Trader Anonymity and Market Characteristics

Abstract

We study the impact of trader anonymity on trading behavior and price characteristics. Revealing the identity of informed traders allows the market to better disaggregate the source of orders, but does not guarantee more informative prices. When markets are less anonymous, informed traders protect their information by adopting “bluffing” strategies, e.g., buying overvalued assets. This behavior decreases the price impact of trading, even to the point where informed traders may in fact prefer to trade in less anonymous markets. We extend our analysis to consider implications for price efficiency, information production, and the effects of anonymizing events on market stability.
1 Introduction

Traders have traditionally had little ability to influence the anonymity of their trades. However, this is no longer the case. An expanding landscape of venues now offers traders considerable heterogeneity in the amount of anonymity they are afforded, from platforms offering complete anonymity such as INET or Euronext to ones offering none, such as the Hong Kong or Australian Stock Exchanges.\footnote{In the latter two markets, broker identities are nearly always disclosed when the trade is initiated.} Characteristics of traders themselves also play roles in how much the market can infer about their identities. Many informed traders are likely to be hedge funds or other relatively unregulated entities that not only escape disclosure requirements, but also have significant flexibility to disguise their identity with anonymizing strategies, e.g., breaking up large orders or routing through multiple brokers. Although these observations suggest that anonymity likely plays an important role in trading, research has had little to say about its impact on market characteristics.

This paper is an attempt to improve our theoretical understanding of this issue. We develop a simple model where an informed trader faces a non-zero probability of having his trades revealed to the market. For example, consider a hedge fund, investment bank, or other relatively informed trader who wishes to unwind a position by selling securities or to establish a position by buying securities. In our model there is some chance that the trades will be executed anonymously and some chance that the trades will be identified as coming from the informed party.\footnote{In either case the market is aware that an informed trader may be participating in the market and prices adjust to order flow accordingly.} When the probability of revelation is low (high), the market is said to be more (less) anonymous. We characterize the optimal strategies of an informed trader, solve for the expected trading profits, and describe the resulting price dynamics under different levels of trader anonymity. Our analysis yields some interesting results.

First, we show that more anonymous markets are not necessarily more liquid, as defined by...
the price sensitivity of order flow (Kyle (1985)). The intuition for this result is as follows. All else equal, reducing anonymity decreases liquidity, simply because informed trades are more likely to be revealed as such, producing more extreme price adjustments. However, there is an indirect effect stemming from an informed trader’s response to an increase in the chance that his trades are made public. The informed trader may use the increased visibility to his advantage, attempting to confuse the market maker by “bluffing” and trading against his information. This occurs when: 1) the stock is not badly mispriced (because trading against one’s information is costly in proportion to the mispricing), and 2) when there is a high enough chance that the bluff is revealed to the market maker (otherwise the bluff is useless). Because an informed trader’s demand may not reflect the nature of her information, the market price becomes less responsive to order flow, i.e., liquidity is increased. On balance, decrease anonymity may increase or decrease market liquidity.

Our second main result examines the profitability of informed trade strategies. The liquidity boost associated with bluffing strategies results in higher expected profits for the informed trader, but not directly - bluffing itself does not increase the trader’s expected profits. Rather, the possibility that bluffing may occur increases liquidity which increases the profitability of informed trade when no bluffing is done. To better understand this, consider how bluffing is beneficial in poker. When a player has poor cards in a given hand, she may bet more aggressively than her cards warrant. Although she will almost certainly sustain a loss if her bluff is called during that round, her opponents learn that aggressive betting is not always backed up by good cards. Thus, her opponents are more likely to match her future aggressive betting, increasing her gains substantially when she has the cards to justify the aggressive bet, i.e., the potential of a bluff, rather than the bluff itself, generates the abnormal profits. Likewise in a market where bluffing is anticipated, the market is deeper and price is less sensitive to order flow. The benefits to the informed trader of increased market depth are realized at times when the informed trader does not bluff (though an informed trader with different information might).
A third class of closely related implications speaks to the impact of trader anonymity on market stability and efficiency. Reducing anonymity creates incentives for informed traders to adopt destabilizing strategies that move prices away from fundamentals. To the extent that trader anonymity is determined by the relative proportions of informed and uninformed traders in the market (as in our model), our findings imply that this effect may be self-reinforcing. Previous research has shown that when liquidity traders have discretion over the timing or location of their trades, they will tend to avoid trading where or when liquidity is low or there exists a high proportion of informed traders.\(^3\) Therefore, factors that increase informed trader profits will increase liquidity trader losses and tend to drive liquidity traders toward alternative trading venues. This reasoning suggests that if less anonymity increases the expected profits of informed traders, the long-term stability of less anonymous markets is called into question.

Finally, our analysis speaks to the relation between anonymity protection, information production, and price efficiency. We show that an informed trader is more likely to bluff when anonymity protection is low and when prices more accurately reflect fundamentals, i.e., the informational gap between the informed trader and market maker is narrow. This suggests that the relationship between information production and price efficiency isn’t necessarily straightforward. When more information is produced (due, for example, to an increased number of analysts following a stock), prices will tend to be closer to fundamental value, all else equal. However, because prices are closer to fundamental value all else is not equal: informed traders have a greater incentive to trade against their information and push prices away from fundamental value. Since bluffing is more likely in earlier periods when losses (or foregone gains) can be recovered, early information production may actually decrease \textit{average} price efficiency, i.e., over the course of the entire trading game. This temporary reduction in price efficiency is magnified when trader anonymity is poorly protected.

Our model is based on Kyle (1985), but differs in important ways that allow us to study varying degrees of trader anonymity. The most important departure is the introduction of a parameter ("informational trade transparency") that captures the chance that the informed trader's behavior is detected by the market maker. A single informed trader is endowed with a binary (e.g., bullish or bearish) signal of the liquidation value of a risky asset. The informed trader is known to exist, but her information is private. There are three rounds of trade and in each round the informed trader can submit an order to buy or sell a single share (or not trade). The market also includes a cohort of liquidity traders who trade for exogenous reasons and a risk-neutral competitive market maker. The aggregate liquidity trade in each round is independent and drawn from a random distribution. The competitive market marker sets the market price in each trade round equal to the expected liquidation value of the asset, conditional on the observed aggregate order flow (and all previous trades). The model is solved by backward induction.

Our paper contributes to a small but growing literature on trader anonymity in financial markets. For example, Foucault, Moinas, and Theissen (2007) investigate how anonymity influences information content of prices about future volatility, and Simaan, Weaver, and Whitcomb (2003) and Benveniste, Marcus, and Wilhelm (1992) explore how anonymity can reduce collusive equilibria among dealers. To our knowledge, ours is the first paper to explore how varying degrees of anonymity protection influence optimal trading behavior, price dynamics, and incentives to collect private information. In addition, part of our analysis (particularly with regard to optimal trading strategies) overlaps with a larger literature on

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4 Our assumption of a single informed trader is important, but need not be interpreted literally. Even if there exists multiple informed traders, each is likely to possess some degree of unique information. In this sense, the model may be viewed as studying the marginal component of an informed trader’s order, that which is orthogonal to the information-based trade of other informed traders. Section 3 discusses the impact of multiple informed traders in more detail. Holden and Subrahmanyam (1992) study a multi-period Kyle (1985) model with multiple identically informed insiders. Foster and Viswanathan (1996) study a multi-period Kyle model with multiple differently informed insiders. Dridi and Germain (2004) study a one period Kyle-type model with multiple identically informed insiders with binary signals.

5 Our assumption of net unit demand is innocuous because we allow the informed trader to adopt mixed strategies.
“trade-based” manipulation (Allen and Gale (1992)). Back and Baruch (2004) show that bluffing can arise in both Kyle (1985) and Glosten and Milgrom (1985) type settings, implying that our particular assumptions are not crucial for appreciating the results directly related to bluffing. Huddart, Hughes, and Levine (2001) show that insiders have an incentive to “dissimulate” their orders following disclosure, with an intent similar to bluffing of reducing the link between order flow and information. Chakraborty and Yilmaz also study trade-based manipulation incentives in a Kyle-type (2004a) and Glosten-Milgrom (2004b) setting with finite discrete order flow and liquidation value. Our study not only provides a theoretical link between this literature and trader anonymity, but also extends the analysis to consider a broad range of market characteristics including efficiency, market stability, and information production.

2 The Model

2.1 Economic Environment

This setting is similar to Kyle (1985), but with different distributional assumptions that suit our purpose of studying markets differing in anonymity. A single risky asset is traded in a market among three types of agents: a single risk-neutral informed trader, a competitive market maker, and noise traders. There are three successive rounds of trade. The asset pays a single cash flow $\tilde{v}$ after the final round of trade. Prior to trade $E[\tilde{v}] = p_0$. For simplicity, the discount rate between successive trade rounds is assumed to be zero. Prior to the market opening for trade an informed trader receives a binary signal $s \in \{l, h\}$ that is perfectly correlated with the asset payoff. Without loss of generality, we set $E[\tilde{v}|l] = 0$ and $E[\tilde{v}|h] = 0$.

6Other relevant papers include Fishman and Hagerty (1995), who study trade-based manipulation from uninformed traders and John and Narayanan (1997), who shows that even a known informed trader may choose to manipulate the market.
$E[\bar{e}|h] = 1$.

In each round of trade the informed trader can buy one share, sell one share, or not trade (i.e., sit out of the market). The informed trader’s order flow in round $n$ is denoted $x_n$. Therefore, $x_n \in \{-1, 0, +1\}$ for $n = 1, 2, 3$. $x_n$ may be the outcome of a mixed trading strategy. We denote the informed trader’s trading strategy in round $n$ as $X_n(s; p_{n-1})$. The per trade round order flow from noise traders, denoted $u_n$, is i.i.d. discrete uniform $[-w, +w]$.

The parameter $w$ captures the anonymity of the market - when $w$ is large (small), the informed trader has a smaller (larger) chance of being revealed as informed. It is only for modeling convenience that the informed trader’s trade itself is responsible for revealing her identity. Although this may be the case for large, informed traders whose trades themselves may provide market makers with information about their identities, all that is needed is a technology that allows for the informed trader’s actions to be observed with positive probability. A competitive market maker observes the aggregate order flow in each round and sets price equal to the expected value of the asset. The aggregate order flow is denoted $z_n = x_n + u_n$ for $n = 1, 2, 3$ and the market price set by the market maker in each round is denoted $p_n$. We denote the market maker’s pricing function in round $n$ as $P_n(z_n; p_{n-1})$.

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7 All the results presented hold for $w > 3$. Some results need to be modified for $w \leq 3$. We don’t present detailed results for $w \leq 3$.

8 Overall, our assumptions equate to a discretization of the model. With a discrete-space model we can explore and solve for all equilibria. The discretization is the key departure; the specific discrete distributions chosen are less consequential. The model would be more tedious to solve, for example, if the informed trader were permitted to submit orders ranging from $-k$ to $+k$ shares, but the qualitative nature of the results would remain. Similarly, if the informed trader’s information were, e.g., binomial rather than binary, the qualitative nature of our results would not change. We continue our discussion of the implications of our distributional assumptions in Section 3.

9 $w$ also controls the level of noise trade in the market and therefore has a direct influence on market liquidity. Where appropriate we “liquidity adjust” our results to isolate the impact of changing trader anonymity and emphasize the role of $w$ as a measure of trader anonymity.

10 The informed trading strategy and pricing function are more properly denoted as $X_n(s, p_{n-1}, \ldots, p_0)$ and $P_n(z_n, \ldots, z_1, p_0)$. However, market efficiency dictates that prices follow a martingale which justifies the notation used in the text.
2.2 Definition of Equilibrium

An equilibrium for the model comprises an informed trader trade strategy, $X = (X_1, X_2, X_3)$, and a market maker pricing function, $P = (P_1, P_2, P_3)$ such that the informed trader maximizes her expected future profits:

$$\sum_{m=n}^{3} E[(\tilde{v} - p_m)x_m(X, P)|s, p_0, \ldots, p_{m-1}] \geq \sum_{m=n}^{3} E[(\tilde{v} - p_m)x_m(X^*, P)|s, p_0, \ldots, p_{m-1}] \quad \forall \ X^* \neq X \text{ and } n = 1, 2, 3$$

and price equals the expected future asset payoff conditional on the observed order flow:

$$p_n = E[\tilde{v}|p_0, z_1, \ldots, z_{n-1}] \quad \text{for } n = 1, 2, 3.$$

2.3 Optimal Strategies

The model is solved by backward induction. For exposition, the equilibrium is presented and discussed from the perspective of an informed trader with a high signal ($s = h$). Given this perspective, when the informed trader buys a share she is trading with her information and when an informed trader sells a share she is trading against her information. The following proposition presents an equilibrium to the 3-period model, details and a proof of which is contained in the appendix.\(^{11}\)

**Proposition 1** *In the first round of trade, the informed trader’s trading strategy $X_1(h; p_0)$*  

\(^{11}\)We prove existence, but not uniqueness, of the equilibrium. There exist, at least, additional equilibria that differ from the presented equilibrium in ways that are economically insignificant. For example, if the informed trader’s information is fully reflected in market price prior to the last round of trade (as can happen), in later rounds the informed trader is indifferent between all feasible trading strategies as each and every one has exactly zero expected profits.
is a mixed strategy that depends on the initial price of the risky asset $p_0$. Specifically,

$$X_1(h; p_0) = \begin{cases} 
-1 \text{ w.p. } \phi_1^h(p_0) \\
0 \text{ w.p. } \theta_1^k(p_0) \\
+1 \text{ w.p. } 1 - \phi_1^h(p_0) - \theta_1^k(p_0),
\end{cases}$$

where $\phi_1^h(p_0)$ and $\theta_1^k(p_0)$ are non-decreasing functions. There exists a non-empty set of prices $p^{C3} < p_0 \leq 1$ for which $\phi_1^h(p_0) > 0$, i.e., for some prices the informed trader trades against her information with strictly positive probability. There exists a larger set of prices $p^{C1} < p_0 \leq 1$ where $p^{C1} < p^{C3}$ for which $\theta_1^k(p_0) > 0$, i.e., the informed trader does not trade with some probability during the first round. In the second round of trade, the informed trader’s trading strategy $X_2(h; p_1)$ is a mixed strategy that depends on the first period price of the risky asset $p_1$. Specifically,

$$X_2(h; p_1) = \begin{cases} 
0 \text{ w.p. } \theta_2^h(p_1) \\
+1 \text{ w.p. } 1 - \theta_2^h(p_1),
\end{cases}$$

where $\theta_2^h(p_1)$ is a non-decreasing function. There exists a non-empty set of prices for which the informed trader will sit out during the second round of trading. In the third and final round of trade, the informed trader’s trading strategy is the following pure strategy:

$$X_3(h; p_2) = 1$$

In all trading rounds $n = 1, 2, 3$, the market maker sets prices equal to the expected liquidation
value of the asset, given the insider’s trading strategy and total order flow, i.e.,

\[
P_n(z_n; p_{n-1}) = \begin{cases} 
\phi^h_n(p_{n-1}) + \phi^h(p_{n-1})p_{n-1} & \text{for } z_n = -w - 1 \\
\frac{\phi^h_n(p_{n-1}) + \phi^h(p_{n-1})}{(1-p_n-1)} & \text{for } z_n = -w \\
\frac{1 - \phi^h_n(p_{n-1})p_{n-1} + \phi^h_n(p_{n-1})(1-p_n-1)}{(1-p_n-1)} & \text{for } -w + 1 \leq z_n \leq w - 1 \\
\frac{1 - \phi^h_n(p_{n-1})p_{n-1} + \phi^h_n(p_{n-1})(1-p_n-1)}{(1-p_n-1)} & \text{for } z_n = w \\
\frac{1 - \phi^h_n(p_{n-1})p_{n-1} + \phi^h_n(p_{n-1})(1-p_n-1)}{(1-p_n-1)} & \text{else.}
\end{cases}
\]

Hagerty

3 Discussion

3.1 Optimal Strategies

Proposition 1 characterizes the optimal strategies of the informed trader in each of the three trading rounds. In the final period, it is trivial that she always trades with her information. This is no longer the case in the second-to-last round where, when the price gets sufficiently high, the informed trader begins to mix between trading with her information and not trading on her information. Sitting out the market is at least partially an artifact of having a discrete order size: there are prices for which the informed trader would prefer to trade a small fraction of a share rather than none at all. Nevertheless, the following intuition is very useful in understanding if, when, and why an informed trader will not always trade with her information.

The informed trader’s opportunity cost of not trading is proportional to the difference between the asset’s value and the price, \(1 - p_0\). Offsetting this is the opportunity benefit of not (on average) moving prices toward fundamental value. While the marginal cost clearly
decreases in price, the marginal benefit is relatively constant in price.\textsuperscript{12} When prices are far from fundamentals therefore, it is never worthwhile to sit out. When prices are close to fundamentals, the marginal benefit of sitting out can be made equal to the marginal cost by choosing the appropriate mixing probabilities of each strategy.

In the first period, the same logic applies, except that the informed trader’s motivation to influence future prices is stronger. For prices very close to fundamentals, in addition to sitting out with some probability, the informed trader plays a mixed strategy involving trading against her information. The \textit{combined} probability of not trading on one’s information, or trading against one’s information, increases in price. This combined probability, \((\theta + \phi)\), is shown in Figure 1. Notice that these probabilities decrease in \(w\), the inverse of the market’s anonymity.\textsuperscript{13}

Intuitively, one expects informed traders to prefer markets with high anonymity where traders can transact undetected and earn large profits. In such markets manipulation is also less likely. The benefit of trading against one’s information is in moving prices away from fundamentals to increase future expected profits. In markets with high trader anonymity (and low informational trade transparency generally) the likelihood of an informed trader’s order moving prices is lessened, so the incentives to manipulate are reduced. This raises an interesting tension.

On the one hand, higher informational trade transparency increases market efficiency via a

\textsuperscript{12}See the appendix for a formal proof of this claim.

\textsuperscript{13}As previously noted, \(w\) also determines the amount of noise in the market during each trade round. Because our interest is in the direct role of anonymity, we normalize our main results to negate the direct effects of \(w\) on liquidity. More generally, because increasing \(w\) raises liquidity and increases anonymity, one can interpret \(w\) as parameterizing the market’s informational trade transparency. A market with high informational trade transparency is able to extract more information from the order flow. High liquidity and strong trader anonymity both inhibit the informational trade transparency of a market and lessen the information content of order flow. Ambiguous motives for trade (i.e., where a given trader’s motives may be information-based or liquidity-based) also lessen informational trade transparency. Ambiguous motives for trade drive the results in Allen and Gale (1992), Fishman and Hagerty (1995), and Chakraborty and Yilmaz (2004a,b).
more direct link between informed order flow and market price adjustments. On the other hand, higher informational trade transparency increases the incentives for informed traders to manipulate prices by trading against their information such that the “informed” order flow becomes less informative. This point has been discussed by Fishman and Hagerty (1995) as it pertains to mandatory disclosure laws. We demonstrate that this is a general consideration that pertains to any aspect of the market mechanism that impacts informational trade transparency.

### 3.2 Informed Trader Profits

We now present a corollary to Proposition 1 that quantifies the expected profits of an informed trader with a high signal of the risky asset’s value. Expected profits conditional on a low signal are symmetric.

**Corollary 1** The informed trader’s expected trading profits for the 3-period game are a
decreasing, piecewise continuous, and linear function in price. Below are the expected profits for an insider receiving the high signal \((s = h)\) prior to the first period’s trading activity.

\[
E(\pi|p_0) = \begin{cases} 
    \pi_{\text{base}} = \frac{(1-p_0)(2w-1)(1+12w^2)}{(1+2w)^3}, & \text{if } 1 \geq p_0 \geq 1 - p_0^{C_4} \\
    \pi_a = \pi_{\text{base}} + \frac{(1+12w^2)(8w^2-p_0(-1+2w)(1+2w)^2)}{16w^2(1+2w)^3}, & \text{if } 1 - p_0^{C_4} \geq p_0 \geq 1 - p_0^{C_3} \\
    \pi_b = \pi_a + \frac{(1+12w^2)(8w^2-p_0(1+2w)^3)}{16w^2(1+2w)^3}, & \text{if } 1 - p_0^{C_3} \geq p_0 \geq \frac{1}{1+2w} \\
    \pi_c = \pi_b + \frac{4w(-1+2w)(1-p_0-2p_0w)}{(1+2w)^3}, & \text{if } \frac{1}{1+2w} \geq p_0 \geq 1 - p_0^{C_2} \\
    \pi_d = \pi_c + \frac{(1+12w^2)(4w(-1-4w+4w^2)-p_0(1+2w)^2(1+12w^2))}{4w(1+2w)^3(-1-4w+12w^2)}, & \text{if } 1 - p_0^{C_2} \geq p_0 \geq 1 - p_0^{C_1} \\
    \pi_e = \pi_d + \frac{(1+12w^2)(8w^2(-1-8w+4w^2)-p_0(1+2w)^2(1+12w^2))}{(1+2w)^3(1+12w+16w^2-112w^3+48w^4)}, & \text{if } 1 - p_0^{C_1} \geq p_0 \geq 0.
\end{cases}
\]

The expressions for the price region boundaries \(p_0^{C_1}, p_0^{C_2}, p_0^{C_3}, \text{ and } p_0^{C_4}\) are given in the appendix.

Figure 2 plots the expected 3-period profits of the informed trader as a function of pre-trade price, \(p_0\), for various values of \(w\). Larger values of \(w\) correspond to a market that is more liquid with a lower degree of informational trade transparency. Because liquidity increases with \(w\), profits also increase with \(w\). More interesting is the shape of the expected
profit curves for each $w$. The curves are drawn for an informed trader with a high signal. Expected profits decrease in $p_0$ as expected: informed trader profits are lower on average when she has a smaller informational advantage. What is striking is the region in which expected profits are elevated (relative to a non-manipulation linear benchmark). Recall that an informed trader with a high signal may bluff when price is close to fundamental value, but does not do so when price is far from fundamental value. In contrast, Figure 2 shows that the informed trader earns excess profits when price is far from fundamental value, and not when price is close to fundamental value. That is, in price regions where the informed trader bluffs, her expected profits simply match those she would earn from not bluffing and always trading with her information. While in price regions where the informed trader exclusively trades with her information, she earns excess expected profits. This means that the high-type informed trader earns excess profits in the price region where bluffing would occur if a low-type informed trader were in the market and the low-type informed trader earns excess profits in the price region where bluffing would occur if a high-type trader were in the market.

Consider the case when price is close to zero. A price close to zero indicates that the market maker believes there is a relatively high probability that a low-type informed trader is in the market. The market maker also recognizes that when prices are close to zero, and when trader anonymity is poorly protected, a low-type informed trader may trade against her information and submit a buy order. Therefore, if the market maker infers that an informed trader submitted a buy order, the market maker updates his beliefs based on the relative likelihood that the order came from a low-type informed trader trading against her information versus from a high-type informed trader trading with her information. Because the market maker has a high prior that the informed trader has a low signal, the market maker is reluctant to raise price too much even when he is certain that the informed trader submitted a buy order.
This is an ideal situation for a high-type informed trader. Like a card player who has bluff ed in the past when her cards were poor but now has a good hand, she can trade with her information and not cause the price to move too far toward fundamental value even when the market maker perfectly infers the informed trader order flow. Therefore, the expected profits for a high-type informed trader are elevated due to the likelihood that a low-type informed trader may be bluffing the market.

To summarize: (i) the direct effect of bluffing on informed trader profits is simply to break even, (ii) the benefits of bluffing accrue to informed traders who don’t bluff, and (iii) the effects of bluffing are most pronounced in less anonymous markets or markets generally characterized by a high degree of informational trade transparency. Thus, the impact of anonymity on the informed trader’s expected profits are indirect. The potential of bluffing changes the market dynamics to the favor of the informed trader. Specifically, the possibility of bluffing increases market liquidity by making prices more sticky and less responsive to order flow. An informed trader creates (but doesn’t profit from) the price stickiness by bluffing when she has a small informational advantage. An informed trader earns excess profits from the increased liquidity by trading with her information when her informational
advantage is large.

This lends support to Allen and Gale’s (1992) claim that trade-based manipulation is difficult to detect and eradicate. Our model suggests that, in general, an informed trader will not earn excess profits and engage in manipulation concurrently. Trade reversals occur when an informed trader has a small informational advantage and could credibly claim to have ‘changed their mind’ about the asset value. Excess profits occur when an informed trader trades consistently in one direction. If both trade reversals and excess profits are needed to prove manipulation, proof will be difficult.\textsuperscript{14} Perhaps more importantly, our model identifies the types of markets most likely to foster such trade-based manipulation (i.e., those market settings where trader anonymity is poorly protected) as well as those most likely to adopt bluffing strategies (i.e., those with small information advantages).

Figure 3 shows expected profits of the informed trader on a liquidity-adjusted basis for different values of $w$. It is perhaps counterintuitive that, all else equal, informed traders profit more, and would prefer to trade, in a less anonymous market. This is because one generally expects informational trade transparency and market liquidity to be inversely related (as they are through the joint effect of our $w$ parameter): orders are easier to disaggregate and likely to be less anonymous in markets with low liquidity. But even in such a case the profit curves in Figure 2 indicate that when an informed trader has a large informational advantage she may be willing to sacrifice market depth to gain higher informational trade transparency so long as the market maker believes that the likelihood that an informed trader might bluff is sufficiently high.\textsuperscript{15} This is seen, for example, by noticing that the bluffing based profit curve for $w = 4$ would exceed a non-bluffing based (i.e., linear) profit curve for $w = 6$ for prices near 0. In any case, our model suggests that there are circumstances in which an

\textsuperscript{14}Note, our focus is on understanding the feasibility, dynamics, and profitability of trade-based manipulation. While our work may be relevant to legal and policy discussions regarding trade-based manipulation, we explicitly are not making any arguments or claims about whether trade-based manipulation is or should be legal or illegal.

\textsuperscript{15}Of course the market maker’s beliefs can be rational if the informed trader is not expected to have so large an information advantage as she actually does.
informed trader would wish to make her actions more transparent by “leaking” her trading activity, not breaking up a large order into multiple smaller orders, and the like.

3.3 Market Liquidity and Price Efficiency

Figure 4 shows the pre-trade expectation of the post-trade residual variance of the asset’s liquidation value (i.e., after the final round of trading but before the liquidation value of the asset is announced). The figure is drawn conditional on the informed trader having received a high signal, which creates an asymmetric residual variance profile. Absent bluffing the informed trader trades in the direction of her information each period and the residual variance plots would be parabolas. In our setting there is a constant probability, $\frac{2}{(2w+1)}$, in each round that the informed trader’s information will be revealed, independent of initial price. The parabolic profiles therefore simply represent a constant scaling of the initial price variance, which is parabolic owing to the binomial distribution of the informed signal. For very large $w$ it is unlikely that the market maker will perfectly infer the informed trader’s information prior to the final trade date and the post-trade residual variance is very close to the ex-ante uncertainty. As $w$ decreases there is an increasing probability that the informed trader’s information will be revealed and residual uncertainty profiles are scaled appropriately.

Reducing trader anonymity gives rise to bluffing, which changes the residual variance profile in a very significant way. For prices near zero the residual variance plots are not parabolic and, in fact, it is expected that the uncertainty regarding the liquidation value will increase over the three rounds of trade. Figure 5 shows a close-up of this price region. This region

\[^{16}\text{Unconditionally, the figure would be symmetric around } p = 0.5.\]

\[^{17}\text{The scalloped shape of the price efficiency curves arises from the discrete changes in market maker beliefs represented by the different price regions in Proposition 1. Within each region there is uncertainty about whether a price change will occur by exiting the region to the right and raising the price, or exiting the region to the left and lowering the price. The uncertainty about the direction of the next price update is greatest in the middle of each region, which produces the scalloping.}\]
reflects the change in market dynamics attributable to the bluffing strategy.

Specifically, an informed trader with a high signal can expect, when price is far from fundamental value, to trade in a more liquid market owing to the effect of bluffing. The price is less responsive to order flow because the market maker is uncertain whether to attribute an informed buy order to bluffing by a low-type informed trader or to profitable trade by a high-type informed trader. In this situation, when price is close to 0, a buy order is very rare: a low-type informed trader is likely to exist, but she only trades against her information with low probability. A high-type informed trader always trades with her information, but her very existence is rare when price is close to zero. Absent bluffing, price responsiveness could be quite extreme. In particular, an inferred buy order from the informed trader would move price all the way to 1, no matter how close to zero the previous price had been. When markets are less anonymous, the informed trader’s incentive to adopt strategies intended to confuse the market maker ensures that this doesn’t happen.

Also note that the expected increase in residual uncertainty over the trading horizon is most
pronounced for low values of $w$, when trader anonymity is poorly protected - so much so that the effects of bluffing outweigh the effects of increasing liquidity in $w$. Thus there are significant price regions for which markets with higher levels of noise trade (bigger $w$) are expected to be more informationally efficient. We provide a new rationale for this result. Naively, one might expect that increasing levels of noise trade would make prices less efficient. Grossman and Stiglitz (1980) argued, on the contrary, that if information production is costly, then prices can become more efficient when noise trade increases because it allows more profitable trading opportunities for informed traders and thereby stimulates information production. Kyle (1985) showed that even absent costly information production, increasing levels of noise trade needn’t impact price efficiency because the intensity of informed trade may increase proportionally.

We show that even in the absence of costly information production, increasing noise trade may increase price efficiency by diminishing the incentives for bluffing. This, again, is why an informed trader may actually prefer to trade in a less liquid market versus a more liquid market, provided concerns about bluffing are larger in the less liquid market. The
implications of Figure 5 thus suggests a reassessment of the claim that anonymous markets ultimately improve price efficiency. For example, in early 2004, the Sydney Futures Exchange (SFE) announced that all broker identifiers (pre-trade mnemonics) be removed, preventing traders from identifying their counterparties in the electronic limit order book. Two of the SFE’s stated reasons for this regulatory change were to: 1) reduce the risk of “price slippage” of large orders and, 2) “facilitate efficient price discovery.” Our analysis indicates that such a conclusion may be premature. Although anonymizing markets will increase efficiency and liquidity if *trader strategies are held constant*, an equilibrium analysis suggests that this may not be the case.

### 3.4 Additional Considerations

#### 3.4.1 Information Production

The informed trader in our model is endowed with her information. Here we discuss the interplay between anonymity, bluffing, price efficiency, and costly information production. Grossman and Stiglitz (1980) show that if information production is costly, markets must be sufficiently ‘noisy’ for traders who invest in information to profitably trade on their information. If the market is not ‘noisy,’ price is a sufficient statistic for private information and uninformed free-riding undermines the incentive to collect costly private information. Market noise is often assumed to come from liquidity-based demand or other supply shocks. Our paper shows that an informed trader can also generate market noise endogenously via bluffing. All else equal, reducing trader anonymity increases the expected profits of informed traders (through bluffing) and should therefore lead to more information production. Additional information production will offset the negative price efficiency effects of bluffing. Therefore, in a setting with costly information production it is not clear whether the net effect of increasing anonymity on expected price efficiency will be positive or negative.
Also recall that the excess profits due to bluffing are convex in the magnitude of the informed trader’s informational advantage, as shown in Figure 2. This has several potentially interesting implications. First, this may create increasing returns to scale for information production. Second, if different methods of producing information have different risks with respect to the amount of information produced, the convexity of the expected profits creates a bias toward risk-taking in information production. Last, because there is a higher marginal benefit to generating a lot versus a little information, but because bluffing occurs when an informed trader has a little versus a lot of information, it is possible that a model with endogenous information production may have multiple equilibria or no equilibrium. For example, excess expected profits accruing from a market with bluffing may dictate that an informed trader should collect a lot of information. But if the informed trader does collect a lot of information then her presumption of excess profits is unjustified because no bluffing will occur in equilibrium. However, if the informed trader collects only a little information owing to the lack of excess expected profits, then in equilibrium the informed trader will bluff and will have been better off having collected more information.

3.4.2 Endogenous Liquidity Trade

The amount of liquidity trade in our model is exogenously specified via $w$. The effect of endogenizing the liquidity trade is uncertain. On the one hand, endogenous noise trade may reinforce bluffing. All else equal, the potential for bluffing leads to higher expected informed trader profits, which are financed by liquidity trader losses. Therefore, if liquidity traders are given some degree of control over when or where they trade, they will choose to avoid times or markets when the potential for bluffing are high. As shown above, bluffing strategies are more likely to be adopted in illiquid markets because illiquid markets are expected to have a higher degree of informational trade transparency. Therefore, it might be the case that low liquidity and bluffing strategies are mutually reinforcing.
On the other hand, bluffing is more likely when the expected informational advantage of informed traders is small. All else equal, liquidity traders prefer to trade in a market where the degree of information asymmetry is small. Therefore, if we take the ex ante degree of information asymmetry between informed traders and liquidity traders as exogenous, it may be the case that high liquidity trade and bluffing will be coincident in markets with low information asymmetry while low liquidity and no bluffing will be coincident in markets with high information asymmetry.\footnote{See Dow (2004) for a market model with endogenous liquidity trade and multiple equilibria. Spiegel and Subrahmanyam (1992) show that inferences drawn from models with exogenous liquidity trade may not hold up if liquidity traders are replaced by rational maximizers trading to satisfy hedging demands.}

3.4.3 Multiple Informed Traders

In our model there is a single informed trader. The existence of multiple informed traders would effect the model significantly. Multiple informed traders would mitigate, if not eliminate, bluffing incentives due to free-riding issues. Trading against one’s information creates a public good (for the other informed traders), but a personal bad. Informed traders may collectively be better off if they could commit to a trading strategy admitting bluffing, but absent a commitment mechanism, each individual trader may find it in her best interest not to engage in bluffing.

It’s likely that the correlation among the information of different informed traders may play a significant role. If multiple informed traders have heterogeneous information, then it is possible that the informed traders will compete away the common component of their information and then adopt a bluffing strategy with respect to the unique component of their information. This intuition is based on the results of Foster and Viswanathan (1996). A more formal treatment of the impact of multiple informed traders is beyond the scope of the current paper and left for future research.
4 Conclusion

We develop a model to study how trader anonymity impacts optimal trading behavior and price characteristics. Our main result is that revealing trader identities does not necessarily speed up the revelation of fundamental information about the traded securities. The reason is that markets that afford less trader anonymity create incentives for informed traders to adopt more complex trading strategies, including bluffing, such that market makers are less able to draw clean inferences from the order flow of an informed trader. Bluffing is costless to an informed trader (in terms of her expected profits relative to a non-bluffing benchmark), but changes market dynamics so that expected profits are higher when an informed trader does not bluff and trades with her information. The indirect benefits of bluffing can be sufficiently large that an informed trader would choose to trade in a less anonymous market with fewer liquidity traders relative to a more anonymous market with more liquidity traders. This is a surprising result because informed traders are generally presumed to prefer to trade in markets with both more anonymity and more liquidity trade.

The relationship between trader anonymity and market price dynamics is ambiguous. More anonymous markets may be more or less liquid than less anonymous markets and may have more or less efficient prices. In general, decreasing trader anonymity will raise liquidity and lower efficiency when informed traders are likely to have small informational advantages. Thus, low trader anonymity may bound the informational efficiency a market can achieve. However, if the excess profits attributable to adoption of bluffing strategies encourage additional information production, the net effect on market efficiency is unclear. Furthermore, the indirect nature of the profits to bluffing strategies make them susceptible to free-rider problems in settings with multiple informed traders. As such, concerns about bluffing strategies and their potentially negative impact on the price discovery process may be more relevant to smaller, less liquid markets with fewer sophisticated participants.
References


5 Appendix

5.1 Detailed Strategies and Pricing Rules in Proposition 1 and Corollary 1

In the first round of trade, the informed trader’s trading strategy is the following mixed strategy:

\[ X_1(h; p_0) = \begin{cases} 
-1 \; \text{w.p.} \quad \phi^h_1(p_0) \\
0 \; \text{w.p.} \quad \theta^h_1(p_0) \\
+1 \; \text{w.p.} \quad 1 - \phi^h_1(p_0) - \theta^h_1(p_0) 
\end{cases} \]

where

\[ \phi^h_1(p_0) = \begin{cases} 
0 & \text{for } 0 \leq p_0 \leq p^C_3 \\
-\left( \frac{1+8w+32w^2+32w^3+48w^4}{p_0(1+4w+16w^2+80w^3+48w^4-p_0(1+2w)^2(1+12w^2))} \right) & \text{for } p^C_3 \leq p_0 \leq p^C_4 \\
0 & \text{else} 
\end{cases} \]

\[ \theta^h_1(p_0) = \begin{cases} 
0 & \text{for } 0 \leq p_0 \leq p^C_1 \\
\frac{8w^2-8p_0w^2-(-1+p_0)^2(-1+2w)(1+2w)^2}{8p_0w^2-(-1+p_0)p_0(-1+2w)(1+2w)^2} & \text{for } p^C_1 \leq p_0 \leq p^C_2 \\
1 - \frac{4w^2+1}{(2w+1)^2} \cdot \frac{1}{p_0} & \text{for } p^C_2 \leq p_0 \leq p^C_3 \\
-8w \left( -1 - 6w - 12w^2 - 24w^3 + p_0(1+2w)^3 \right) & \text{for } p^C_3 \leq p_0 \leq p^C_4 \\
\frac{8w}{12w^2+1} \cdot \frac{1}{p_0} & \text{else} 
\end{cases} \]

and

\[ p^C_1 = -1 - 2w - 4w^2 + 8w^3 \]
\[ p^C_2 = 1 + 6w + 4w^2 + 8w^3 \]
\[ p^C_3 = 1 + 8w + 32w^2 + 32w^3 + 48w^4 \]
\[ p^C_4 = 1 + 6w + 32w^2 + 144w^3 + 112w^4 + 96w^5 \]
$X_1(l; p_0)$ is symmetric. The market maker’s pricing rule is:

$$P_1(z_1; p_0) = \begin{cases} 
\frac{\phi_h^1(p_0) - 1}{\phi_t^1(p_0) + [1 - \theta_t^1(p_0) + \phi_t^1(p_0)](1 - p_0)} & \text{for } z_1 = -w - 1 \\
\frac{\theta_t^1(p_0) - 1}{\theta_t^1(p_0) + [1 - \phi_t^1(p_0)](1 - p_0)} & \text{for } z_1 = -w \\
\frac{[\theta_t^1(p_0) - 1] + [1 - \phi_t^1(p_0)](1 - p_0)}{[\theta_t^1(p_0) - 1] + [1 - \phi_t^1(p_0)](1 - p_0)} & \text{for } -w + 1 \leq z_1 \leq w - 1 \\
\frac{\theta_t^1(p_0) - 1}{\theta_t^1(p_0) + [1 - \phi_t^1(p_0)](1 - p_0)} & \text{for } z_1 = w \\
\frac{\phi_t^1(p_0) - 1}{\phi_t^1(p_0) + [1 - \theta_t^1(p_0) + \phi_t^1(p_0)](1 - p_0)} & \text{else}
\end{cases}$$

In the second round of trade, the informed trader’s trading strategy is the following mixed strategy:

$$X_2(h; p_1) = \begin{cases} 
0 \text{ w.p. } \theta_h^2(p_1) \\
+1 \text{ w.p. } 1 - \theta_h^2(p_1)
\end{cases}$$

where

$$\theta_h^2(p_1) = \begin{cases} 
0 & \text{for } 0 \leq p_1 \leq \frac{2w}{2w+1} \\
1 - \frac{2w}{2w+1} \cdot \frac{1}{p_1} & \text{else}
\end{cases}$$

$X_2(l; p_1)$ is symmetric. The market maker’s pricing rule is:

$$P_2(z_2; p_1) = \begin{cases} 
0 & \text{for } z_2 = -w - 1 \\
\frac{\theta_h^2(p_1)}{(1 - p_1 + \theta_h^2(p_1))p_1} & \text{for } z_2 = -w \\
p_1 & \text{for } -w + 1 \leq z_2 \leq w - 1 \\
\frac{\theta_h^2(1 - p_1) + p_1}{1} & \text{for } z_2 = w \\
\frac{\theta_h^2(p_1)}{(1 - p_1 + \theta_h^2(p_1))p_1} & \text{else}
\end{cases}$$

In the third and final round of trade, the informed trader’s trading strategy is the following pure strategy:

$$X_3(h; p_2) = 1$$
$$X_3(l; p_2) = -1$$

and the market maker’s pricing rule is:

$$P_3(z_3; p_2) = \begin{cases} 
0 & \text{for } z_3 \leq -w \\
p_2 & \text{for } -w + 1 \leq z_3 \leq w - 1 \\
1 & \text{for } z_3 \geq w
\end{cases}$$
5.2 Proof of Proposition 1

The model is solved by backward induction. The equilibrium is presented and discussed from the perspective of an informed trader with a high signal, $s = h$, without loss of generality. We adopt the following notation. The price in trade round $n$ is $p_n$. The informed trader’s order flow in round $n$ is $x_n$. The noise trader order flow in round $n$ is $u_n$. The aggregate order flow is $z_n = x_n + u_n$. The informed trader trade strategy in round $n$, is:

$$X_n(h; p_{n-1}) = \begin{cases} 
-1 \text{ w.p. } & \phi_n^h(p_{n-1}) \\
0 \text{ w.p. } & \theta_n^h(p_{n-1}) \\
+1 \text{ w.p. } & 1 - \phi_n^h(p_{n-1}) - \theta_n^1(p_{n-1})
\end{cases}$$

5.2.1 Period Three

In the last round of trade before the liquidation value of the risky asset is announced, the informed trader submits his order $x_3 \in \{-1, 0, +1\}$. There are no successive period profits to be considered so the informed trader maximizes expected profit in the current trading round. He is free to choose a mixed strategy over the feasible orders, but trading with his information ($x_3 = +1$) is a (weakly) dominant strategy. The terminal period payoffs $\pi_2(p_2, p_1, p_0, s)$ for each pure strategy $x_3 \in \{-1, 0, +1\}$ are given by $p_2 - 1$, $0$, and $1 - p_2$ respectively. Since $0 \leq p_2 \leq 1$, it is trivial to see that $x_3 = +1$ weakly dominates all other strategies. The final round expected profits equal $\frac{2w-1}{2w+1}(1-p_2)$; there are two possible order flows (out of the $2w+1$ order flows possible when $x_3 = 1$) in which the insider’s signal is revealed, $z_3 = +w$ and $z_3 = +w + 1$. In these states, $p_3 = 1$ and the informed trader earns zero profit.

5.2.2 Period Two

Prices

During the second period, the market maker sets prices equal to the expected value of the risky asset, taking as given the strategy of the informed trader. Suppose that observed aggregate order flow were $z_2 = -w - 1$. Since the minimum value of the pure noise component is $-w$, the market maker knows that $x_2 = -1$ has been submitted. Only the underlying signal $s \in \{l, h\}$ of the insider is uncertain. Either an insider with $s = h$ (probability = $p_1$) submitted $x_2 = -1$ which occurs with conditional probability $\phi_2(p_1)$, or an insider with $s = l$ (probability = $1 - p_1$) submitted $x_2 = -1$ which occurs with conditional probability $1 - \phi_2(1 - p_1) - \theta_2(1 - p_1)$. Applying Bayes Rule, the expected value of the asset given

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\[ z_2 = -w - 1 \] is given by:

\[
p_{2+}^+ = p_2 | \{ z_2 = -w - 1 \} = \frac{p_1 \phi_2(p_1)}{p_1 \phi_2(p_1) + (1 - p_1)[1 - \theta_2(1 - p_1) - \phi_2(1 - p_1)]}
\]

All other prices are identically formed, and may be interpreted as the probability that \( s = h \) given \( z_2 \). The remainder of the prices are given below:

\[
p_2^- = p_2 | \{ z_2 = -w \} = \frac{p_1[\theta_2(p_1) + \phi_2(p_1)]}{p_1[\theta_2(p_1) + \phi_2(p_1)] + (1 - p_1)[1 - \phi_2(1 - p_1)]}
\]

\[
p_2^- = p_2 | \{ z_2 = +w \} = \frac{p_1[1 - \phi_2(p_1)]}{p_1[1 - \phi_2(p_1)] + (1 - p_1)[\theta_2(1 - p_1) + \phi_2(1 - p_1)]}
\]

\[
p_2^- = p_2 | \{ z_2 = +w + 1 \} = \frac{p_1[1 - \phi_2(p_1) - \theta_2(p_1)] + (1 - p_1)\phi_2(1 - p_1)}{p_1[1 - \phi_2(p_1) - \theta_2(p_1)] + (1 - p_1)\phi_2(1 - p_1)}
\]

**Expected Profits**

The insider’s expected profits include those from the second and third rounds of trade. Consider each the strategies \( x_2 \in \{ +1, 0, -1 \} \) in turn. The expected profits from submitting \( x_2 = +1 \) allow for \(-w + 1 \leq z_2 \leq +w + 1\), which eliminate two of the five possible prices. If \( u_2 \leq +w - 2 \) (which occurs with probability \( \frac{2w - 1}{2w + 1} \)), then \( p_2^0 = p_1 \) as indicated above. Likewise, if \( u_2 = +w - 1 \) (which occurs with probability \( \frac{1}{2w + 1} \)), \( p_2^+ = p_2 \). Finally, \( p_2^{++} \) is possible if \( u_2 = +w \). Period two expected profits, conditional on \( x_2 = +1 \), written as a function of possible second period prices \( p_2^0, p_2^+, \) and \( p_2^{++} \) are:

\[
E[\Pi_2 | x_2 = +1] = \frac{2w - 1}{2w + 1} \left[ (1 - p_2^0) + \frac{2w - 1}{2w + 1}(1 - p_2^0) \right] + \cdots
\]

\[
= \frac{1}{2w + 1} \left[ (1 - p_2^+) + \frac{2w - 1}{2w + 1}(1 - p_2^+) \right] + \frac{1}{2w + 1} \left[ (1 - p_2^{++}) + \frac{2w - 1}{2w + 1}(1 - p_2^{++}) \right]
\]

A similar expression results for \( x_2 = 0 \). In this case, the most extreme prices \( p_2^{++} \) and \( p_2^- \) are precluded:

\[
E[\Pi_2 | x_2 = 0] = \frac{2w - 1}{2w + 1} \left[ 0 + \frac{2w - 1}{2w + 1}(1 - p_2^0) \right] + \cdots
\]

\[
= \frac{1}{2w + 1} \left[ 0 + \frac{2w - 1}{2w + 1}(1 - p_2^+) \right] + \frac{1}{2w + 1} \left[ 0 + \frac{2w - 1}{2w + 1}(1 - p_2^-) \right]
\]

Finally, expected profits given \( x_2 = -1 \) are provided for an insider with signal \( s = h \). Now the two highest price regions are impossible, resulting in the following:

\[
E[\Pi_2 | x_2 = -1] = \frac{2w - 1}{2w + 1} \left[ (p_2^0 - 1) + \frac{2w - 1}{2w + 1}(1 - p_2^0) \right] + \cdots
\]
in the second period, the following condition necessarily holds:

\[
\frac{1}{2w+1} \left[ (p_2^- - 1) + \frac{2w-1}{2w+1} (1 - p_2^-) \right] + \frac{1}{2w+1} \left[ (p_2^- - 1) + \frac{2w-1}{2w+1} (1 - p_2^-) \right] = 0
\]

Characterization of Optimal Trading Strategy for Two-Period Trading Game

We conjecture the two-period equilibrium strategy given in Proposition 1, and verify that no profitable deviations exist. For the entire price region, \( x_2 = +1 \) is submitted with positive probability; the profits from this strategy, therefore, represent the relevant comparison for any potentially profitable deviation. We begin by demonstrating that \( x_2 = -1 \) is strictly dominated over the possible price range, and can be eliminated from consideration.

Under the conjectured equilibrium, insiders never trade against their information. That is \( \phi_2(p_1) = 0 \) and \( \phi_2(1 - p_1) = 0 \). There are still five possible prices, but they are greatly simplified. In particular, both price extremes are now fully revealing, i.e. \( p_2^+ = 1 \) and \( p_2^- = 0 \). Under the market maker’s belief that insiders never trade against their information in the second period, the following condition necessarily holds:

\[
\frac{2w-1}{2w+1} \left[ (1 - p_1) + \frac{2w-1}{2w+1} (1 - p_1) \right] + \frac{1}{2w+1} \left[ \frac{(1-p_1)\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1} + \frac{2w-1}{2w+1} \frac{(1-p_1)\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1} \right] \geq \frac{2w-1}{2w+1} \left[ (p_1 - 1) + \frac{2w-1}{2w+1} (1 - p_1) \right] + \frac{1}{2w+1} \left[ \frac{p_1-1}{(1-p_1)+p_1\theta(p_1)} + \frac{2w-1}{2w+1} \frac{1-p_1}{(1-p_1)+p_1\theta(p_1)} \right] + \frac{1}{2w+1} \left[ (0 - 1) + \frac{2w-1}{2w+1} (1 - 0) \right]
\]

The left-hand side, representing the insider’s expected two-period profits from trading with his information, is weakly positive for the entire set of possible prices \( p \in [0, 1] \). The expected profits from trading against one’s information are always weakly less than zero for the two-period model, which can never exceed the profits from trading with one’s information. The right hand side simplifies to the following, whose value is bounded from above at zero:

\[
\frac{2w-1}{2w+1} \left[ \frac{-2}{2w+1} (1 - p_1) \right] + \frac{1}{2w+1} \left[ \frac{-2}{2w+1} \left( \frac{1 - p_1}{1 - p_1} + p_1\theta(p_1) \right) \right] + \frac{1}{2w+1} \left[ \frac{-2}{2w+1} \right]
\]

By iterated deletion of weakly dominated strategies, we eliminate \( x_2 = -1 \) from further consideration, and restrict our attention to the mixed strategy space spanned by \( x_2 \in \{0, +1\} \).

To show that \( x_2 = +1 \) is strictly dominant for \( p_1 \leq \frac{2w}{2w+1} \), we set the expected profits from submitting \( x_2 = 0 \) and \( x_2 = +1 \) respectively, simplify, and equate.

\[
\frac{(2w-1)(1-p_1)}{(2w+1)^2} \left[ \frac{1}{(1-p_1)+p_1\theta(p_1)} + (2w - 1) \frac{\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1} \right] = \frac{4w(1-p_1)}{(2w+1)^2} \left[ 2w - 1 + \frac{\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1} \right]
\]

Taking advantage of the symmetric structure of \( \theta(p) \), we note that \( p_1 \leq \frac{2w}{2w+1} \Rightarrow \theta(p_1) = 0 \) necessarily implies that \( p_1 \leq \frac{1}{2w+1} \Rightarrow \theta(1 - p_1) = 0 \). Making this substitution and solving
for $\theta(p_1)$ easily results in the expected profits for the two-period game:

$$E[\Pi_2] = \begin{cases} 
\frac{4w(2w-1)}{(2w+1)^2} (1 - p_1), & \text{if } p_1 \geq \frac{1}{2w+1} \\
\frac{8w^2}{(2w+1)^2} (1 - 2p_1), & \text{if } p_1 < \frac{1}{2w+1}
\end{cases}$$

Thus, in the penultimate round of trade, the informed trader may, depending on the price, choose to not trade rather than trade with his information, but he will never trade against his information.

5.2.3 Period One

We show that the informed insider’s strategy is optimal given the market maker’s pricing rule and that the pricing rule sets price equal to the expected value of the asset conditional on the aggregate order flow and trade strategy of the informed trader.

Prices

The expressions for prices are identical to those presented in the last section. We present only $p_3^{++}$, noting that only the time subscripts have been advanced by one position:

$$p_1^{++} = p_1|\{z_1 = -w - 1\} = \frac{p_0\phi_1(p_0)}{p_0\phi_1(p_0) + (1 - p_0)[1 - \theta_1(1 - p_0) - \phi_1(1 - p_0)]}$$

All other prices are formed identically.

Trading Strategy and Expected Profits

Period one and period two prices are functions of the probability of manipulation (or sitting out), and expected profits, of course, depend on these prices. The functional form of the informed trader mixing probabilities change over the price region $p \in [0, 1]$. Consequently, when evaluating the expected payoffs to each strategy, we must consider each region independently. We begin by describing the expected profits conditional on each pure strategy, $E[\Pi_1|x_1 = +1]$, $E[\Pi_1|x_1 = 0]$, and $E[\Pi_1|x_1 = -1]$, and apply these payoffs to each region. The price regions correspond to different mixing probabilities, although informed traders with different signals manipulate at opposite ends of the price spectrum.

$$E[\Pi_1|x_1 = +1] = \frac{2w - 1}{2w + 1} [(1 - p_0) + \Pi_2(p_0)] + \frac{1}{2w + 1} [(1 - p_1^+) + \Pi_2(p_1^+)] + \cdots$$

$$E[\Pi_1|x_1 = 0] = \frac{2w - 1}{2w + 1} [(0) + \Pi_2(p_0)] + \frac{1}{2w + 1} [(0) + \Pi_2(p_1^+)] + \frac{1}{2w + 1} [(0) + \Pi_2(p_1^-)]$$

$$E[\Pi_1|x_1 = -1] = \frac{2w - 1}{2w + 1} [(p_0 - 1) + \Pi_2(p_0)] + \frac{1}{2w + 1} [(p_1^- - 1) + \Pi_2(p_1^-)] + \cdots$$
\[
\frac{1}{2w + 1} \left[ (p_1^- - 1) + \Pi_1(p_0^-) \right]
\]

**Region 1:** \( \frac{1 + 8w + 32w^2 + 32w^3 + 48w^4}{(1 + 2w)^2 (1 + 12w^2)} \leq p_0 \leq 1 \)

Given the proposed manipulation strategies in Proposition 1, and the pricing schedule above, the expected profits to each pure strategy are given, after substitution and simplification, as:

\[
E[\Pi_1 | x_1 = +1] = E[\Pi_1 | x_1 = 0] = E[\Pi_1 | x_1 = -1] = \frac{(1-p_0)(-1+2w)(1+12w^2)}{(1+2w)^3}
\]

Facing the same pricing rule, any mixed strategy \( \Lambda \in \mathbb{R}^+_3 \equiv \{ \gamma_1, \rho_1, 1 - \gamma_1 - \rho_1; 0 \leq \gamma_1 = Pr(x_1 = 1) \leq 1 \text{ and } 0 \leq \rho_1 = Pr(x_1 = 0) \leq 1 \} \) over the pure strategy space will also yield the identical payoff given above. Since the informed insider’s manipulation schedule \( \{ \phi_1(p_0), \theta_1(p_0), 1 - \phi_1(p_0) - \theta_1(p_0) \} \in \Lambda \), then a rational expectations equilibrium exists at the proposed equilibrium strategy. Thus, the insider will manipulate with the schedule given by Proposition 1, and the market maker will set a price that the insider anticipates. Note that this pricing region encompasses *two* regions where all three pure strategies are utilized with positive probability.

**Region 2:** \( \frac{1 + 6w + 4w^2 + 8w^3}{(1 + 2w)^3} \leq p_0 \leq \frac{1 + 8w + 32w^2 + 32w^3 + 48w^4}{(1 + 2w)^2 (1 + 12w^2)} \)

Given the proposed manipulation strategies in Proposition 1, and the pricing schedule above, the expected profits to each pure strategy are given, after substitution and simplification, as:

\[
E[\Pi_1 | x_1 = +1] = E[\Pi_1 | x_1 = 0] = \frac{(1-p_0)(-1+2w)(1+12w^2)}{(1+2w)^3}
\]

\[
E[\Pi_1 | x_1 = -1] = \frac{-1 + p_0 + 4(-3+2p_0)w + 8(-3+p_0)w^2 + 16(-5+6p_0)w^3 - 48(-1+p_0)w^4}{4w(1+2w)^3}
\]

After some algebra, one can show that for

\[
p_0 < \frac{(1-p_0)(-1+2w)(1+12w^2)}{(1+2w)^3}
\]

it is the case that:

\[
E[\Pi_1 | x_1 = +1] = E[\Pi_1 | x_1 = 0] < E[\Pi_1 | x_1 = -1].
\]
Since the region of interest excludes this range of prices, $x_1 = -1$ is strictly dominated in this region, and cannot be part of any equilibrium strategy.

For the two remaining undominated pure strategies, their identical payoffs allow us to argue with the same reasoning applied to region 1. Since the market maker anticipates both $x_1 = +1$ and $x_1 = 0$ to be played with positive probability in region 2, the proposed equilibrium strategy represents a rational expectations equilibrium in region 2.

**Region 3:**

$$\frac{-1-2 w-4 w^2+8 w^3}{(-1+2 w)(1+2 w)^2} \leq p_0 \leq \frac{1+6 w+4 w^2+8 w^3}{(1+2 w)^3}$$

In this region:

$$E[\Pi_1|x_1 = +1] = E[\Pi_1|x_1 = 0] = \frac{(1-p_0) (-1+2 w) (1+12 w^2)}{(1+2 w)^3}$$

$$E[\Pi_1|x_1 = -1] = -\frac{-1-6 w+16 w^2+272 w^4-224 w^5+p_0 (1+6 w+8 w^2+48 w^3-304 w^4+224 w^5)}{16 w^2 (1+2 w)^3}$$

For $p_0 > \frac{-1-6 w+80 w^3+80 w^4+160 w^5}{(1+2 w)^2 (-1-2 w+12 w^2+40 w^3)}$, one can show that $E[\Pi_1|x_1 = -1] > E[\Pi_1|x_1 = +1]$. However, for $w > 0 | w \in \mathbb{R}$:

$$\frac{-1-6 w+80 w^3+80 w^4+160 w^5}{(1+2 w)^2 (-1-2 w+12 w^2+40 w^3)} > \frac{1+6 w+4 w^2+8 w^3}{(1+2 w)^3},$$

which is strictly outside region 3. Therefore, for the prices within region 3, $E[\Pi_1|x = -1]$ is strictly dominated, and cannot be part of any equilibrium strategy.

For the two remaining undominated pure strategies, their identical payoffs allow us to argue with the same reasoning applied to region 1 and 2. Since the market maker anticipates both $x_1 = +1$ and $x_1 = 0$ with positive probability in region 3, the proposed equilibrium strategy represents a rational expectations equilibrium in region 3.

**Region 4:**

$$\frac{8 w^2}{(-1+2 w)(1+2 w)^2} \leq p_0 \leq \frac{-1-2 w-4 w^2+8 w^3}{(-1+2 w)(1+2 w)^2}$$

For this and all remaining regions, the claim is that both manipulation ($x_1 = -1$) and sitting out ($x_1 = 0$) are strictly dominated for an insider facing prices governed by the proposed equilibrium strategies in Proposition 1. In region 4:

$$E[\Pi_1|x_1 = +1] = -\frac{(-1+p_0) (-1+2 w)(1+12 w^2)}{(1+2 w)^3}$$

$$E[\Pi_1|x_1 = 0] = -\frac{-4 w (-1+p_0 (1-2 w)^2+2 w-4 w^2)}{(1+2 w)^3}$$
\[
E[\Pi_1|x_1 = -1] = -\frac{1+2 w+12 w^2-8 w^3+p_0 \left(1+6 w-20 w^2+8 w^3\right)}{(1+2 w)^3}
\]

It follows that:

\[
E[\Pi_1|x = -1] > E[\Pi_1|x = +1] \iff p_0 > \frac{2 \left(w + 4 w^3\right)}{(-1 + 2 w) \left(1 + 2 w\right)^2},
\]

which for any \( w \in \mathbb{R} \) is impossible within region 4. Therefore, \( x_1 = -1 \) is strictly dominated by \( x_1 = +1 \) in this region. Also

\[
E[\Pi_1|x = 0] > E[\Pi_1|x = +1] \iff p_0 > \frac{-1 - 2 w - 4 w^2 + 8 w^3}{(-1 + 2 w) \left(1 + 2 w\right)^2},
\]

which by inspection is revealed as the upper border on region 4. Therefore, \( x_1 = 0 \) is strictly dominated by \( x_1 = +1 \). Only \( x_1 = +1 \) survives iterated deletion of strictly dominated strategies.

**Region 5:** \( \frac{8 w^2}{(1+2 w)^3} \leq p_0 \leq \frac{8 w^2}{(-1+2 w) \left(1 + 2 w\right)^2} \)

In this region:

\[
E[\Pi_1|x_1 = +1] = -\frac{\left(1+12 w^2\right) \left(8 \left(1-4 w\right) w^2+p_0 \left(-1-2 w-12 w^2+40 w^3\right)\right)}{16 w^2 \left(1+2 w\right)^3}
\]

\[
E[\Pi_1|x_1 = 0] = \frac{8 w^2 \left(3+8 w+20 w^2-32 w^3\right)+p_0 \left(1+10 w+8 w^2+16 w^3-304 w^4+288 w^5\right)}{16 w^2 \left(1+2 w\right)^3}
\]

\[
E[\Pi_1|x_1 = -1] = -\frac{1+2 w+12 w^2-8 w^3+p_0 \left(1+6 w-20 w^2+8 w^3\right)}{(1+2 w)^3}
\]

It follows that \( E[\Pi_1|x_1 = 0] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{8 w^2 \left(1+6 w+4 w^2+8 w^3\right)}{-1-6 w-16 w^2+80 w^4+96 w^6} \). However, for \( w > 1 \), \( \frac{8 w^2 \left(1+6 w+4 w^2+8 w^3\right)}{-1-6 w-16 w^2+80 w^4+96 w^6} > \frac{8 w^2}{\left(-1+2 w\right) \left(1+2 w\right)^2} \), which is the upper bound on region 5. Thus for \( p_0 \leq \frac{8 w^2}{\left(-1+2 w\right) \left(1+2 w\right)^2} \), \( x_1 = 0 \) is strictly dominated by \( x_1 = +1 \).

Similarly, \( E[\Pi_1|x_1 = -1] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{4 w \left(1+2 w+8 w^2\right)}{-1-6 w+12 w^2+56 w^3} \), which for all \( w > 2 \), is strictly greater than the upper bound of region 5.

**Region 6:** \( \frac{1}{1+2 w} \leq p_0 \leq \frac{8 w^2}{(1+2 w)^3} \)

In this region:

\[
E[\Pi_1|x_1 = +1] = -\frac{\left(1+12 w^2\right) \left(p_0-8 w^2+12 p_0 w^2\right)}{4 w \left(1+2 w\right)^3}
\]
\[
\begin{align*}
E[\Pi_1 | x_1 = 0] &= \frac{8 w (-1 - 4 w^2 + 8 w^3) + p_0 (1 + 12 w + 16 w^2 + 80 w^3 - 80 w^4)}{4 w (1 + 2 w)^3} \\
E[\Pi_1 | x_1 = -1] &= -\frac{1 + 2 w + 12 w^2 - 8 w^3 + p_0 (1 + 6 w - 20 w^2 + 8 w^3)}{(1 + 2 w)^3}
\end{align*}
\]

\[E[\Pi_1 | x_1 = 0] > E[\Pi_1 | x_1 = +1] \iff p_0 > \frac{4 w (1 + 5 w + 4 w^2 + 4 w^3)}{(1 + 2 w)^2 (1 + 2 w + 8 w^2)}\] However, for \(w > 2\), \(\frac{4 w (1 + 5 w + 4 w^2 + 4 w^3)}{(1 + 2 w)^2 (1 + 2 w + 8 w^2)}\) is strictly greater than the upper bound for region 6, \(p_0 = \frac{8 w^2}{(1 + 2 w)^2}\). Therefore, \(x_1 = 0\) is strictly dominated in this region, and cannot be part of any equilibrium strategy.

\[E[\Pi_1 | x_1 = -1] > E[\Pi_1 | x_1 = +1] \iff p_0 > \frac{4 w (1 + 2 w + 8 w^2)}{1 - 6 w + 12 w^2 + 56 w^3}\] However, for \(w > 0\), \(\frac{4 w (1 + 2 w + 8 w^2)}{1 - 6 w + 12 w^2 + 56 w^3}\) is strictly greater than the upper bound for region 6. Therefore, \(x_1 = -1\) is strictly dominated in this region, and cannot be part of any equilibrium strategy.

**Region 7:** \(\frac{4 w (-1 - 4 w^2 + 4 w^3)}{(1 + 2 w)^2 (1 + 12 w^2)} \leq p_0 \leq \frac{1}{1 + 2 w}\)

In this region:

\[
\begin{align*}
E[\Pi_1 | x_1 = +1] &= -\frac{p_0 + 8 (1 + p) w^2 - 32 w^3 + 16 (-6 + 13 p) w^4}{4 w (1 + 2 w)^3} \\
E[\Pi_1 | x_1 = 0] &= \frac{8 w (-1 - 6 w + 8 w^3) + p_0 (1 + 12 w + 32 w^2 + 80 w^3 - 144 w^4)}{4 w (1 + 2 w)^3} \\
E[\Pi_1 | x_1 = -1] &= -\frac{1 + 6 w + 4 w^2 - 8 w^3 + p_0 (1 - 2 w)^2 (1 + 6 w)}{(1 + 2 w)^3}
\end{align*}
\]

\[E[\Pi_1 | x_1 = 0] > E[\Pi_1 | x_1 = +1] \iff p_0 > \frac{4 w (1 + 5 w + 4 w^2 + 4 w^3)}{(1 + 2 w)^2 (1 + 2 w + 8 w^2)}\] However, for \(w > 0\), \(\frac{4 w (1 + 5 w + 4 w^2 + 4 w^3)}{(1 + 2 w)^2 (1 + 2 w + 8 w^2)} > \frac{1}{1 + 2 w}\), indicating that \(x_1 = 0\) is strictly dominated by \(x_1 = +1\) in region 7.

\[E[\Pi_1 | x_1 = -1] > E[\Pi_1 | x_1 = +1] \iff p_0 > \frac{4 w (1 + 2 w + 8 w^2)}{1 - 6 w + 12 w^2 + 56 w^3}\] However, for \(w > 0\), \(\frac{4 w (1 + 2 w + 8 w^2)}{1 - 6 w + 12 w^2 + 56 w^3} > \frac{1}{1 + 2 w}\), indicating that \(x_1 = -1\) is strictly dominated by \(x_1 = +1\) in region 7.

**Region 8:** \(\frac{8 w^2 (-1 - 8 w + 4 w^2)}{(1 + 2 w)^2 (1 + 12 w^2)} \leq p_0 \leq \frac{4 w (-1 - 4 w + 4 w^2)}{(1 + 2 w)^2 (1 + 12 w^2)}\)

In this region:

\[
E[\Pi_1 | x_1 = +1] = -\frac{1 + 2 w + 4 \left( p_0 + 2 p w^2 + 4 (1 + p) w^2 - 2 (3 + p) w^3 + 12 (-3 + 8 p_0) w^4 \right)}{(-1 + 2 w) (1 + 2 w)^3 (1 + 6 w)}
\]
E[\Pi_1|x_1 = 0] = \frac{8 w \left(-1-6 w+8 w^3\right)+p_0 \left(1+12 w+32 w^2+80 w^3-144 w^4\right)}{4 w (1+2 w)^3}
E[\Pi_1|x_1 = -1] = -\frac{1+6 w+4 w^2-8 w^3+p_0 (1-2 w)^2 (1+6 w)}{(1+2 w)^3}
E[\Pi_1|x_1 = 0] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{4 w \left(-3-16 w+32 w^3+48 w^4\right)}{-1-14 w-8 w^2+16 w^3+304 w^4+672 w^5}
E[\Pi_1|x_1 = -1] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{2 \left(-1-6 w-12 w^2-32 w^3+64 w^4+96 w^5\right)}{(1+2 w)^2 \left(1+10 w-52 w^2+120 w^3\right)}

\text{Region 9: } 0 \leq p_0 \leq \frac{8 w^2 \left(-1-8 w+4 w^2\right)}{(1+2 w)^3 (1+12 w^2)}

In this region:
E[\Pi_1|x_1 = +1] = -\frac{1+6 w+192 w^3+112 w^4-96 w^5+p_0 \left(1+2 w+20 w^2+88 w^3\right)}{(1+2 w)^4 \left(-1-8 w+4 w^2\right)}
E[\Pi_1|x_1 = 0] = \frac{8 w \left(-1-6 w+8 w^3\right)+p_0 \left(1+12 w+32 w^2+80 w^3-144 w^4\right)}{4 w (1+2 w)^3}
E[\Pi_1|x_1 = -1] = -\frac{1+6 w+4 w^2-8 w^3+p_0 (1-2 w)^2 (1+6 w)}{(1+2 w)^3}
E[\Pi_1|x_1 = 0] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{4 w \left(-3-34 w-88 w^2-128 w^3+16 w^4+32 w^5\right)}{(1+2 w)^2 \left(-1-20 w-16 w^2-112 w^3+208 w^4\right)}
E[\Pi_1|x_1 = -1] > E[\Pi_1|x_1 = +1] \iff p_0 > \frac{-\left(1+10 w+24 w^2+96 w^3+16 w^4+32 w^5\right)}{8 w (1-4 w^2)^2}

Q.E.D.

This completes the proof of the equilibrium strategies proposed in Proposition 1.