC and ABC are special cases of this model. As stated in Section 3.1, both labour hours and machine hours are expandable resources. Therefore, we separate them from the four categories of overhead activity costs for describing in the following models. We first present an enhanced ABC product-mix decision model based on Kee and Schmidt (2000) and Tsai and Lin (2004) as follows:

- Maximise  $\pi$  = total revenue
- total direct material cost
- total direct labour cost
- total direct machine cost
- total unit-, batch-, product- & facility-level overhead activity costs.

$$\begin{aligned} \pi = & \left( \sum_{i=1}^n p_i Y_i + \sum_{i=n+1}^m p_i q_i \right) - \left( \sum_{r=1}^s l_r A_r + \sum_{r \in D} l D_r A D_r \right) - (C_1 \alpha_1 + C_2 \alpha_2) \\ & - \left( \sum_{i=1}^n a_{ih} \varphi_{ih} Y_i + \sum_{i=n+1}^m a_{ih} \varphi_{ih} q_i \right) - \left( \sum_{i=1}^n \sum_{j \in U} a_j \varphi_{ij} Y_i + \sum_{i=n+1}^m \sum_{j \in U} a_j \varphi_{ij} q_i \right) \\ & - \sum_{i=1}^m \sum_{j \in B} a_j \mu_{ij} B_{ij} - \sum_{i=1}^m \sum_{j \in P} a_j \rho_{ij} Z_i - \sum_{k=0}^l F_k \theta_k. \end{aligned} \tag{1}$$

Subject to:

Direct material quantity constraints:

$$\sum_{i=1}^n b_{ir} Y_i + \sum_{i=n+1}^m b_{ir} q_i \leq A_r, \quad r = 1, 2, \dots, s, \quad r \notin D \tag{2}$$

$$A_r \leq W_r, \quad r = 1, 2, \dots, s, \quad r \notin D \tag{3}$$

$$\sum_{i=1}^n b_{ir} Y_i + \sum_{i=n+1}^m b_{ir} q_i \leq A_r + A D_r, \quad r = 1, 2, \dots, s, \quad r \in D \tag{4}$$

$$A D_r \geq T D_r S D_r, \quad r = 1, 2, \dots, s, \quad r \in D \tag{5}$$

$$A_r < T D_r N D_r, \quad r = 1, 2, \dots, s, \quad r \in D \tag{6}$$

$$A D_r \leq W_r S D_r, \quad r = 1, 2, \dots, s, \quad r \in D \tag{7}$$

$$ND_r + SD_r = 1, \quad r = 1, 2, \dots, s, \quad r \in D \quad (8)$$

$$A_r \geq 0, \quad r = 1, 2, \dots, s. \quad (9)$$

Direct labour hour constraints:

$$TL = G_1\alpha_1 + G_2\alpha_2 \quad (10)$$

$$\alpha_0 - \eta_1 \leq 0 \quad (11)$$

$$\alpha_1 - \eta_1 - \eta_2 \leq 0 \quad (12)$$

$$\alpha_2 - \eta_2 \leq 0 \quad (13)$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1 \quad (14)$$

$$\eta_1 + \eta_2 = 1. \quad (15)$$

Machine hour constraints:

$$\sum_{i=1}^n \varphi_{ih} Y_i + \sum_{i=n+1}^m \varphi_{ih} q_i \leq \sum_{k=0}^t M_k \theta_k \quad (16)$$

$$\sum_{k=0}^t \theta_k = 1. \quad (17)$$

Unit-level activity constraints:

$$\sum_{i=1}^n \varphi_{ij} Y_i + \sum_{i=n+1}^m \varphi_{ij} q_i \leq N_j, \quad j \in U. \quad (18)$$

Batch-level activity constraints:

$$Y_i \leq \sigma_{ij} B_{ij}, \quad i = 1, 2, \dots, n, \quad j \in B \quad (19)$$

$$q_i \leq \sigma_{ij} B_{ij}, \quad i = n + 1, n + 2, \dots, m, \quad j \in B \quad (20)$$

$$\sum_{i=1}^m \mu_{ij} B_{ij} \leq T_j, \quad j \in B. \quad (21)$$

Product-level activity constraints:

$$Y_i \leq D_i Z_i, \quad i = 1, 2, \dots, n \quad (22)$$

$$q_i \leq D_i Z_i, \quad i = n + 1, n + 2, \dots, m \quad (23)$$

$$\sum_{i=1}^m \rho_{ij} Z_i \leq Q_j, \quad j \in P \quad (24)$$

$$Y_i \geq 0, \quad i = 1, 2, \dots, n \quad (25)$$

$$q_i \geq 0, \quad i = n + 1, n + 2, \dots, m. \quad (26)$$

Where:

$(ND_r, SD_r)$  is an SOS1 set of 0–1 variables,  $r = 1, 2, \dots, s, r \in D$ ;

$(\alpha_0, \alpha_1, \alpha_2)$  is an SOS2 set of non-negative variables;

$(\eta_1, \eta_2)$  is an SOS1 set of 0–1 variables;

$(\theta_0, \theta_1, \dots, \theta_t)$  is an SOS1 set of 0–1 variables;

$Z_i$  is 0–1 variable,  $i = 1, 2, \dots, m$ ; and

$B_{ij}$  is non-negative integer variable,  $i = 1, 2, \dots, m, j \in B$ .

The following is the detailed description of the above model:

- **Total revenue**

The terms in the first set of parentheses in Equation (1) represent total revenue of old and new products. The first term is the revenue of  $n$  original products and the second term is that of the new product.

- **Total direct material cost**

The terms in the second set of parentheses in Equation (1), i.e.,  $\sum_{r=1}^s l_r A_r + \sum_{r \in D} l D_r A D_r$ , represent total direct material cost with ( $r \in D$ ) and without ( $r \notin D$ ) a purchase discount, respectively. Equations (2)–(9) are the constraints associated with the various kinds of materials. For a material with a purchase discount condition ( $r \in D$ ), Equation (4) describes that the quantity of this material that either did qualify or did not qualify of a purchase discount should satisfy its necessary amount for producing each product. Equations (5) and (6) describe the conditions that a purchase discount either did qualify or not. Equation (7) sets a maximum quantity of a material with a purchase discount that can be ordered. At last, Equation (8) ensures that one, and only one of the conditions described by Equations (5) and (6) is in effect for each material.

- **Total direct labour cost**

The terms in the third set of parentheses in Equation (1), i.e.,  $C_1 \alpha_1 + C_2 \alpha_2$ , represent total direct labour cost of both old and new products. Equations (10)–(15) are the constraints associated with direct labour. TL, in Equation (10), is total direct labour hours we need, and its function depends on the case under study. In this paper, we assume that the direct labour resource can be expanded by using overtime work or additional night shift with a higher wage rate. Thus, the total direct labour cost function will be a piecewise linear function composed of two segments with different wage rates as shown in Figure 1. In Equations (11)–(15),  $(\eta_1, \eta_2)$  is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero;  $(\alpha_0, \alpha_1, \alpha_2)$  is an SOS2 set of non-negative variables within which at most two adjacent variables, in the order given to the

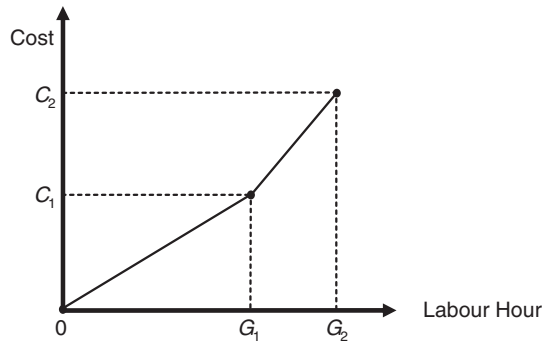


Figure 1. Direct labour cost in enhanced ABC product-mix decision model.

set, can be non-zero (Williams 1985). If  $\eta_1 = 1$ , then  $\eta_2 = 0$ ,  $\alpha_2 = 0$ ,  $\alpha_0, \alpha_1 \leq 1$ , and  $\alpha_0 + \alpha_1 = 1$ . Thus, total direct labour hours needed and total direct labour cost are  $G_1\alpha_1$  and  $C_1\alpha_1$ , respectively. This means that: (1) the point  $(G_1\alpha_1, C_1\alpha_1)$  is on the first segment of the piecewise linear direct labour cost function; and (2)  $(G_1\alpha_1, C_1\alpha_1)$  is the linear combination of  $(0, 0)$  and  $(G_1, C_1)$ . This also means that we will not need the overtime work. On the other hand, if  $\eta_2 = 1$ , then  $\eta_1 = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_1, \alpha_2 \leq 1$ , and  $\alpha_1 + \alpha_2 = 1$ . Thus, total direct labour hours needed and total direct labour cost are  $G_1\alpha_1 + G_2\alpha_2$  and  $C_1\alpha_1 + C_2\alpha_2$ , respectively. This means that: (1) the point  $(G_1\alpha_1 + G_2\alpha_2, C_1\alpha_1 + C_2\alpha_2)$  is on the second segment of the piecewise linear direct labour cost function; and (2)  $(G_1\alpha_1 + G_2\alpha_2, C_1\alpha_1 + C_2\alpha_2)$  is the linear combination of  $(G_1, C_1)$  and  $(G_2, C_2)$ . This also means that we will need the overtime work.

- **Total direct machine cost**

The terms in the fourth set of parentheses in Equation (1), i.e.,  $\sum_{i=1}^n a_h \varphi_{ih} Y_i + \sum_{i=n+1}^m a_h \varphi_{ih} q_i$ , represent total direct machine cost. As stated in Section 3.1, we assume that the facility-level activity cost function is a stepwise function, discussed later, which varies with the machine hours.

- **Total unit-level overhead activity cost**

The term  $(\sum_{i=1}^n \sum_{j \in U} a_j \varphi_{ij} Y_i + \sum_{i=n+1}^m \sum_{j \in U} a_j \varphi_{ij} q_i)$  in Equation (1) is total unit-level overhead activity cost of various unit-level activities ( $j \in U$ ) needed by both old and new products. Equation (18) is the capacity constraints for various unit-level activities.

- **Total batch-level overhead activity cost**

The term  $\sum_{i=1}^m \sum_{j \in B} a_j \mu_{ij} B_{ij}$  in Equation (1) is total batch-level overhead activity cost of various batch-level activities ( $j \in B$ ) needed by both old and new products. Equations (19)–(21) are the constraints associated with various batch-level activities, where Equation (21) is the capacity constraints for various batch-level activities. For example, we may use ‘setup hours’ as the activity driver of the batch-level activity ‘setup’ because each product needs different setup hours. In this case,  $T_j$  are the available setup hours,  $\mu_{ij}$  are the needed setup hours for product  $i$ ,  $B_{ij}$  is the number of setups needed for product  $i$ , and  $\sigma_{ij}$  is the average

number of units in each setup batch. In fact, there may be a different number of units in each setup batch for a specific product. However, we can use the average number of units for the purpose of planning. On the other hand, we may use ‘number of batches’ as the activity driver of the batch-level activity ‘setup’ because the setup hours needed is the same for each product. In this situation,  $\mu_{ij}$  can be set to 1 and  $T_j$  is the available number of setups.

- **Total product-level overhead activity cost**

The term  $\sum_{i=1}^m \sum_{j \in P} a_j \rho_{ij} Z_i$  in Equation (1) is total product-level overhead activity cost of various product-level activities ( $j \in P$ ) needed by both old and new products. Equations (22)–(24) are the constraints associated with various product-level activities. Equations (22) and (23) are the market demand constraints for old and new products, respectively and Equation (24) is the capacity constraints for various product-level activities. For example, we may use ‘number of drawings’ as the activity driver of the product-level activity ‘product design’. In this case,  $Q_j$  is the available number of drawings for the firm’s capacity, and  $\rho_{ij}$  is the number of drawings needed for product  $i$ . Since  $Z_i$  is a 0–1 variable, the market demand constraint for product  $i$  (in Equation (22) or (23)) will exist if  $Z_i = 1$ .

- **Total facility-level overhead activity cost**

The term  $\sum_{k=0}^t F_k \theta_k$  in Equation (1) is total facility-level overhead activity cost and Equations (16)–(17) are the associated constraints. For the purpose of planning, we assume that the cost function of facility-level activity is a stepwise function that varies with the machine-hours range.  $(\theta_0, \theta_1, \dots, \theta_t)$  is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero (Williams 1985). If  $\theta_0 = 1$ , the current machine hour capacity  $M_0$  will not be expanded and the current total facility-level activity cost is still  $F_0$ . If  $\theta_v = 1$  ( $v \neq 0$ ), we know that the capacity needs to be expanded to the  $v$ th level, i.e.,  $M_v$  machine hours, and total facility-level activity cost will be increased to  $F_v$ .

### 3.4 The product-mix decision model under TOC

In Kee (1995), and Kee and Schmidt (2000), a product-mix decision model under TOC selects a product mix based on maximising throughput, which is defined as a product’s price less its direct material cost, while direct labour and overhead are treated as a fixed operating expense. However, since the cost of capacity expansion is not a committed resource initially, the cost of capacity expansion should be treated as a variable. Therefore, for TOC, the product mix is selected by maximising the objective function shown in Equation (27) subject to the constraints in Equations (2)–(26) except Equations (10)–(15) which are replaced with Equations (28) and (29).

For the terms in the third set of parentheses in Equation (27), the cost of normal direct labour hours,  $C_1$ , becomes fixed, but the direct labour resource can be expanded from  $G_1$  to  $G_2$  as shown in Figure 2.  $\beta$  is a non-negative variable, where  $(G_2 - G_1)\beta$  and  $(C_2 - C_1)\beta$  are the total expanded direct labour hours needed and the total expanded direct labour cost, respectively.

The terms from the fifth set to the last set of parentheses in Equation (27) represent the total overhead activity cost of the four levels respectively. The cost function of total direct

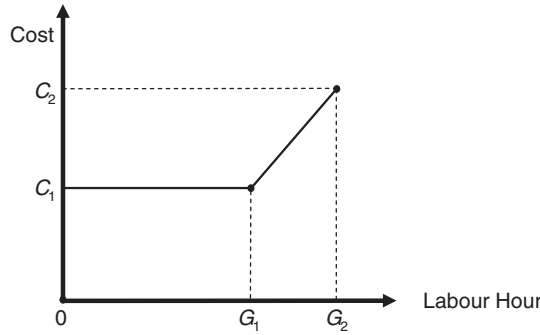


Figure 2. Direct labour cost in enhanced TOC product-mix decision model.

machine cost, i.e.,  $\sum_{k=0}^t a_h M_k \theta_k$ , and facility-level activity, i.e.,  $\sum_{k=0}^t F_k \theta_k$ , are assumed to be stepwise functions that vary with the machine-hours range.

$$\begin{aligned} \pi = & \left( \sum_{i=1}^n p_i Y_i + \sum_{i=n+1}^m p_i q_i \right) - \left( \sum_{r=1}^s l_r A_r + \sum_{r \in D} l D_r A D_r \right) \\ & - (C_1 + (C_2 - C_1)\beta) \\ & - \sum_{k=0}^t a_h M_k \theta_k - \sum_{j \in U} a_j N_j - \sum_{j \in B} a_j T_j - \sum_{j \in P} a_j Q_j - \sum_{k=0}^t F_k \theta_k \end{aligned} \tag{27}$$

$$TL = G_1 + (G_2 - G_1)\beta \tag{28}$$

$$\beta \geq 0. \tag{29}$$

### 3.5 The enhanced general product-mix decision model

As indicated by Kee and Schmidt (2000), Equations (1) and (27) imply that management has either complete control or has no control over labour and overhead resources that are somewhat extreme. Therefore, we propose an enhanced general model which incorporates management’s degree of control over resources together with capacity expansions and purchase discount into the product-mix decision model as follows:

$$\begin{aligned} \pi = & \left( \sum_{i=1}^n p_i Y_i + \sum_{i=n+1}^m p_i q_i \right) - \left( \sum_{r=1}^s l_r A_r + \sum_{r \in D} l D_r A D_r \right) \\ & - (NC_1 + DC_1 \alpha_1 + (C_2 - NC_1) \alpha_2) \\ & - \sum_{k=0}^t a_h (NM_k + DM_k^*) \theta_k - \sum_{j \in U} (a_j (NN_j + DN_j^*)) \\ & - \sum_{j \in B} (a_j (NT_j + DT_j^*)) \\ & - \sum_{j \in P} (a_j (NQ_j + DQ_j^*)) - \sum_{k=0}^t F_k \theta_k. \end{aligned} \tag{30}$$

Subject to:

Direct material quantity constraints:

$$\sum_{i=1}^n b_{ir} Y_i + \sum_{i=n+1}^m b_{ir} q_i \leq A_r, \quad r = 1, 2, \dots, s, \quad r \notin D \quad (31)$$

$$A_r \leq W_r, \quad r = 1, 2, \dots, s, \quad r \notin D \quad (32)$$

$$\sum_{i=1}^n b_{ir} Y_i + \sum_{i=n+1}^m b_{ir} q_i \leq A_r + AD_r, \quad r = 1, 2, \dots, s, \quad r \in D \quad (33)$$

$$AD_r \geq TD_r SD_r, \quad r = 1, 2, \dots, s, \quad r \in D \quad (34)$$

$$A_r < TD_r ND_r, \quad r = 1, 2, \dots, s, \quad r \in D \quad (35)$$

$$AD_r \leq W_r SD_r, \quad r = 1, 2, \dots, s, \quad r \in D \quad (36)$$

$$ND_r + SD_r = 1, \quad r = 1, 2, \dots, s, \quad r \in D \quad (37)$$

$$A_r \geq 0, \quad r = 1, 2, \dots, s. \quad (38)$$

Direct labour hour constraints:

$$TL = (NG_1 + DG_1 \alpha_1 + (G_2 - NG_1) \alpha_2) \quad (39)$$

$$\alpha_0 - \eta_1 \leq 0 \quad (40)$$

$$\alpha_1 - \eta_1 - \eta_2 \leq 0 \quad (41)$$

$$\alpha_2 - \eta_2 \leq 0 \quad (42)$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1 \quad (43)$$

$$\eta_1 + \eta_2 = 1. \quad (44)$$

Machine hour constraints:

$$\sum_{i=1}^n \varphi_{ih} Y_i + \sum_{i=n+1}^m \varphi_{ih} q_i \leq \sum_{k=0}^l (NM_k^* + DM_k^*) \theta_k \quad (45)$$

$$\sum_{k=0}^l NM_k^* \theta_k \leq \sum_{k=0}^l NM_k \theta_k \quad (46)$$

$$\sum_{k=0}^l DM_k^* \theta_k \leq \sum_{k=0}^l DM_k \theta_k \quad (47)$$

$$\sum_{k=0}^l (NM_k + DM_k) \theta_k \leq \sum_{k=0}^l M_k \theta_k \quad (48)$$

$$\sum_{k=0}^t \theta_k = 1. \quad (49)$$

Unit-level activity constraints:

$$\sum_{i=1}^m \varphi_{ij} Y_i + \sum_{i=1}^m \varphi_{ij} q_i \leq (NN_j^* + DN_j^*), \quad j \in U \quad (50)$$

$$NN_k^* \leq NN_k \quad (51)$$

$$DN_k^* \leq DN_k. \quad (52)$$

Batch-level activity constraints:

$$Y_i \leq \sigma_{ij} B_{ij}, \quad i = 1, 2, \dots, n, \quad j \in B \quad (53)$$

$$q_i \leq \sigma_{ij} B_{ij}, \quad i = n+1, n+2, \dots, m, \quad j \in B \quad (54)$$

$$\sum_{i=1}^m \mu_{ij} B_{ij} \leq (NT_j^* + DT_j^*), \quad j \in B \quad (55)$$

$$NT_j^* \leq NT_j, \quad j \in B \quad (56)$$

$$DT_j^* \leq DT_j, \quad j \in B. \quad (57)$$

Product-level activity constraints:

$$Y_i \leq D_i Z_i, \quad i = 1, 2, \dots, n \quad (58)$$

$$q_i \leq D_i Z_i, \quad i = n+1, n+2, \dots, m \quad (59)$$

$$\sum_{i=1}^n \rho_{ij} Z_i \leq (NQ_j^* + DQ_j^*), \quad j \in P \quad (60)$$

$$NQ_j^* \leq NQ_j, \quad j \in P \quad (61)$$

$$DQ_j^* \leq DQ_j, \quad j \in P. \quad (62)$$

Where:

$(ND_r, SD_r)$  is an SOS1 set of 0–1 variables,  $r = 1, 2, \dots, s$ ,  $r \in D$ ;

$(\alpha_0, \alpha_1, \alpha_2)$  is an SOS2 set of non-negative variables;

$(\eta_1, \eta_2)$  is an SOS1 set of 0–1 variables;

$(\theta_0, \theta_1, \dots, \theta_t)$  is an SOS1 set of 0–1 variables;

$Z_i$  is 0–1 variable,  $i = 1, 2, \dots, m$ ; and

$B_{ij}$  is non-negative integer variable,  $i = 1, 2, \dots, m$ ,  $j \in B$ .



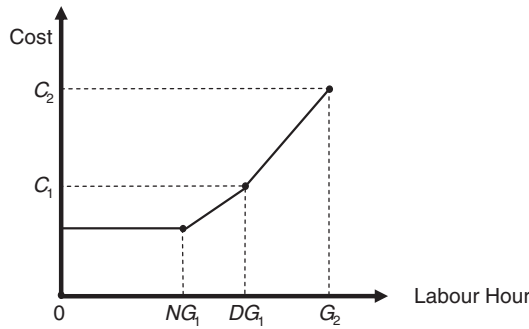


Figure 3. Direct labour cost in enhanced general product-mix decision model.

We assume that direct material is a variable cost and facility-level activity is a fixed cost as prior studies did. For other resources, management may have different discretionary power. Therefore, we use  $DC_1$ ,  $DM_k$ ,  $DN_j$ ,  $DT_j$ , and  $DQ_j$  to denote the amount of resources which subject to management control, and use  $NC_1$ ,  $NM_k$ ,  $NN_j$ ,  $NT_j$ , and  $NQ_j$  to denote the amount of resources which do not subject to management control, while  $DC_1^*$ ,  $NC_1^*$ ,  $DM_k^*$ ,  $NM_k^*$ ,  $DN_j^*$ ,  $NN_j^*$ ,  $DT_j^*$ ,  $NT_j^*$ ,  $DQ_j^*$ , and  $NQ_j^*$  represent the amount of those resources that are consumed in production.

The objective function in Equation (30) incorporates the resources that management has no control over as a fixed cost and the resources over which management has control that is used in production as a product cost. The constraints in Equations (45)–(62) state that different kinds of resources used in production must under the sum of the amount of those resources which are non-discretionary and the amount of those resources which are discretionary and are used in production. The amounts of resources which have no control over and control over are also restricted by the constraints in Equations (45)–(62).

The normal direct labour hours are separated into two parts: non-discretionary labour hours and discretionary labour hours as shown in Figure 3. The terms in the third set of parentheses in Equation (30), i.e.,  $(NC_1 + DC_1\alpha_1 + (C_2 - NC_1)\alpha_2)$ , represent total direct labour cost of both old and new products, where  $NC_1$  represents the cost of non-discretionary labour hours, and  $DC_1\alpha_1 + (C_2 - NC_1)\alpha_2$  represents the sum of the cost of discretionary labour hours and the cost of overtime work. The functions of Equations (40)–(44) are similar to the Equations (11)–(15) in the enhanced ABC model.

#### 4. A numerical illustration

##### 4.1 Data and description of a numerical example

This paper provides a numerical example to apply the product-mix decision models. The data and background of this example are described as follows.

Let us assume that a company produces two products, A ( $i=1$ ) and B ( $i=2$ ), and plans to launch a new product C ( $i=3$ ). Each product requires the processing of five primary activities, of which, two activities are performed at the unit-level, two at the batch-level and one at the product-level. Three products consume two types of direct material. The vendor of material 1 allows a purchase discount of 10% if the amount of the purchase reaches \$450,000. Table 1 shows the data in this example. In Table 1,  $M_0$  stands for the current machine capacity, 200,000 machine hours, and  $F_0$  represents the facility-level

Table 1. Data for the numerical example.

	Product <i>i</i>			Available capacity
	A <i>i</i> =1	B <i>i</i> =2	C <i>i</i> =3	
Maximum demand	$D_i$ 500,000	300,000	250,000	
Selling price	$p_i$ 28	49	100	
Direct material	$b_{r1}$ 1	2	1	$W_1 = 800000$
	$b_{r2}$ 1	1	2	$W_2 = 600000$
Unit-level activity	Cost/driver			
1 Machining	Activity driver	$a_j$		
2 Finishing	Machine hours	\$1		
Batch-level activity	Production order	\$100		
	Scheduling	Setup hours	\$100	
Setup	Drawings	\$300		
Product-level activity	Cost	$F_0 = \$3,000,000$	$F_1 = \$4,500,000$	$F_2 = \$6,000,000$
Design	Machine hours	$M_0 = 200,000$	$M_1 = 240,000$	$M_2 = 280,000$
	Facility-level cost	Cost	$C_1 = \$1,600,000$	$C_2 = \$2,400,000$
Direct labour constraint	Labour hours	$G_1 = 400,000$	$G_2 = 560,000$	$NG_1 = 300,000$
	Wage rate	$R_1 = \$4/hr$	$R_2 = \$5/hr$	$R_1 = \$4/hr$
				$DC_1 = \$400,000$
				$DG_1 = 100,000$
				$R_1 = \$4/hr$
				$Q_5 = 600$

Note: 1. For material 1, a purchase discount of 10% will be offered, if the amount of the purchase reaches \$450,000;  
 2. Management has complete control over drawings, but has no control over production order and setup hours.

activity cost, \$3,000,000 at this capacity level. To increase the machine capacity from  $M_0$  to  $M_1$  or  $M_2$ , the company needs to lease machines from vendors and, as a result, increases the facility-level cost to \$4,500,000 ( $F_1$ ) or \$6,000,000 ( $F_2$ ). Normal direct labour hours,  $G_1$ , is 400,000 hours, and the wage rate is \$4 per hour. In the 400,000 direct labour hours, 300,000 of them come from formal employees that are non-discretionary, while the other 100,000 come from temporary workers that are discretionary. The direct labour hours can be expanded to  $G_2 = 560,000$  hours by using overtime work and the wage rate will be \$5 per hour. In this case, total direct labour hours (TL), can be depicted by the following equation:

$$\sum_{i=1}^n \varphi_{i2} Y_i + \sum_{i=n+1}^m \varphi_{i2} q_i = G_1 \alpha_1 + G_2 \alpha_2. \quad (63)$$

In this example, management has complete control over drawings, but has no control over production order and setup hours.

## 4.2 Three-part analysis

### 4.2.1 Part 1: determining the optimal product-mix and capacity expansion before adding new products

The objective of this part is to find the optimal product-mix that maximises the company's profit prior to adding a new product into the product-mix. Based on the information provided in Table 1, this part applies the three proposed models, which are 0–1 mixed-integer programming models and can be solved by the software 'LINGO', and ignores the new product C's information. Table 2 shows the objective function, related constraints before adding a new product into the product-mix of the three models.

A comparison of the optimal solutions of the three models is shown in Table 3. In Table 3, panel 1, the product mix, the resources used in production, the unused resources, and the capacity expansions are compared, while an income statement for the product mix selected with the three models is shown in Table 3, panel 2. According to Table 3, panel 2, the product mix selected with ABC has the highest income based on the resources used in production. However, when the cost of unused non-discretionary resources was deducted from revenue, the product mix selected with the enhanced general model leads to the highest income.

### 4.2.2 Part 2: determining the optimal product-mix after adding new products

Generally, the initial production volume of a new product is lower because the company is unaware of the condition of market acceptance. Over a period of time, when the new product receives positive evaluation from customers, the company then takes expanded action. Additionally, the company will gradually transfer production resources from old products to new products. However, the company's production policy in old products still maintains at specific volumes to keep the old customers.

The company should evaluate carefully for new product's price orientation in the market. Based on the information provided in Table 1, this part applies our enhanced general model, Equations (30) to (62), and other constraints (i.e., new product's price is equal to 70 ( $p_3 = 70$ ) and old products' production volumes are greater than or equal to 500 units respectively ( $Y_1 \geq 500$ ,  $Y_2 \geq 500$ )) to explore the optimal product-mix.

Table 2. Decision analysis prior to adding the new product into the product-mix (only two original products).

**Enhanced ABC product-mix decision model**

Maximise

$$\begin{aligned} \pi = & 28Y_1 + 49Y_2 - 5M_1 - 4.5MD_1 - 3M_2 - 1,600,000\alpha_1 - 2,400,000\alpha_2 \\ & - 0.5Y_1 - Y_2 - 100B_{13} - 100B_{23} - 200B_{14} - 200B_{24} - 30,000Z_1 - 60,000Z_2 \\ & - 3,000,000\theta_0 - 4,500,000\theta_1 - 6,000,000\theta_2 \end{aligned}$$

Subject to—direct material

$$\begin{aligned} Y_1 + 2Y_2 - M_1 - MD_1 & \leq 0 \\ Y_1 + Y_2 - M_2 & \leq 0 \end{aligned}$$

$$M_1 \geq 0$$

$$M_1 < 4500,000ND_1$$

$$MD_1 \geq 450,000SD_1$$

$$MD_1 \leq 800,000SD_1$$

$$ND_1 + SD_1 = 1$$

$$M_2 \geq 0$$

$$M_2 \leq 600,000$$

Subject to—direct labour

$$Y_1 + 0.5Y_2 - 400,000\alpha_1 - 560,000\alpha_2 \leq 0$$

$$\alpha_0 - \eta_1 \leq 0$$

$$\alpha_1 - \eta_1 - \eta_2 \leq 0$$

$$\alpha_2 - \eta_2 \leq 0$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1$$

$$\eta_1 + \eta_2 = 1$$

Subject to—machine hour

$$\begin{aligned} 0.5Y_1 + Y_2 - 200,000\theta_0 - 240,000\theta_1 - 280,000\theta_2 & \leq 0 \\ \theta_0 + \theta_1 + \theta_2 & = 1 \end{aligned}$$

Subject to—batch-level activity (scheduling)

$$Y_1 - 120B_{13} \leq 0$$

$$Y_2 - 120B_{23} \leq 0$$

$$B_{13} + B_{23} \leq 4000$$

Subject to—batch-level activity (setup)

$$Y_1 - 100B_{14} \leq 0$$

$$Y_2 - 100B_{24} \leq 0$$

$$2B_{14} + 2B_{24} \leq 8000$$

Subject to—product-level activity (design)

$$Y_1 - 500,000Z_1 \leq 0$$

$$Y_2 - 300,000Z_2 \leq 0$$

$$100Z_1 + 200Z_2 \leq 600$$

**Enhanced TOC product-mix decision model**

Maximise

$$\begin{aligned} \pi = & 28Y_1 + 49Y_2 - 5M_1 - 4.5MD_1 - 3M_2 - 1,600,000 - 800,000\beta - 200,000\theta_0 \\ & - 240,000\theta_1 - 280,000\theta_2 - 400,000 - 800,000 - 180,000 - 3,000,000\theta_0 \\ & - 4,500,000\theta_1 - 6,000,000\theta_2 \end{aligned}$$

Subject to—direct material

$$Y_1 + 2Y_2 - M_1 - MD_1 \leq 0$$

$$Y_1 + Y_2 - M_2 \leq 0$$

$$M_1 \geq 0$$

$$M_1 < 4500,000ND_1$$

$$MD_1 \geq 450,000SD_1$$

$$MD_1 \leq 800,000SD_1$$

$$ND_1 + SD_1 = 1$$

$$M_2 \geq 0$$

$$M_2 \leq 600,000$$

Subject to—machine hour

$$\begin{aligned} 0.5Y_1 + Y_2 - 200,000\theta_0 - 240,000\theta_1 - 280,000\theta_2 & \leq 0 \\ \theta_0 + \theta_1 + \theta_2 & = 1 \end{aligned}$$

Subject to—batch-level activity (scheduling)

$$Y_1 - 120B_{13} \leq 0$$

$$Y_2 - 120B_{23} \leq 0$$

$$B_{13} + B_{23} \leq 4000$$

Subject to—batch-level activity (setup)

$$Y_1 - 100B_{14} \leq 0$$

$$Y_2 - 100B_{24} \leq 0$$

$$2B_{14} + 2B_{24} \leq 8000$$

Subject to—product-level activity (design)

$$Y_1 - 500,000Z_1 \leq 0$$

$$Y_2 - 300,000Z_2 \leq 0$$

$$100Z_1 + 200Z_2 \leq 600$$

Subject to—direct labour

$$Y_1 + 0.5Y_2 - 400,000 - 160,000\beta \leq 0$$

$$\beta \geq 0$$

(continued)

Table 2. Continued.

**Enhanced general product-mix decision model**

Maximise

$$\pi = 28Y_1 + 49Y_2 - 5M_1 - 4.5MD_1 - 3M_2 - 1,200,000 - 400,000\alpha_1 - 1,200,000\alpha_2 - 0.5Y_1 - Y_2 - 1,200,000 - 30,000Z_1 - 60,000Z_2 - 3,000,000\theta_0 - 4,500,000\theta_1 - 6,000,000\theta_2$$

Subject to—direct material

$$Y_1 + 2Y_2 - M_1 - MD_1 \leq 0$$

$$Y_1 + Y_2 - M_2 \leq 0$$

$$M_1 \geq 0$$

$$M_1 < 4500,000ND_1$$

$$MD_1 \geq 450,000SD_1$$

$$MD_1 \leq 800,000SD_1$$

$$ND_1 + SD_1 = 1$$

$$M_2 \geq 0$$

$$M_2 \leq 600,000$$

Subject to—direct labour

$$Y_1 + 0.5Y_2 - 300,000 - 100,000\alpha_1 - 260,000\alpha_2 \leq 0$$

$$\alpha_0 - \eta_1 \leq 0$$

$$\alpha_1 - \eta_1 - \eta_2 \leq 0$$

$$\alpha_2 - \eta_2 \leq 0$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1$$

$$\eta_1 + \eta_2 = 1$$

Subject to—machine hour

$$0.5Y_1 + Y_2 - 200,000\theta_0 - 240,000\theta_1 - 280,000\theta_2 \leq 0$$

$$\theta_0 + \theta_1 + \theta_2 = 1$$

Subject to—batch-level activity (scheduling)

$$Y_1 - 120B_{13} \leq 0$$

$$Y_2 - 120B_{23} \leq 0$$

$$B_{13} + B_{23} \leq 4000$$

Subject to—batch-level activity (setup)

$$Y_1 - 100B_{14} \leq 0$$

$$Y_2 - 100B_{24} \leq 0$$

$$2B_{14} + 2B_{24} \leq 8000$$

Subject to—product-level activity (design)

$$Y_1 - 500,000Z_1 \leq 0$$

$$Y_2 - 300,000Z_2 \leq 0$$

$$100Z_1 + 200Z_2 \leq 600$$

Table 4 shows the objective function, related constraints, and optimal solution after adding a new product into the product-mix. It is a 0–1 mixed-integer non-linear programming model and can be solved by the software ‘LINGO’.

In Table 4, the optimal production volumes for products A, B and C (new product) are 59,000, 500 and 250,000 units, respectively. The company achieves the maximal profit \$7,088,000. Under this solution, machine capacity is expanded to  $M_2 = 280,000$  hours, and neither discretionary labour hours nor overtime work are used in production.

*4.2.3 Part 3: simulating the impact on the company’s profit of reducing price of a product with different price elasticity of demand*

Suppose that the company finds that the profit of a new product exceeds that of an old product. In the short run, management needs to simulate and evaluate whether adopting lower pricing for a new product to expand market share is beneficial to the company’s profit or not.

Table 4 shows the company’s optimal profit is \$7,088,000 when the production volumes of products A, B, and C are 59,000, 500 and 250,000 units, respectively.

However, the above example does not consider price elasticity of demand ( $\epsilon^D$ ) in new product C. The following example will simulate the impact on the company’s profit of price change of a new product C with various  $\epsilon^D$ .

The determinations of elasticity can be either in the form of an arc or point elasticity. Arc elasticity is the elasticity of demand/supply between the distances of two points on

Table 3. A comparison between the optimal solutions of the three models.

	TOC	ABC	General-model
<b>Panel 1: product-mix</b>			
Product mix			
A	400,000	0	240,000
B	0	200,000	120,000
Resource used in production			
Direct material 1	400,000	400,000	480,000
Direct material 2	400,000	200,000	360,000
Non-discretionary direct labour hours	300,000	100,000	300,000
Discretionary direct labour hours	100,000	0	0
Direct machine hours	200,000	200,000	240,000
Production order	3334	1667	3000
Setup hours	8000	4000	7200
Engineering drawings	100	200	300
Unused resources			
Non-discretionary direct labour hours	0	200,000	0
Discretionary direct labour hours	0	100,000	100,000
Direct machine hours	0	0	0
Production order	666	2333	1,000
Setup hours	0	4000	800
Engineering drawings	500	400	300
Capacity expansion			
Machine capacity	0	0	40,000
Overtime work	0	0	0
<b>Panel 2: product-mix income</b>			
Revenue	11,200,000	9,800,000	12,600,000
Cost of resource used in production			
Direct material 1	2,000,000	2,000,000	2,160,000
Direct material 2	1,200,000	600,000	1,080,000
Non-discretionary direct labour hours	1,200,000	400,000	1,200,000
Discretionary direct labour hours	400,000	0	0
Direct machine hours	200,000	200,000	240,000
Production order	333,400	166,700	300,000
Setup hours	800,000	400,000	720,000
Engineering drawings	30,000	60,000	90,000
Facility-level cost	3,000,000	3,000,000	4,500,000
Income based on resources used	2,036,600	2,973,300	2,310,000
Cost of unused non-discretionary resource	66,600	1,433,300	180,000
Net income	1,970,000	1,540,000	2,130,000

a demand/supply curve. In this paper, the concept of arc elasticity is used to measure price elasticity of demand. The price elasticity of demand  $\varepsilon^D$  is defined as the following equation:

$$\begin{aligned}\varepsilon^D &= -\left(\frac{\Delta q / \frac{1}{2}(q' + q)}{\Delta p / \frac{1}{2}(p' + p)}\right) \\ &= -\left(\frac{\Delta q / \bar{q}}{\Delta p / \bar{p}}\right).\end{aligned}\tag{64}$$

Table 4. Decision analysis of the enhanced general product-mix decision model after adding a new product.

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Maximise

$$\pi = 28Y_1 + 49Y_2 + p_3q_3 - 5M_1 - 4.5MD_1 - 3M_2 - 1,200,000 - 400,000\alpha_1 - 1,200,000\alpha_2 - 0.5Y_1 - Y_2 - q_3 - 400,000 - 800,000 - 30,000Z_1 - 60,000Z_2 - 90,000Z_3 - 3,000,000\theta_0 - 4,500,000\theta_1 - 6,000,000\theta_2$$

<p>Subject to—direct material</p> $Y_1 + 2Y_2 + q_3 - M_1 - MD_1 \leq 0$ $Y_1 + Y_2 + 2q_3 - M_2 \leq 0$ $M_1 \geq 0$ $M_1 < 450,000ND_1$ $MD_1 \geq 450,000SD_1$ $MD_1 \leq 800,000SD_1$ $ND_1 + SD_1 = 1$ $M_2 \geq 0$ $M_2 \leq 600,000$ <p>Subject to—direct labour</p> $Y_1 + 0.5Y_2 + 0.5q_3 - 300,000 - 100,000\alpha_1 - 260,000\alpha_2 \leq 0$ $\alpha_0 - \eta_1 \leq 0$ $\alpha_1 - \eta_1 - \eta_2 \leq 0$ $\alpha_2 - \eta_2 \leq 0$ $\alpha_0 + \alpha_1 + \alpha_2 = 1$ $\eta_1 + \eta_2 = 1$ <p>Subject to—machine hour</p> $0.5Y_1 + Y_2 + q_3 - 200,000\theta_0 - 240,000\theta_1 - 280,000\theta_2 \leq 0$ $\theta_0 + \theta_1 + \theta_2 = 1$	<p>Subject to—batch-level activity (scheduling)</p> $Y_1 - 120B_{13} \leq 0$ $Y_2 - 120B_{23} \leq 0$ $Y_3 - 90B_{33} \leq 0$ $B_{13} + B_{23} + B_{33} \leq 4000$ <p>Subject to—batch-level activity (setup)</p> $Y_1 - 100B_{14} \leq 0$ $Y_2 - 100B_{24} \leq 0$ $Y_3 - 300B_{34} \leq 0$ $2B_{14} + 2B_{24} + 4B_{34} \leq 8000$ <p>Subject to—product-level activity (design)</p> $Y_1 - 500,000Z_1 \leq 0$ $Y_2 - 300,000Z_2 \leq 0$ $Y_3 - 250,000Z_3 \leq 0$ $100Z_1 + 200Z_2 + 300Z_3 \leq 600$ <p>Other constraint (1)—<math>p_3 = 100</math></p> <p>Other constraint (2)—<math>Y_1 \geq 500</math></p> $Y_2 \geq 500$
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Optimal solution is as follows:  $\pi = 7,088,000$ ,  $Y_1 = 59,000$ ,  $Y_2 = 500$ ,  $p_3 = 70$ ,  $q_3 = 250,000$ ,  $M_1 = 310,000$ ,  $MD_1 = 0$ ,  $M_2 = 559,500$ ,  $ND_1 = 1$ ,  $SD_1 = 0$ ,  $\alpha_0 = 1$ ,  $\alpha_1 = 0$ ,  $\alpha_2 = 0$ ,  $B_{13} = 492$ ,

In calculating the price elasticity, we use the average of the original and new price, i.e.,  $\bar{p}$ ; similarly for the quantity demanded, we use  $\bar{q}$ . An elasticity computed by this method is called an arc elasticity of demand (Pindyck and Rubinfeld 1998). Where  $p$  represents the selling price, and  $p'$  represents that of after change;  $\Delta p$  (i.e.,  $p' - p$ ) is a change in price;  $q$  represents the quantity demanded of new product before change, and  $q'$  represents that of after change;  $\Delta q$  (i.e.,  $q' - q$ ) is induced change in the quantity demanded.

This paper assumes that  $\varepsilon^D$  value of new product C is varied from 0.25 to 1000. Additionally, the company adopts a lower pricing strategy to stimulate demand, and the company's plant is able to supply the product as needed. Thus, the following equation is added into the product-mix decision model:

$$\varepsilon^D = - \left( \frac{\Delta q_3 / \frac{1}{2}(q'_3 + q_3)}{\Delta p_3 / \frac{1}{2}(p'_3 + p_3)} \right) = k, \quad k = 0.25, 1, 5.286, 20, 40, 1000. \tag{65}$$

Under a different  $k$  value, we could formulate the constraint of  $\varepsilon^D$ . Assume  $\varepsilon^D$  of product C has six different situations (see column (4) of Table 5), and reducing price of product C has eight possibilities (see columns (5) and (6) of Table 5). According to this information, we could obtain demand quantity information of product C while reducing the price at different  $\varepsilon^D$  from column (7) of Table 5.

For most goods, the lower the price the more customers will demand or purchase, because the demand curve is downward sloping. Furthermore, the greater  $\varepsilon^D$  value in the product the more customers will demand or purchase by a lower price. On columns (4) to (7) of Table 5, if new pricing ( $p'_3$ ) of product C is 99.98 (reducing price \$0.02), then demand quantity for product C will be 250,017 to 333,347 units in  $\varepsilon^D$  varied from 0.25 to 1000. On column (8) of Table 5, we can see that even the demand quantity of product C increases with the price of product C decrease, the maximal production volumes of product C is 279,250 under the capacity and production constraints listed in Table 4. In columns (8) to (10) of Table 5, this symbol (—) represents the demand quantity of product C exceeds the boundary of general integer of LINGO.

In general, the company's production policy will be based on the maximising profit rule instead of only considering customers' demand. However, management needs to simulate and evaluate whether adopting lower pricing for a new product is beneficial to the company's profit or not.

It is important for management to not only consider the impact of  $\varepsilon^D$  and reducing the price for new product C, but to also evaluate the impact on the company's aggregate profit of price change. The details about a new product's new price ( $p'_3$ ), price difference ( $\Delta p$ ), demand quantity ( $q''_3$ ), production quantity ( $q'_3$ ), new product mix ( $q'_3, Y'_1, Y'_2$ ), profit ( $\pi'$ ) and profit change percentage are shown in Table 5. Table 5 shows product C's initial selling price is 70 when the company produces 250,000 units of product C. Suppose that the company plans to increase product C's production volumes from 250,000 to at least 262,500 units ( $q'_3 \geq 262,500$ ), i.e., increasing 5% market share. In Table 5, left and right sides for  $\varepsilon^D$  column represent the information before and after a price change in product C, respectively. According to the earlier eighth assumption, this study implies that the company produces the old and new products based on aggregate market demand and related cost information of products. Column (8) of Table 5 represents product C's production quantity maximising company's profit after a price change. For example, when  $\varepsilon^D$  of product C is equal to 20 and the new pricing of product C is 69.10 (reducing the price by \$0.9), the customer will demand 324,318 units of product C. In this situation, the production quantity of products A, B, and C are 500, 500, and 279,250 units, respectively, and the company could increase profits compared to before the price change (increasing profit 4.29%).

On the contrary, observing  $\varepsilon^D = 1$  situation in Table 5, if the company produces 268,240 units of product C (selling price \$65.24), 22,520 units of product A ( $Y'_1 = 22,520$ ) and 500 units of product B ( $Y'_2 = 500$ ), then this company's profit is \$6,157,738, lower than the product-mix's profit ( $\pi = \$7,088,000$ ) before the price change. The same situation happens on  $\varepsilon^D = 0.25$  of Table 5. Suppose that the company plans to increase product C's production volume from 250,000 to at least 262,500 units ( $q'_3 \geq 262,500$ ), i.e., increasing 5% market share. In Table 5 for  $\varepsilon^D = 0.25$  for  $p'_3 = 52.22$  the values for  $q'_3, Y'_1, Y'_2$ , (the optimal product-mix) and  $\pi'$  are 268,870, 21,260, 500 and \$2,666,021, respectively, lower than the product-mix's profit ( $\pi = \$7,088,000$ ) before the price change. This situation indicates product C having  $\varepsilon^D = 0.25$  in this case is not appropriate for reducing price from profit view. If the current strategy of the company is to upgrade market share



Table 5. New product's new pricing ( $p'_3$ ), price difference ( $\Delta p$ ), demand quantity ( $q'_3$ ), production quantity ( $q_3$ ), new product mix ( $q_3$ ,  $Y_1$ ,  $Y_2$ ), profit ( $\pi'$ ) and profit change percentage ( $\Delta\pi/\pi$ ) after the price change of the new product with various  $\varepsilon^D$ .

Before price change		After price change											
(1)	(2)	(3)	(4)	(5)	(6) = (5) - (1)	(7)	(8)	(9)	(10)	(11)	(12)		
$p_3$	$q_3$	$\pi$	$\varepsilon^D$	$p'_3$	$\Delta p$	$q'_3$	$q'_3$	$Y'_1$	$Y'_2$	$\pi'$	$(\pi' - \pi)/\pi = \Delta\pi/\pi$		
70	250,000	\$7,088,000	0.25	69.98	-0.02	250,017	250,017	58,966	500	\$7,083,323	-0.07%		
				69.55	-0.45	250,403	250,403	58,194	500	\$6,982,976	-1.48%		
				69.10	-0.90	250,810	250,810	57,380	500	\$6,877,661	-2.97%		
				69.08	-0.92	250,828	250,828	57,344	500	\$6,872,970	-3.03%		
				68.79	-1.21	251,092	251,092	56,816	500	\$6,804,927	-3.99%		
				65.24	-4.76	254,438	254,438	50,124	500	\$5,961,197	-15.90%		
				60.70	-9.30	259,055	259,055	40,890	500	\$4,850,834	-31.56%		
				52.22	-17.78	268,870	268,870	21,260	500	\$2,666,021	-62.39%		
70	250,000	\$7,088,000	1	69.98	-0.02	250,071	250,071	58,858	500	\$7,084,348	-0.05%		
				69.55	-0.45	251,617	251,617	55,766	500	\$7,005,495	-1.16%		
				69.10	-0.90	253,256	253,256	52,488	500	\$6,921,934	-2.34%		
				69.08	-0.92	253,329	253,329	52,342	500	\$6,918,188	-2.40%		
				68.79	-1.21	254,397	254,397	50,206	500	\$6,863,723	-3.16%		
				65.24	-4.76	268,240	268,240	22,520	500	\$6,157,738	-13.12%		
				60.70	-9.30	288,303	279,250	500	500	\$5,046,725	-28.80%		
				52.22	-17.78	335,120	279,250	500	500	\$2,678,685	-62.21%		
70	250,000	\$7,088,000	5	69.98	-0.02	250,357	250,357	58,286	500	\$7,089,775	0.03%		
				69.55	-0.45	258,193	258,193	42,614	500	\$7,127,480	0.56%		
				69.10	-0.90	266,716	266,716	25,568	500	\$7,165,560	1.09%		
				69.08	-0.92	267,102	267,102	24,796	500	\$7,167,204	1.12%		
				68.79	-1.21	272,788	272,788	13,424	500	\$7,190,899	1.45%		
				65.24	-4.76	356,783	279,250	500	500	\$6,314,520	-10.91%		
				60.70	-9.30	526,128	279,250	500	500	\$5,046,725	-28.80%		
				52.22	-17.78	1,584,034	279,250	500	500	\$2,678,685	-62.21%		

70	250,000	\$7,088,000	20	69.98	-0.02	251,432	251,432	56,136	500	\$7,110,179	0.31%
				69.55	-0.45	284,469	279,250	500	500	\$7,518,088	6.07%
				69.10	-0.90	324,318	279,250	500	500	\$7,392,425	4.29%
				69.08	-0.92	326,234	279,250	500	500	\$7,386,840	4.22%
				68.79	-1.21	355,593	279,250	500	500	\$7,305,858	3.07%
				65.24	-4.76	1,438,811	279,250	500	500	\$6,314,520	-10.91%
				60.70	-9.30	—	—	—	—	—	—
				52.22	-17.78	—	—	—	—	—	—
70	250,000	\$7,088,000	40	69.98	-0.02	252,874	252,873	53,254	500	\$7,137,530	0.70%
				69.55	-0.45	324,043	279,250	500	500	\$7,518,088	6.07%
				69.10	-0.90	424,587	279,250	500	500	\$7,392,425	4.29%
				69.08	-0.92	429,898	279,250	500	500	\$7,386,840	4.22%
				68.79	-1.21	517,728	279,250	500	500	\$7,305,858	3.07%
				65.24	-4.76	—	—	—	—	—	—
				60.70	-9.30	—	—	—	—	—	—
				52.22	-17.78	—	—	—	—	—	—
70	250,000	\$7,088,000	1,000	69.98	-0.02	333,347	279,250	500	500	\$7,638,165	7.76%
				69.55	-0.45	—	—	—	—	—	—
				69.10	-0.90	—	—	—	—	—	—
				69.08	-0.92	—	—	—	—	—	—
				68.79	-1.21	—	—	—	—	—	—
				65.24	-4.76	—	—	—	—	—	—
				60.70	-9.30	—	—	—	—	—	—
				52.22	-17.78	—	—	—	—	—	—

Note: The symbol (—) represents the demand quantity of product C exceeds the boundary of general integer of LINGO.

at a goal volume (i.e., at least  $q_3' \geq 262,500$ ), then the company will decrease more profit to trade-off market share comparing to before price change (decreasing profit 62.39%). Note that for a price of  $p_3' = 69.98$  and  $p_3' = 52.22$  in  $\varepsilon^D = 0.25$ , the profits of both product-mix are lower than the profit before the price change. In this case, the company ought not to adopt reducing price strategy for product C having  $\varepsilon^D = 0.25$  to expand market share, because it will decrease the company's total profit.

The above situation illustrates that the company cannot arbitrarily adopt lower pricing strategy to expand market share because the company's profit growth rate ( $\Delta\pi/\pi$ ) is negative when  $\varepsilon^D$  value of a new product is lower than a specific value. This numerical example shows the importance of considering the price elasticity of demand. The more management realises the elasticity of a company's product, the more management adopts an appropriate price strategy. Some economics professors often use Starbucks as an example of a company whose product seems to have little price elasticity. Starbucks' management agrees with those professors and thinks the demand of Starbucks' product is inelastic, meaning that a price change will cause less of a change in quantity demanded. Maybe, that is one reason behind Starbucks' success, because Starbucks' management fully realises their products' characteristics. Generally, the price elasticity of demand is affected by factors such as those listed below:

- *Availability of substitutes*: the more possible substitutes, the greater the elasticity.
- *Degree of necessity or luxury*: luxury products tend to have greater elasticity. Some products that initially have a low degree of necessity are habit forming and can become 'necessities' to some consumers.
- *Proportion of the purchaser's budget consumed by the item*: products that consume a large portion of the purchaser's budget tend to have greater elasticity.
- *Time period considered*: elasticity tends to be greater over the long run because consumers have more time to adjust their behaviour.
- *Permanent or temporary price change*: a one-day sale will elicit a different response than a permanent price decrease.

According to available information, the company can examine price elasticity of related products, and develop an appropriate pricing strategy for a particular product to increase its market share and profit.

## 5. Concluding remarks

This paper developed an enhanced general model that incorporates all four factors: capacity constraint, management's degree of control over resource, capacity expansions, and purchase discount to determine the optimal product-mix.

Establishing a general model related to ABC product-mix decisions is still limited in the current ABC literature. This paper tried to develop a more realistic product-mix decision model under ABC and discussed the related-analysis for adding a new product into the present product-mix. This paper integrated factors proposed in related works and considered an important feature in practice: price elasticity of demand and the capacity expansion. This paper also provided a comprehensive illustration on how a company uses the mathematical programming approach to obtain the optimal product-mix and the capacity expansion to achieve profit goals. Additionally, this paper used a numerical example to simulate the impact on the company's profit for reducing the price of a product with different  $\varepsilon^D$ .

This paper was based on some specific assumptions. For example, this paper assumed that potential competitors' prices are not sensitive to temporary price changes of a company's product in the short term. In future studies, researchers can relax assumptions to explore more complicated and realistic situations.

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### Note

1. For example, when  $\varepsilon^D = 1$ , we then have the following equation by using Equation (65):

$$\begin{aligned} & (q'_3 - q_3)(p_3 + p'_3) = (q_3 + q'_3)(p_3 - p'_3) \\ \rightarrow & (q'_3 p_3 + q'_3 p'_3 - q_3 p_3 - q_3 p'_3) = (q_3 p_3 - q_3 p'_3 + q'_3 p_3 - q'_3 p'_3) \\ \rightarrow & 2q'_3 p'_3 = 2q_3 p_3 \\ \rightarrow & q'_3 p'_3 = q_3 p_3. \end{aligned}$$

Assuming that  $p_3 = 70$ ,  $q_3 = 250,000$ , then  $q'_3 p'_3 = 17,500,000$  (the constraint when  $\varepsilon^D = 1$ ).

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