Corporate Real Estate Holdings and the Cross Section of Stock Returns*

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August 2009

Abstract

This paper explores the link between the composition of firms' capital holdings and stock returns. I develop a general equilibrium production economy where firms use two factors: Real estate capital and other capital. Investment is subject to asymmetric adjustment costs that make cutting the capital stock costlier than expanding it. Because real estate depreciates slowly, firms with high real estate holdings are more vulnerable to bad productivity shocks, and real estate investment is riskier than investment in other capital. In equilibrium, investors demand a premium to hold these firms. This prediction is supported empirically. I find that the returns of firms with a high share of real estate capital compared to other firms in their industry exceed that of low real estate firms by 3-6% annually, adjusted for exposures to the market return, size, value, and momentum factors. Moreover, conditional beta estimates reveal that these firms indeed have higher market betas, and the spread between the betas of high and low real estate firms is countercyclical.

*This paper is based on chapter 2 of my UCLA PhD dissertation. I am truly indebted to Mark Grinblatt and Monika Piazzesi for their generous support, guidance, and encouragement. I would like to thank Fernando Alvarez, Tony Bernardo, Michael Brennan, Harold Cole, Wayne Ferson, Lars Hansen, Ayse Imrohoroglu, Selahattin Imrohoroglu, Chris Jones, Bruno Miranda, Oguzhan Ozbas, Stavros Panageas, Vincenzo Quadrini, Richard Roll, Pedro Santa-Clara, Martin Schneider, Clifford Smith, Avanidhar Subrahmanyam, seminar participants at Boston College, Carnegie-Mellon, Chicago GSB, Chicago macro working group, Columbia GSB, Duke, Emory, HBS, Maryland, Notre Dame, Oregon, Rochester, UC Berkeley, UCLA, UNC, USC Marshall, USC Real Estate Symposium, Wharton, and WFA 2005 meeting for their helpful comments and discussions. I gratefully acknowledge financial support from the Allstate and UCLA Dissertation Year Fellowships.

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1 Introduction

Firms own and use many different capital goods. Capital is heterogeneous; a building is not a computer. Even if in some extreme cases one can be substituted with the other in the firm's production (Barnes & Noble versus BarnesandNoble.com), other characteristics still distinguish them, such as the rates of depreciation. Commercial real estate and equipment naturally emerge as two major classes of capital goods. Their dollar values in the U.S. economy are comparable. The Bureau of Economic Analysis (BEA) estimates approximately 8.8 trillion dollars worth of nonresidential structures (value of buildings excluding the value of the land) and 4.7 trillion dollars worth of nonresidential equipment at the end of 2005. While most firms own and use both capital types in their operations, there is considerable variation in firms' capital composition. When firms are sorted on the share of buildings and capital leases in their total physical capital (PPE), firms in the first quintile have an average share of buildings and capital leases that is 22% lower than the average firm in that industry. Firms in the fifth quintile have 25% more buildings and capital leases than the average firm.

In addition to the obvious differences in their roles in firm operations, structures and equipment are different in their durability. Structures, on the average, depreciate much more slowly than equipment. Bureau of Economic Analysis rates of depreciation for private nonresidential structures range between 1.5-3%, whereas the depreciation rates for private nonresidential equipment are in the range of 10-30% (Fraumeni, 1997). Additionally, Glaeser and Gyourko (2005) point out the extremely durable nature of residential real estate. Since structures depreciate at a slower rate than equipment, structures require less replacement investment than equipment. This introduces significant heterogeneity into the capital stock of firms. The value of a firm depends on the underlying value of its assets, i.e. its capital stock. Therefore, the dynamics of a firm's value (return) is fundamentally linked to the changes in the firm's capital stock, both its size and composition.

In this paper, I study the link between the composition of the firm's capital holdings and stock returns.[1] Specifically, I explore the role of real estate holdings in the firm's investment decisions and capital. I develop a general equilibrium model in which a representative agent invests in the firms in the economy and consumes all wages and dividends. A continuum of firms use two factors, real estate capital (buildings/structures)[2] and other capital (equipment). Heterogeneity among firms arises endogenously as a result of stochastic productivity shocks. Investment in either form of capital is subject to convex adjustment costs, which are asymmetric; i.e., decreasing the capital stock is costlier than expanding it. Numerical solutions of the model suggest that a firm that owns a substantial amount of real estate as part of its capital is riskier than a firm that holds a smaller fraction of its capital in the form of real estate; therefore in equilibrium, investors demand a premium to hold such firms. I consequently verify this prediction with firm level data. I find that the returns of firms with a high share of real estate compared to other firms in their industry exceed that of low real estate firms by 3-6% annually,

[1] The composition of the firm's capital is different from the *composition risk* in Piazzesi, Schneider, and Tuzel (2007). The composition risk, measured by the changes in the expenditure share of housing in household's consumption, is part of the pricing kernel in that paper.

[2] Throughout this paper, I use the real estate/buildings/structures terms interchangibly. The BEA reports the quantity of structures, whereas Compustat reports the value of buildings.

adjusted for exposures to the market return, size, value, and momentum factors. Moreover, conditional beta estimates reveal that these firms indeed have higher market betas, and the spread between the betas of high and low real estate firms is countercyclical.

Capital heterogeneity is the first building block of this paper. Even though many different capital inputs enter the firm's production process, for simplicity, capital is overwhelmingly modeled as homogeneous in the literature. However, this assumption implies that different capital goods are perfect substitutes; i.e. personal computers can be replaced by factory space. Furthermore, aggregation across different capital goods eliminates many interesting investment dynamics.[3]

In the presence of capital heterogeneity, the real investment decisions of firms determine not only the size of the firm's capital but also its composition. If the capital holdings can be costlessly adjusted at any time then the composition of the firm's capital becomes trivial. The firm always holds the *optimal* capital mix for a given level of output, i.e. the mix of capital inputs that minimizes the firm's costs for a given level of output. Nevertheless, capital adjustment is rarely costless. Frictions in capital adjustment can distort the firm's investment decisions and its capital composition, reducing the flexibility of the firms to accommodate exogenous shocks by increasing or decreasing their investments and capital holdings. Frictions force firms to operate with too much unproductive capital in bad times, leading to low cash flows.[4] In good times, firms cannot quickly invest and increase their capital stock. Therefore, the cash flows and returns of firms covary more with economic cycles in the presence of adjustment costs.

I assume a particular form of friction in capital adjustment, namely that investment is subject to convex but asymmetric adjustment costs. This is the second building block of this paper. This form of friction captures the idea that it is more costly to cut the existing capital stock (reverse investment) than to add new capital to the current stock. In the presence of capital heterogeneity, the implications of costly reversibility can be starkly asymmetric for different types of capital. For a simple example, take a manufacturing firm with two types of capital: a plant, which depreciates very slowly, and cutting tools, which wear out (depreciate completely) after a few hours of heavy use. The firm dreads bad exogenous shocks and tries to mitigate the effect of a bad shock, primarily by decreasing the investment in short-lived cutting tools. This change in investment policy distorts the capital composition of the firm, increasing the share of plant in the firm's capital holdings. Positive exogenous shocks have the opposite effect, reducing the share of plant in the firm's capital. As the ratio of plant to total capital increases, the firm becomes more vulnerable to bad productivity shocks. In equilibrium, the investors demand a premium to hold this type of firm.

Asymmetric adjustment costs find strong support in the empirical literature. The sale price of used capital is generally substantially lower than its replacement cost, even after taking depreciation into account.[5] Many factors contribute to these low resale prices, including installation costs, capital specificity, thin markets, and adverse selection problems. Furthermore, Eisfeldt

[3] Goolsbee and Gross (1997), Doms and Dunne (1998), Nilsen and Schiantarelli (2003), Cummins and Dey (1998) and Abel and Eberly (2002) are among the papers that empirically investigate investment dynamics using plant or firm level data disaggregated to capital types. Their common finding is that firms do not invest nor disinvest in all types of capital every period and aggregation leads to very smooth investment patterns.

[4] Throughout the paper, cash flows refer to cash flows after investment.

[5] Ramey and Shapiro (2001), by collecting and analyzing data from aerospace industry auctions, find that reallocating capital entails substantial costs due to the loss of value and time. They estimate that the average market value of equipment sold in auctions is 28 cents per dollar of replacement cost.

and Rampini (2006) find that capital reallocation is procyclical, even though the benefits to reallocation are countercyclical, implying substantial countercyclical costs of reallocation. Firms are stuck with excess capital when they most need to reverse their investments, which is during economic downturns.

In the literature, costly reversibility is introduced in different forms. Abel and Eberly (1996) do so directly by introducing a difference between the price at which the firm can purchase capital and the price at which it can sell it. Hall (2001) and Zhang (2005), like this paper, introduce costly reversibility through a piecewise quadratic adjustment cost function, which allows cutting capital to be costlier than expanding the capital stock through parameter asymmetry. Recently, several papers have studied the asset pricing implications of models with irreversible investment (Cooper, 2006; Gomes, Kogan, and Zhang, 2003; Kogan, 2004; Berk, Green, and Naik, 1999), which prohibit disinvestment completely.

Although real estate benefits from the presence of more established secondary markets,[6] investment in real estate can nevertheless be very costly to reverse. Toward the end of 1981, Ford Motor Company announced that it would close its huge Michigan Casting Center in Flat Rock, which was built only 12 years previously at a cost of more than $150 million. The company spokesman said that, "Slow sales of large cast-iron auto engines and the fact that the plant cannot easily be adapted to newer products have forced the closing" (*New York Times*, Sep. 15, 1981). After staying idle for more than three years, in 1985, the Ford casting plant was demolished, and Mazda Motor Manufacturing Corporation built a factory on the same site.

The risks associated with investing in and holding real estate capital are well understood and frequently mentioned in the business press:

> A number of analysts express concerns about Hilton and Starwood in particular, because the two companies' real estate poses additional recession risks ... Owning hotels is more risky than managing or franchising them because of the cost of carrying and maintaining property ... Hilton in particular could be hard hit by the economic slowdown. Hilton owns many of its hotels, unlike Marriott, which mostly franchises and manages properties owned by others.
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> \- WSJ, 3/26/01

Different business cycle implications of investment in real estate capital and other, less durable capital types are also cited in the business press:

> Yet the aftereffects of overinvestment in technology are likely to be less pronounced than those of previous investment busts. In the 1980s, a frenzy of real estate investment saddled the U.S. with commercial office space that took years to fill. During that time, new investment in such properties almost ground to a halt. By contrast, business equipment and software depreciate in just a few years, if not months. Rapid depreciation means that any excess capacity should be eliminated relatively quickly.
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> \- WSJ, 1/5/01

[6] Some types of equipment, such as photocopy machines, laboratory equipments, microscopes, etc. have relatively established secondary markets.

The paper proceeds as follows. Section 2 discusses the related work. Section 3 presents the model and derives the pricing equations. Section 4 briefly explains the computational solution, which is detailed in Appendix A. Section 5 explains the quantitative results. Section 6 ties the quantitative results to the data. The paper is concluded in Section 7.

2 Related Work

The main contribution of this paper is to investigate the implications of capital heterogeneity within the firm in the asset pricing context. A somewhat related line of literature is concerned with intangible capital (Hall, 2001; Hansen, Heaton, and Li, 2004; Cummins, 2003; Li, 2004; Atkeson and Kehoe, 2005). Even though the existence and importance of intangible capital is widely agreed upon, interpreting and accounting for intangible capital is inherently difficult. Interpretations of intangible capital range from being a capital input in addition to physical capital to being a form of adjustment cost. Considering the difficulties with interpreting, measuring, and modeling intangible capital, I choose to concentrate on heterogeneity in physical capital.

This paper belongs to the growing strand of papers that studies the interactions between business cycles and asset returns with production economies and investment frictions. Jermann (1998) introduces capital adjustment costs to the standard business cycle model to mitigate the endogenous consumption smoothing mechanism inherent in production economies. Boldrin, Christiano, and Fisher (2001) consider a two sector economy with limited labor mobility that makes the short term supply of capital completely inelastic, limiting the firm's ability to smooth its net cash flows. Both of these papers consider habit formation preferences. Panageas and Yu (2006) also features habits and a two sector economy with limited factor mobility. The paper considers two types of shocks, where the large technological innovations are embodied into new vintages of the capital stock and generates a mechanism that make consumption based asset pricing more successful at lower frequencies. Recently, several papers have worked with these types of economies in an attempt to link stock returns to the book-to-market ratio (Cooper, 2006; Zhang, 2005; Kogan, 2004; Gourio, 2004; Gala, 2006). Their general idea is that firms with high B/M ratios are burdened with excess capital in bad times. Frictions in capital adjustment mechanisms (irreversibilities, costly reversibility) prevent the firms from achieving their desired capital holdings, leading to discrepancies between the market and book values of assets and time varying stock returns. These papers mainly differ along the frictions they assume in capital adjustment mechanisms.

The link between real investment and stock returns is explored by Cochrane (1991, 1996). Cochrane considers a production based asset pricing model with quadratic adjustment costs in which the first order conditions of the producers describe the relationship between asset returns and real investment returns in a partial equilibrium framework. Recently, Jermann (2006) studies the determinants of the equity premium as implied by producers' first order conditions in the presence of a multi-input aggregate production technology. In his model, sectoral investment (structures, equipment) plays the key role; asymmetries across these sectors are crucial in matching the first two moments of the risk free rate and the aggregate equity returns.

The paper is also part of a small but growing literature that incorporates real estate into the asset pricing framework. Even though real estate is an important component of aggregate wealth, it is generally omitted from the empirical and theoretical work in the asset pricing literature. A few notable exceptions include Stambaugh (1982); Kullman (2003); Flavin and Yamashita (2002); Piazzesi, Schneider, and Tuzel (2007); and Lustig and Nieuwerburgh (2006). Stambaugh (1982) constructs market portfolio as a combination of several asset groups, some of which includes proxies for residential real estate in his tests of CAPM. Kullman (2003) includes measures of both residential real estate returns and commercial real estate returns (as measured from REITs) in the market portfolio. Flavin and Yamashita (2002) consider portfolio choice with exogenous returns in the presence of housing. Piazzesi, Schneider, and Tuzel (2007) construct an equilibrium asset pricing model with housing and show that the composition of the consumption bundle appears in the pricing kernel, and matters for asset pricing. The expenditure share of housing predicts stock returns. Lustig and Nieuwerburgh (2006) find that the ratio of housing wealth to human wealth is related to the market price of risk and therefore has asset pricing implications.

3 Setup

The economy is populated with many firms and many infinitely lived identical agents who maximize expected discounted utility. There is a single consumption/investment good that is produced by the firms that use two types of capital. The investment is subject to convex adjustment costs.

3.1 Firms

There are many firms that produce a homogeneous good. The firms use two types of capital: structures and equipment. The firms are subject to different productivity shocks. The investment in either form of capital is subject to convex adjustment costs. Structures depreciate at rate μ and equipment depreciates at rate δ. I assume that structures depreciate more slowly than other capital ($\mu < \delta$), consistent with the depreciation rates in the BEA tables (1-3% for nonresidential structures, 10-30% for nonresidential equipment, annually).

The production function for firm i is given by:

$$\begin{aligned} Y_{it} &= F(A_t, Z_{it}, K_{it}, H_{it}, L_{it}) \\ &= A_t Z_{it} (K_{it}^{\alpha_1} H_{it}^{\alpha_2})^{\alpha} L_{it}^{1-\alpha}. \end{aligned}$$

H_{it} and K_{it} denote the beginning of period t stock of structures and equipment of firm i, respectively, α, α_1 and $\alpha_2 \in (0,1)$. L_{it} denotes the labor used in production by firm i during period t. $a_t = \log(A_t)$ denotes aggregate productivity. At steady-state, productivity A_t grows at rate g. a_t has a stationary and monotone Markov transition function, denoted by $p_a(a_{t+1}|a_t)$, as follows:

$$a_{t+1} = \overline{a} + \rho_a a_t + \sigma_a \varepsilon_{t+1}^a \tag{1}$$

where ε_{t+1}^a is an IID normal shock. The firm productivity, denoted $z_{it} = \log(Z_{it})$, has a stationary and monotone Markov transition function, denoted $p_{z_i}(z_{i,t+1}|z_{it})$, as follows:

$$z_{i,t+1} = \rho_z z_{it} + \sigma_z \varepsilon_{i,t+1}^z \tag{2}$$

where $\varepsilon_{i,t+1}^z$ is IID normal shock. $\varepsilon_{i,t+1}^z$ and $\varepsilon_{j,t+1}^z$ are uncorrelated for any pair (i,j) with $i \neq j$.

The capital accumulation rule is

$$K_{i,t+1} = (1-\delta)K_{it} + I_{it}^k \tag{3}$$
$$H_{i,t+1} = (1-\mu)H_{it} + I_{it}^h$$

where I_{it}^k and I_{it}^h denote investment in equipment and structures, respectively.

The investment is subject to quadratic adjustment costs on gross investment, g_{it}^k and g_{it}^h:

$$g^k\left(I_{it}^k, K_{it}\right) = \frac{1}{2}\eta^k \frac{I_{it}^k}{K_{it}} I_{it}^k \tag{4}$$
$$g^h\left(I_{it}^h, H_{it}\right) = \frac{1}{2}\eta^h \frac{I_{it}^h}{H_{it}} I_{it}^h$$

and

$$\eta^j = \left\{ \begin{array}{c} \eta_{low}^j \text{ if } I_{it}^j > 0 \\ \eta_{high}^j \text{ otherwise} \end{array} \right\}, \; j = h, k.$$

The adjustment costs are allowed to be asymmetric $\left(\eta_{low}^j \leq \eta_{high}^j \,, \, j = h, k\right)$ as in Zhang (2005), Hall (2001), and Abel and Eberly (1996) to capture the intuition that reversing an investment is costlier than expanding the capital stock of the firm. The investor incurs no adjustment cost when gross investment is zero, and disinvestment (negative gross investment) leads to higher adjustment costs than investment (positive gross investment).[7]

Firms are equity financed. The dividend to shareholders is equal to

$$D_{it} = Y_{it} - \left[I_{it}^k + I_{it}^h + g_{it}^k + g_{it}^h\right] - w_{it}L_{it}, \tag{5}$$

where w_{it} is the wage payment to labor services. Labor markets are competitive, so wage payments are determined by the marginal product of labor. Labor is free to move between firms; therefore, the marginal product of labor is equalized among firms.

At each date t, firms choose $\{K_{i,t+1}, H_{i,t+1}\}$ to maximize the net present value of their expected dividend stream,

$$E_t\left[\sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{i,t+k}\right], \tag{6}$$

subject to (Eq.1-4), where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is the marginal rate of substitution of the firm's owners between time t and $t+k$.

The pricing equations that come out of the firm's optimization problem are:

[7] This is different from the adjustment costs used by Jermann (1998), which are on net investment. Jermann's adjustment cost specification would penalize the investor whenever her investment deviates from replacement of depreciated capital (zero net investment) and inaction or disinvestment (zero or negative gross investment) is not allowed. Jermann's model has a representative firm, therefore, investment in his model is aggregate investment. In the data aggregate investment rates are always positive. Therefore, the adjustment cost form used by Jermann does not contradict the empirical observation. In this paper there is a continuum of firms and firms make individual investment decisions. At the firm level disinvestment does not happen too often, but it is not uncommon either. Therefore, I allow disinvestment, and calibrate the model to generate realistic disinvestment frequency.

$$\Lambda_t = \int\int \beta\Lambda_{t+1} \frac{F_{K_{i,t+1}} + (1-\delta)q^k_{i,t+1} + \frac{1}{2}\eta^k \left(\frac{I^k_{i,t+1}}{K_{i,t+1}}\right)^2}{q^k_{it}} \times p_{z_i}(z_{i,t+1}|z_{it})p_a(a_{t+1}|a_t)d_{z_i}d_a \tag{7}$$

$$\Lambda_t = \int\int \beta\Lambda_{t+1} \frac{F_{H_{i,t+1}} + (1-\mu)q^h_{i,t+1} + \frac{1}{2}\eta^h \left(\frac{I^h_{i,t+1}}{H_{i,t+1}}\right)^2}{q^h_{it}} \times p_{z_i}(z_{i,t+1}|z_{it})p_a(a_{t+1}|a_t)d_{z_i}d_a \tag{8}$$

where

$$F_{K_{it}} = F_K(A_t, Z_{it}, K_{it}, H_{it}, L_{it})$$
$$F_{H_{it}} = F_H(A_t, Z_{it}, K_{it}, H_{it}, L_{it})$$

and

$$q^k_{it} = 1 + \eta^k \frac{I^k_{it}}{K_{it}} \tag{9}$$
$$q^h_{it} = 1 + \eta^h \frac{I^h_{it}}{H_{it}}.$$

q^k_{it} and q^h_{it} are Tobin's q, the consumption cost of capital.

Multiplying both sides of pricing equations with $K_{i,t+1}$ and $H_{i,t+1}$, respectively, rearranging, and adding the equations leads to:

$$\begin{aligned} &q^k_{it}K_{i,t+1} + q^h_{it}H_{i,t+1} \\ &= \int\int \frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[\alpha Y_{i,t+1} + (1-\delta)K_{i,t+1}q^k_{i,t+1} + (1-\mu)H_{i,t+1}q^h_{i,t+1} + g^k_{i,t+1} + g^h_{i,t+1}\right] \\ &\qquad \times p_{z_i}(z_{i,t+1}|z_{it})p_a(a_{t+1}|a_t)d_{z_i}d_a. \end{aligned} \tag{10}$$

The (end of period) value of a firm's equity (V_{it}) is equal to the market value of its assets in place:

$$V_{it} = q_{k_it}K_{i,t+1} + q_{h_it}H_{i,t+1}. \tag{11}$$

Replacing equations (11) and (5) in (10) gives the standard Euler equation:

$$1 = \int\int \frac{\beta\Lambda_{t+1}}{\Lambda_t}\frac{V_{i,t+1} + D_{i,t+1}}{V_{it}} p_{z_i}(z_{i,t+1}|z_{it})p_a(a_{t+1}|a_t)d_{z_i}d_a. \tag{12}$$

3.2 Households

The households maximize expected discounted utility. Preferences over consumption take the standard form:

$$E_t\left[\sum_{k=0}^{\infty}\beta^k u(C_{t+k})\right], \text{ with } u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{13}$$

I assume that a complete set of contingent claims are marketed. Risk averse agents trade in claims to their individual labor income so that their intertemporal marginal rate of substitutions are equated. Therefore, the households can be aggregated into a representative agent. The representative agent invests in a one-period riskless discount bond in zero net supply and the risky assets, the equity of firms. At every date t, the representative agent satisfies the following budget constraint:

$$b_{t+1}q_t^{r_f} + \sum_i s_{i,t+1}V_{it} + C_t \leq \sum_i s_{it}(V_{it} + D_{it}) + b_t + \sum_i w_{it} \tag{14}$$

where $b_{t+1}$and $s_{i,t+1}$ denote period t acquisition of riskless bond and risky asset i; and q_t^{rf} and V_{it} denote their prices, respectively. D_{it} denotes period t dividend of the risky asset i as defined in the previous section. At each date t, the agent chooses $b_{t+1}, s_{i,t+1}$ for each firm, and C_t to maximize (13) subject to (14).

The first order conditions for the representative agent's optimization problem are:

$$\begin{aligned} q_t^{r_f} &= E_t\left[\frac{\beta u_C(C_{t+1}, X_{t+1})}{u_C(C_t, X_t)}\right] \\ 1 &= E_t\left[\frac{\beta u_C(C_{t+1}, X_{t+1})}{u_C(C_t, X_t)}\frac{V_{i,t+1} + D_{i,t+1}}{V_{it}}\right]. \end{aligned} \tag{15}$$

3.3 Equilibrium

The state of the economy is characterized by the aggregate productivity a and by the distribution of capital holdings across firms, S. The state variables for any given firm are its own asset holdings (K_i, H_i) and firm productivity, z_i; and the economy wide state (S, a). A competitive equilibrium consists of consumption function $C(S, a)$; investment functions $b'(S, a)$ and $s_i'(K_i, H_i, z_i, S, a)$; policy functions $K_i^{'}(K_i, H_i, z_i, S, a)$ and $H_i^{'}(K_i, H_i, z_i, S, a)$; price functions for installed capital $q_i^k(K_i, H_i, z_i, S, a)$ and $q_i^h(K_i, H_i, z_i, S, a)$; price functions for firms $V_i(K_i, H_i, z_i, S, a)$; and a risk free rate $r^f(S, a)$ that solve the firms' optimization problems (maximize Eq.6 subject to Eq.1-4), solve the representative agent's optimization problem (maximize Eq.13 subject to Eq.14), and satisfy the aggregate resource constraint:

$$\begin{aligned} C(S,a) &+ \sum_i K_i^{'}(K_i, H_i, z_i, S, a) + \sum_i H_i^{'}(K_i, H_i, z_i, S, a) + \sum_i g_i^k(K_i, H_i, z_i, S, a) \\ &+ \sum_i g_i^h(K_i, H_i, z_i, S, a) \leq \sum_i Y_i + (1-\delta)\sum_i K_i + (1-\mu)\sum_i H_i. \end{aligned}$$

4 Computational Solution

The model cannot be solved analytically. I therefore use numerical techniques. The main difficulty in solving the model is in accounting for the distribution of capital holdings across firms. I follow the approximate aggregation idea of Krusell and Smith (1998) and assume that the firms use a small number of moments of the capital distribution when they make their investment decisions. I solve the Euler equations (Eq.7-8) using the parameterized expectations algorithm (PEA) by Marcet (1988). The basic idea in the PEA is to substitute the conditional expectations that appear in the equilibrium conditions with parameterized functions of the state variables. The conditional expectation is parameterized using an exponentiated polynomial, where the exponential guarantees nonnegativity. Once the conditional expectation function is approximated, the policy variables can be expressed as functions of the approximated conditional expectations. The details of the solution are explained in Appendix A.

5 Calibration and Results

I consider asset pricing in a simple production economy with two types of capital (structures and equipment) and adjustment costs. I am particularly interested in whether the *composition* of the capital bundle matters for asset pricing.

The presence of heterogeneous firms in the economy allows me to study the cross sectional implications. The firms receive different productivity shocks, which, over time, lead to heterogeneity between them. Through simulations of the model economy, I show that the productivity shocks effect firms' investment decisions and lead to changes in firms' capital compositions. The composition of the capital bundle determines the *flexibility* of firms to accommodate future productivity shocks. As the share of buildings in the capital bundle of a firm increases, the firm becomes less flexible to accommodate bad shocks in the future, i.e. gets *riskier*. I demonstrate that these riskier firms indeed have higher betas, hence higher expected returns in equilibrium. Futhermore, the riskiness of firms with a high share of buildings is countercyclical; their betas and expected returns go up in recessions.

The model is calibrated to match the key business cycle statistics such as output, consumption and investment volatility. For the parameters that previous empirical studies have guided us with, I use the suggestions of those studies. Following Kydland and Prescott (1982) and Jermann (1998), the annual trend growth rate in the economy, ω, is set to 1.02 and the capital share α is set to 0.36. Similarly, the time discount factor β is set to 0.99 (annually), and the coefficient of relative risk aversion γ is set to 2. The depreciation rates, δ and μ, are set to 0.12 and 0.02 for equipment and buildings, respectively. These are roughly the average BEA depreciation rates for equipment and structures. I set the share of equipment α_1 to 0.6 and the share of buildings α_2 to 0.4. These capital shares approximately yield steady state ratio of 65% for structures in the total fixed capital, consistent with current BEA estimates.

I set the conditional volatility of the aggregate productivity process, σ_a to 0.022 to replicate the US postwar output growth volatility of 2.32%. Consistent with the quarterly parameters used in Cooley and Prescott (1995), the persistence of the aggregate productivity process, ρ_a, is set to 0.8 to replicate the US postwar consumption (nondurables and services) growth volatility of 1.17%.

The persistence and the conditional volatility of the firm productivity process, ρ_z and σ_z, are set to 0.8 and 0.083, respectively. This firm productivity process generates average firm output growth volatility of 24.3%, which is in line with the recent average firm sales growth volatility estimates from Comin and Mulani (2006).[8] The adjustment cost parameters are picked to

[8] Comin and Mulani (2006) estimate firm sales growth volatility as

$$\sigma_{\Delta s_t} = \sqrt{\frac{\sum_{\tau=t-4}^{t+5} \left(\Delta s_t - \overline{\Delta s}_t\right)^2}{10}},$$

generate realistic investment dynamics. In the benchmark case, the adjustment cost parameters for investment, η^k_{low} and η^h_{low} are set to 0.8 and 2.4; the parameters for disinvestment, η^k_{high}, η^h_{high} are set to 4.25 times the parameters for investment. These three parameters are calibrated to match the volatilities of structures and equipment investment growth (6.72% and 7.65%) and the disinvestment frequency of firms (7.1%).

Table 1 summarizes the key parameter values in the model at the annual frequency. Table 2 reports the set of key quantity moments generated using the benchmark parameters. The corresponding moments in the data are also reported for comparison.

Table 1: Model Parameter Values

Parameter	Description	Value
ω	Trend growth rate	1.02
α	Capital share	0.36
β	Discount factor	0.99
γ	Coefficient of relative risk aversion	2
δ	Equipment depreciation rate	0.12
μ	Structures depreciation rate	0.02
α_1	Share of equipment	0.6
α_2	Share of structures	0.4
ρ_a	Persistence of aggregate productivity	0.8
σ_a	Conditional volatility of aggregate productivity	0.022
ρ_z	Persistence of firm productivity	0.8
σ_z	Conditional volatility of firm productivity	0.083
η^h_{low}	Adjustment cost parameter for equipment investment	0.8
η^k_{low}	Adjustment cost parameter for structures investment	2.4
$\eta^{h,k}_{high}/\eta^{h,k}_{low}$	Disinvestment / investment adjustment cost parameter	4.25

Table 2: Calibrated Moments of Quantities (%, annualized)

	Data	Benchmark Model
Volatility of output growth	2.33	2.37
Volatility of consumption growth	1.17	1.20
Volatility of aggregate structures investment	6.72	6.75
Volatility of aggregate equipment investment	7.65	7.65
Average volatility of firm output growth	0.25	0.24
Frequency of negative or zero investment	7.06	7.13

Note: The table reports the volatility of output growth, consumption growth, structures and equipment investment growth, firm output growth and the fraction of disinvestment generated by the model economy using the benchmark parameters from Table 1 and their empirical counterparts.

Figure 1 plots the average Tobin's q (consumption cost of capital) for equipment and structures as functions of the aggregate productivity. Every period, I compute the struc-

and take the average across firms. Their sales data is taken from Compustat over the 1950 and 2002 period. Throughout the sample period, average sales growth volatility rises monotonically to around 25% in 1997.

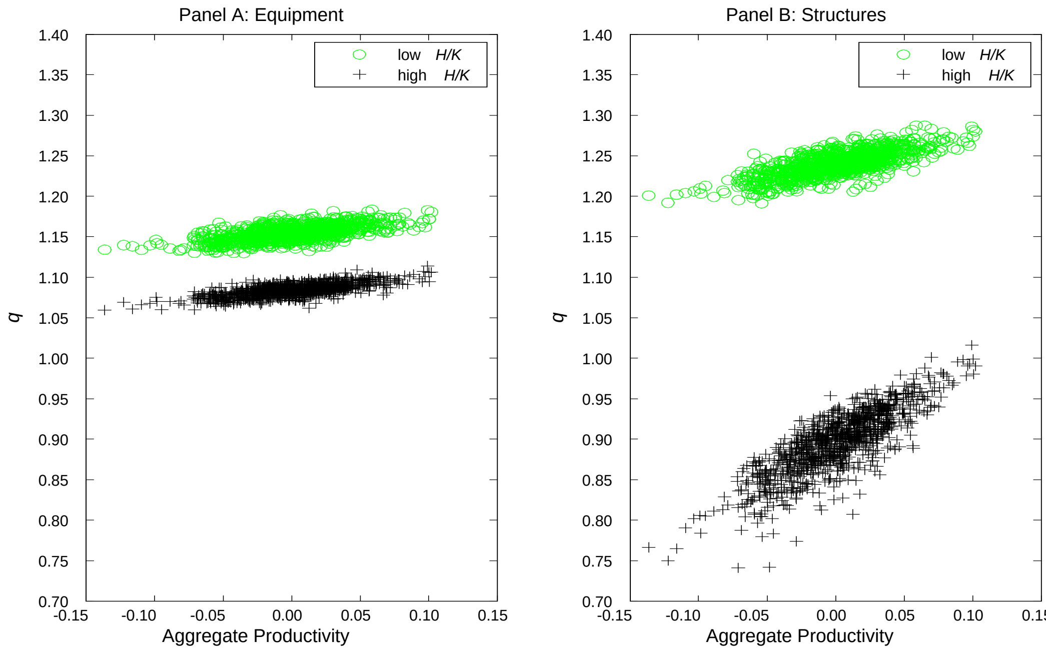

Figure 1: Simulated Tobin's q vs. Aggregate Productivity. Panel A plots the average Tobin's q for equipment as a function of aggregate productivity for low H/K and high H/K firms. Panel B plots the Tobin's q for structures.

tures/equipment (H/K) ratios for firms using quantities of capital and sort firms based on their H/K ratios. Low H/K firms are the firms that are in the lowest quintile with respect to their H/K ratios, and high H/K firms are the firms in the highest H/K quintile. I plot the average Tobin's q for low H/K and high H/K firms in each period with respect to the aggregate productivity in that period.

There is a strong positive relationship between the average consumption cost of capital and aggregate productivity, reflecting the strong link between investment and productivity. Investment (and resale) prices are high during economic booms, and they are depressed during recessions, consistent with the conclusion of Eisfeldt and Rampini (2006) that costs of reallocation are substantially countercyclical. Low H/K firms have a slightly higher consumption cost of equipment than the high H/K firms. The difference between the Tobin's q for structures of low H/K firms and high H/K firms is much bigger, and the gap widens as the aggregate productivity gets lower. The widening gap is due to costly disinvestment of many high H/K firms, which already hold disproportionately high shares of their capital in structures when the aggregate productivity is low. These firms also reduce their equipment investment, but they do not have to disinvest since a considerable portion of their equipment diminishes through depreciation. Therefore, when a firm receives bad productivity shocks, its equipment stock diminishes quickly with little reduction in its stock of buildings, leading to a higher H/K ratio. Few low H/K firms disinvest their structures when they are hit by bad productivity shocks; they have fewer structures compared to equipment to start with.

The main focus of the paper is understanding the link between the firms' capital composition (low H/K versus high H/K) and their risk and stock returns. The rate of return r_i^s on firms and the riskless borrowing rate r^f in the economy are defined as:

$$\begin{aligned} r^s_{i,t+1} &= \log\left(\frac{V_{i,t+1}+D_{i,t+1}}{V_{it}}\right) \\ r^f_t &= -\log q^{rf}_t \\ &= -\log E_t\left[\frac{\beta u_C(C_{t+1})}{u_C(C_t)}\right] \end{aligned}$$

Aggregate stock market return, $r_{vw,t+1}$, is the value-weighted average of the firm returns.

I measure risk with the beta of the firm, where beta is measured with respect to the market portfolio. I.e., the risk of investing in firm i at time t is equal to

$$\beta_{it} = \frac{Cov_t\left(r_{vw,t+1}, r^s_{i,t+1}\right)}{Var_t\left(r_{vw,t+1}\right)}.$$

In this economy the conditional CAPM holds exactly in continuous time and approximately in discrete time. Thus, the cross section of expected excess returns is essentially determined by the distribution of conditional market betas. Since there is a single aggregate shock in the model, return on the market portfolio and the pricing kernel become *instantaneously* perfectly (negatively) correlated (Campbell and Cochrane, 2000; Gomes, Kogan and Zhang, 2003).

Betas are time varying and are computed conditionally. Table 3 presents the model-generated betas of firms sorted on the basis of their H/K ratios for different states of the world. The top panel of Table 3 reports the exact conditional betas from the simulated panels, calculated exactly from the model and but not observable in practice. In practice, betas must be estimated. In order to generate conditional betas that have direct empirical counterparts, I run the following scaled market regression using simulated data:[9]

$$\begin{aligned} r^s_{i,t+1} &= a_0 + (b_0 + b_1 z_t)\, r_{vw,t+1} + \epsilon_{i,t+1} \\ \widehat{\beta}_{it} &= \widehat{b}_0 + \widehat{b}_1 z_t \end{aligned} \tag{16}$$

For conditioning variable z_t, I choose the change in aggregate productivity a_t, which is the state variable that generates the business cycles in this economy.[10] The estimated conditional betas $(\widehat{\beta}_{it})$ implied by the model are reported at the lower panel of Table 3. The results show that the exact and estimated conditional betas are qualitatively and quantitatively similar.

In reporting the exact conditional betas, I classify periods with the highest 10% observations of the aggregate productivity as "peak" periods; the remaining periods with above median productivity as "expansion" periods; periods with below median productivity, except for the lowest 10%, as "contraction" periods; and periods with the lowest 10% productivity as "trough" periods. For the estimated conditional betas, states are defined on the basis of the conditioning variable, the growth in productivity, to maintain consistency with the empirical estimates reported in Section 6.3. Table 3 reports the average conditional betas for the H/K sorted portfolios for these different states.

[9]In the literature betas as commonly modelled as functions of observed macroeconomic variables. (e.g., Shanken, 1990; Ferson and Schadt, 1996; Petkova and Zhang, 2005.)

[10]The empirical counterpart of my conditional variable is the total factor productivity (TFP) growth, which is calculated as Solow residuals. Examining growth rates rather than levels is necessitated by the nonstationarity of TFP.

Table 3: Model Implied Average Conditional Betas

	H/K quintile					
	low	2	3	4	*high*	$5-1$
Exact Betas						
All	0.70	0.80	0.92	1.09	1.49	0.79
Peak	0.75	0.83	0.92	1.06	1.43	0.68
Expansion	0.71	0.81	0.91	1.08	1.49	0.77
Contraction	0.68	0.79	0.92	1.10	1.51	0.83
Trough	0.66	0.78	0.92	1.11	1.53	0.87
Estimated Betas						
All	0.73	0.81	0.87	1.07	1.52	0.79
Peak	0.78	0.82	0.84	1.06	1.50	0.72
Expansion	0.74	0.81	0.86	1.07	1.51	0.77
Contraction	0.71	0.81	0.88	1.08	1.53	0.82
Trough	0.67	0.80	0.90	1.09	1.54	0.87

Note: The table reports the model implied average exact and estimated conditional market betas for the portfolios sorted on H/K. The top panel reports the exact betas calculated inside the model, and the lower panel reports the betas estimated using the scaled beta regressions with simulated data. Averages are reported for all times and for different states of the world. States are defined by sorting on the aggregate productivity (for estimated betas, change in productivity). Periods with the highest 10% observations of the productivity are classified as "peak" periods; the remaining periods with above median productivity as "expansion" periods; periods with below median productivity, except for the lowest 10%, as "contraction" periods; and periods with the lowest 10% productivity as "trough" periods.

Betas of firms in the higher H/K quintiles exceed those of firms in the lower H/K quintiles, confirming that high H/K firms are riskier than low H/K firms. On average, the beta of the highest H/K quintile portfolio is about twice as big as the beta of the lowest H/K quintile portfolio, implying that the expected excess returns of the highest H/K firms are also about twice as high as those of the lowest H/K firms. Furthermore, the difference between the betas of the high H/K and the low H/K portfolios (beta spread) is countercyclical. The beta spread increases from about 0.7 in peak times to 0.87 in troughs. During recessions, firms with a high ratio of structures to equipment (H/K) are hit particularly hard, making them riskier than the firms with low H/K ratios.

The pricing kernel in the model is not volatile enough to generate a high price of risk, therefore, the equity premium and the Sharpe ratio generated by the model are quantitatively low and not reported.[11] The expected returns of firms sorted based on their H/K ratios are

[11] Jermann (1998) generates a sizable equity premium in a general equilibrium production economy with habits in consumption and capital adjustment costs. He shows that the model needs both substantial habits and significant frictions in capital adjustment (elasticity of investment to Tobin's q close to 0.2) to generate the

equal to the conditional betas reported in Table 3, multiplied by the equity premium generated by the model economy.

The conditional betas presented in Table 3 are generated using the parameters in Table 1. The main feature of the adjustment costs considered in this benchmark case is that they are asymmetric; i.e., reversing an investment is costlier than expanding the capital stock. Table 4 presents some key results for alternative adjustment cost parameters, where I change the degree of asymmetry in adjustment costs but keep all other parameters as they are in the benchmark case. The second column ($\eta_{high} / \eta_{low} = 4.25$) presents the investment moments and the beta spread for the benchmark case, which is calibrated to generate the empirical volatilities in structures and equipment investment growth and the frequency of disinvestment observed in the data. The next three cases reduce the asymmetry between the adjustment costs for investment and disinvestment. The final column presents the results for completely symmetric adjustment costs (so that reversing an investment is no more costly than increasing the capital stock). The results show that the estimated beta spread goes up as the asymmetry in adjustment costs increases. The rising asymmetry also leads to lower volatility of investment growth and lower frequency of disinvestment. The beta spread generated by the benchmark model is somewhat higher than the beta spread observed in the data (which is estimated in Section 6.3 using the same estimation procedure and the empirical counterparts of the model generated data) and reported in the first column of Table 4. However, the model generates beta spreads that are similar to the empirical beta spread by lowering the asymmetry in adjustment costs, at the expense of somewhat higher model implied investment growth volatility and frequency of disinvestment.

Table 4: Alternative Adjustment Cost Parameters

Model Implied Moments of Quantities and Excess Stock Returns (%, annualized)

	Data	Model				
		η_{high} / η_{low}				
		4.25	3	2	1.5	1
Volatility of aggregate structures investment	6.72	6.75	6.88	7.03	7.22	7.60
Volatility of aggregate equipment investment	7.65	7.65	8.28	8.54	8.64	8.75
Frequency of negative or zero investment	7.06	7.13	9.57	12.32	14.28	17.16
Estimated beta spread, $\beta_{\text{high } H/K} - \beta_{\text{low } H/K}$	0.16	0.79	0.56	0.30	0.15	0.01

Note: The table reports the volatility of structures and equipment investment growth, fraction of disinvestment, the estimated beta spread generated by the model economy using alternative adjustment cost parameters and their empirical counterparts. The empirical investment moments are from Table 2. The empirical beta estimate is from Table 13, which is the average beta estimate for the $5 - 2$ portfolio.

premium observed in the data. The adjustment cost specification here does not allow an elasticity less than 1. The empirical estimates for the elasticity of investment to q are typically much higher than 0.2. In a recent paper, Groth (2008) estimates an elasticity around 2.4 using data from the U.K.

6 Empirical Results

In this section, I examine the empirical relationship between the composition of firms' physical capital (equipment and the real estate related items that proxy for structures) and stock returns. In the first part, I study the relationship at the firm level. I look at the capital composition of individual firms and try to understand whether there are any cross sectional differences in firm returns with respect to their capital composition. In the second part, I consider the composition of the aggregate capital in the economy. This variable comes out as a state variable in my model economy and therefore is economically meaningful. I use the aggregate H/K ratio as a conditioning variable and try to explain the cross sectional differences in the returns of size and B/M sorted portfolios via the conditional CAPM.

6.1 Data

In order to measure the capital composition of firms, I use data from Compustat. Compustat Industrial Annual provides a breakdown of property, plant, and equipment (PPE) into buildings, capitalized leases, machinery and equipment, natural resources, land and improvements, and construction in progress. Among these items, buildings and capitalized leases are the closest counterparts to "structures" in the model. "Buildings" is the cost of all buildings included in a company's PPE account. "Capital leases" represents the capitalized value of leases and leasehold improvements included in PPE.[12] A capital lease (as opposed to an operating lease) is quite similar to property ownership. In a capital lease, the lessee is exposed to most of the risks and benefits of ownership; therefore, the lease is recognized both as an asset and as a liability on the balance sheet. In addition to buildings and capitalized leases, "construction in progress" and "land and improvements" are also real estate related components of PPE, but they do not have counterparts in the model economy. Construction in progress is not a productive capital for the firm yet, and land cannot be reproduced.[13]

The Compustat data on the composition of the PPE is "net"[14] over 1969-1997 and "historical cost" over 1984-2003. These values are the book values of the assets. In order to make the capital compositions comparable between firms, I calculate a real estate ratio for each firm in every year by dividing the real estate components of PPE that proxy for structures by total PPE. Since neither net nor historical cost series span throughout the whole 1969-2003 period, I use net values until 1984 and switch to historical cost values starting in 1984.[15] My choice of using net versus historical cost values over 1984-1997 is somewhat arbitrary, but the results are insensitive to the choice.

[12] Capitalized leases can be leases of property or equipment. The data does not allow me to distinguish property leases from equipment leases. However, I compared the capital lease data provided by Compustat to the breakdown of PPE in the annual reports for a number of firms and realized that a significant part of the capitalized leases reported by Compustat are leasehold improvements, which are changes to leased property that increase its value. By using data from Census of Manufactures, Eisfeldt and Rampini (2009) find that the fraction of capital leased is much higher for structures than for equipment.

[13] In the model, all capital is reproducible capital, and the consumption goods can be converted into investment goods (capital) in the next period. The model does not have a time to build feature; therefore, there is no construction in progress in the model. Likewise, there is no land in the model because land is not reproducible capital.

[14] "Net" is "at cost" - "accumulated depreciation."

[15] If I have net (gross) real estate holdings in the nominator, I use net (gross) PPE in the denominator.

In the base case, I measure the real estate holdings of the firm as the sum of buildings and capitalized leases. Buildings and capitalized leases are the two biggest real estate related components of PPE; on average, they account for approximately 16% and 10% of PPE, respectively, measured at historical cost over 1984-2003. Construction in progress and land and improvements are much smaller in size; together they account for about 5% of PPE on average. Including construction and land components in real estate holdings does not have a material affect on the results.

The capital composition of firms differ among industries. For example, health service firms (hospitals) and hotels tend to invest heavily in structures, whereas transportation firms tend to hold a lot of equipment. The model assumes that all firms use the same production technology, though in reality the natural composition of capital varies among industries due to their different business needs. In the absence of an industry adjustment, the firms with extremely high or low real estate ratios tend to reflect the characteristics of specific industries, and their returns reflect these industry effects. In order to cancel out industry effects and make firms from different industries comparable, I calculate industry adjusted real estate ratios for firms. Every year, I form industry portfolios using two digit Standard Industrial Classification (SIC) codes and calculate the average real estate ratio within each portfolio. The industry adjusted real estate ratios of firms are the real estate ratios in excess of their industry averages. Throughout the rest of the paper, the "real estate ratio" refers to the "industry adjusted real estate ratio," and it is denoted by RER.[16]

My sample consists of all non-real estate firms with data on buildings and capitalized leases from Compustat Industrial Annual (1969-2003) and stock return data from CRSP (July 1971 - June 2005).[17] Firms that do not have data on assets or net and gross PPE are excluded from the sample. There must be at least 5 firms from each two digit SIC code in order to include firms from that industry in my sample. To ensure that accounting information is already impounded into stock prices, I match CRSP stock return data from July of year t to June of year $t+1$ with accounting information for fiscal year ending in year $t-1$, as in Fama and French (1992, 1993), allowing for a minimum of a six month gap between fiscal year-end and return tests. In other words, I match the RERs calculated using accounting data for fiscal year ending in year $t-1$ to stock returns from July of year t to June of year $t+1$.

In addition to my original sample, I consider two subsamples for empirical tests. In the model presented in this paper, there are no rental markets, i.e., productive assets that are deployed by the firms are owned by the firms. In practice, firms can deploy productive assets through leasing. Accounting rules distinguish between an operating lease and a capital lease,[18] the latter of which is similar to property ownership and is therefore included in firm assets. However, operating leases is a potential concern and might be contaminating the results, hence

[16] Another potentially important concern is that book values do not proxy well for the real quantities of capital firms own. Due to their longevity, book values for buildings are likely to lag book values for equipment and can distort the RER at the firm level. Even though there is no available data for real capital holdings of firms, I construct the real capital holdings for a subperiod when both net and gross book values of PPE components are reported by most firms (1984-1993) using techniques previously used in the macro literature by Hall (1990,1993). I find that the results are fairly insensitive to alternative measurements of the real estate ratio and are qualitatively similar.

[17] I identify real estate firms as firms with two digit SIC code 65. Including these firms in my sample does not change any of the results.

[18] Eisfeldt and Rampini (2009) point out the similarities between the classification of leases for accounting, tax, and legal purposes. Commercial law distinguishes between a "true lease" and a "lease intended as security;" and the tax law distinguishes between a "true lease" and a "conditional sales contract."

I present the results for a subset of firms that do not have a significant amount of operating leases. In this subsample, I normalize the rental expense from Compustat Industrial Annual (which includes only rental payments for operating leases) with the gross PPE and exclude firms that have more than 3% normalized rental expense from the sample.[19] The other subsample that I consider is related to the long term debt holdings of the firms. In my model, all firms are equity financed, there is no debt financing. However, to the extent that buildings are better forms of collateral than equipment, debt financing can be systematically related to the capital composition. In order to isolate the relationship between expected returns and RER while controlling for the leverage of the firm, I perform a double sorting of the firms into RER and long term debt ratio, $LTDR$ portfolios. I calculate a long term debt ratio for firms by dividing their long term debt (LTD) holdings by their total assets calculated as the sum of their LTD and the market value of their equity.

In Section 6.3 I estimate conditional betas for the RER sorted portfolios. My scaling variable is the growth in total factor productivity, which is available quarterly from the Federal Reserve Bank of Cleveland.[20] The calculation of the total factor productivity follows Gomme and Rupert (2007).

6.2 Returns of RER Sorted Portfolios

This section investigates whether a stock's expected return is related to its capital composition, the share of its real estate related items in its total capital. I follow a straightforward portfolio based approach by sorting the firms in my sample every year according to their real estate ratio (RER) and grouping them into quintile portfolios. Table 5 reports the descriptive statistics for RER sorted portfolios. There is significant dispersion in real estate ratios of portfolios. For the firms in the first quintile, the share of real estate-related items in the firms' total physical capital is 22% lower than the average firm in that industry, whereas it is 25% higher for the firms in the fifth quintile. Variation in real estate ratios implies that there is considerable heterogeneity in the capital composition of firms, even within the same industry. The returns of the portfolios are dispersed as well. Table 5 presents the excess returns (firm return - risk free rate) and the industry adjusted returns (firm return - industry return) for RER sorted portfolios. Since the real estate holdings are measured with respect to the firm's industry, I report industry adjusted returns in addition to the standard excess returns. Both excess returns and industry adjusted returns of portfolios increase monotonically with the real estate holdings of firms.

I risk adjust the monthly excess and industry adjusted returns of RER sorted portfolios using a four factor model, where the factors are the three Fama-French factors (MKT, excess market returns; SMB, returns of portfolio that is long in small, short in big firms; HML, returns of portfolio that is long in high B/M, short in low B/M firms) and the momentum factor (MOM, returns of portfolio that is long in short term winners, short in short term losers). The intercepts of the regressions (alphas) represent pricing errors. If the four factor model can account for all the risk in RER sorted portfolios, the alphas should be indistinguishable from zero. Tables 6 and 7 present the alphas and betas of RER sorted portfolios with respect to MKT, SBM, HML, and MOM factors using value and equally weighted portfolios. Betas are estimated by regressing the portfolio excess returns and industry adjusted returns on the

[19] Since this refinement requires data on rental expenses, the firms with missing rental expense data are also excluded from this subsample.

[20] The data and calculations are available at http://clevelandfed.org/research/Models/rbc/Index.cfm .

four factors. The alphas are estimated as intercepts from the regressions of excess portfolio returns and industry adjusted returns on the same factors. Monthly alphas are annualized by multiplying by 12. If real estate risk is priced, risk-adjusted returns (i.e. alphas) should exhibit systematic differences. This is indeed the case in the data. Like the excess returns, risk adjusted returns (alphas) increase monotonically as the RER increases. The value weighted portfolios that are long in high RER portfolios and short in low RER portfolio (5-1) have alphas around 3% over the 1971-2005 period. The equally weighted portfolios produce smaller alphas, which are not statistically significant.

Table 5: Descriptive Statistics for RER Sorted Portfolios (%, annualized)
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
RER	-0.22	-0.09	-0.01	0.07	0.25	0.47
N	435	440	443	442	439	
	Average Excess Returns					
r^e_{VW}	4.64	5.32	5.49	5.30	7.56	2.93
	(1.27)	(1.78)	(1.84)	(1.71)	(2.16)	(1.80)
σ^e_{VW}	21.27	17.40	17.42	18.06	20.47	9.51
r^e_{EW}	11.10	11.19	10.58	10.42	12.37	1.28
	(2.49)	(2.65)	(2.74)	(2.76)	(3.05)	(1.28)
σ^e_{EW}	25.98	24.61	22.52	22.02	23.64	5.82
	Average Industry Adjusted Returns					
r^{IA}_{VW}	-1.15	-0.26	-0.15	-0.11	2.15	3.30
	(-0.95)	(-0.46)	(-0.27)	(-0.21)	(2.76)	(2.28)
σ^{IA}_{VW}	7.09	3.24	3.29	3.11	4.55	8.43
r^{IA}_{EW}	-0.39	0.03	-0.13	-0.49	0.99	1.38
	(-0.50)	(0.07)	(-0.26)	(-0.87)	(1.90)	(1.38)
σ^{IA}_{EW}	4.52	2.76	2.86	3.32	3.04	5.82

Note: The table presents the RER, excess returns and the industry adjusted returns for RER sorted portfolios. For RER, equal-weighted averages are first taken over all firms in that portfolio, then over years. RER is defined as $[(\text{buil.}+\text{cap. leases})\,/PPE]_{firm} - [(\text{buil.} + \text{cap. leases})\,/PPE]_{industry}$. N is the average number of firms in each portfolio. r^e_{VW} is value-weighted monthly average excess returns (excess of risk free rate), r^e_{EW} is equal-weighted monthly average excess returns, annualized; averages are taken over time (%). r^{IA}_{VW} is value-weighted monthly average industry adjusted returns (in excess of industry returns, where industry returns are calculated by value weighting the firms in that industry), r^{IA}_{EW} is equal-weighted monthly average industry adjusted returns (in excess of industry returns, where industry returns are calculated by equally weighting the firms in that industry), annualized; averages are taken over time (%). σ^e_{VW}, σ^e_{EW}, σ^{IA}_{VW}, σ^{IA}_{EW} are the corresponding standard deviations. All returns are measured in the year following the portfolio formation and annualized (%); returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the average returns by their time series standard errors and presented in parentheses.

Table 6: Alphas and Betas of Portfolios Sorted on RER
Dependent Variable: Excess Returns
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
			Value Weighted Portfolios			
$alpha$	-0.64	-0.17	1.37	1.59	2.94	3.58
	(-0.47)	(-0.16)	(1.05)	(1.50)	(2.40)	(2.24)
MKT	1.05	0.98	0.96	0.99	1.05	0.00
	(32.20)	(40.99)	(37.04)	(40.56)	(37.07)	(0.00)
SMB	0.35	0.04	0.04	0.03	0.21	-0.14
	(5.30)	(1.34)	(1.21)	(0.92)	(4.89)	(-1.69)
HML	-0.30	-0.13	-0.13	-0.24	-0.32	-0.01
	(-4.87)	(-3.67)	(-3.31)	(-5.82)	(-6.64)	(-0.20)
MOM	0.01	0.04	-0.07	-0.07	-0.01	-0.03
	(0.28)	(1.53)	(-1.87)	(-2.66)	(-0.47)	(-0.56)
			Equal Weighted Portfolios			
$alpha$	4.92	5.95	4.33	4.08	5.46	0.54
	(2.15)	(2.70)	(3.09)	(2.93)	(3.39)	(0.48)
MKT	1.00	1.01	1.02	1.00	1.00	-0.01
	(24.51)	(27.54)	(31.86)	(32.96)	(30.87)	(-0.32)
SMB	1.23	1.08	0.97	0.94	1.11	-0.11
	(17.19)	(18.98)	(19.93)	(17.98)	(22.09)	(-3.55)
HML	0.02	-0.01	0.15	0.15	0.13	0.11
	(0.21)	(-0.07)	(2.56)	(2.70)	(2.10)	(2.87)
MOM	-0.23	-0.28	-0.25	-0.23	-0.19	0.04
	(-2.72)	(-3.21)	(-5.02)	(-4.46)	(-3.54)	(1.06)

Note: The table presents the regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. The portfolios are sorted on RER. Alphas are annualized (%). Returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the slope coefficient by its time series standard error and presented in parentheses.

Table 7: Alphas and Betas of Portfolios Sorted on RER
Dependent Variable: Industry Adjusted Returns
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
			Value Weighted Portfolios			
alpha	-1.81	-0.17	0.27	0.27	1.19	3.00
	(-1.65)	(-0.30)	(0.45)	(0.49)	(1.53)	(2.23)
MKT	0.04	0.00	-0.02	-0.01	0.04	0.00
	(1.49)	(-0.19)	(-1.52)	(-0.68)	(2.54)	(0.05)
SMB	0.26	-0.01	0.01	-0.04	0.03	-0.22
	(4.71)	(-0.65)	(0.33)	(-2.05)	(1.31)	(-3.42)
HML	-0.09	-0.02	0.01	-0.01	0.00	0.09
	(-1.99)	(-0.79)	(0.32)	(-0.36)	(0.08)	(1.57)
MOM	0.04	0.00	-0.03	-0.02	0.06	0.02
	(1.11)	(0.32)	(-2.04)	(-1.46)	(3.47)	(0.56)
			Equal Weighted Portfolios			
alpha	-0.16	0.57	-0.24	-0.63	0.48	0.64
	(-0.18)	(1.02)	(-0.45)	(-1.07)	(0.91)	(0.55)
MKT	0.00	0.01	0.01	0.00	-0.01	-0.01
	(-0.13)	(0.62)	(1.03)	(-0.14)	(-1.28)	(-0.57)
SMB	0.15	0.01	-0.07	-0.12	0.04	-0.11
	(5.88)	(0.50)	(-4.26)	(-6.68)	(2.31)	(-3.25)
HML	-0.07	-0.05	0.04	0.05	0.03	0.10
	(-2.11)	(-3.72)	(2.11)	(2.39)	(2.03)	(2.47)
MOM	-0.01	-0.03	0.00	0.01	0.03	0.04
	(-0.45)	(-1.70)	(-0.21)	(0.68)	(2.58)	(1.19)

Note: The table presents the regressions of value and equal weighted industry adjusted portfolio returns on FF and momentum factor returns. The portfolios are sorted on RER. Alphas are annualized (%). Returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the slope coefficient by its time series standard error and presented in parentheses.

6.2.1 Refinement: Operating Leases

The general equilibrium model considered in this paper does not allow for the separation of ownership and control of the assets. Productive assets that are deployed by the firms are owned by the firms. In practice, the firms can deploy productive assets through leasing. The finance literature primarily focuses on tax related incentives for leasing. Smith and Wakeman (1985) identify eight non-tax incentives to lease or buy, including asset specificity (assets highly specialized to the firm are generally owned), expected use period compared to asset life (if the expected use period is short compared to the life of the asset, there is an inclination to lease), and comparative advantage in asset disposal (if the lessor has a comparative incentive in disposing of the asset, this provides an incentive to lease). The Financial Accounting Standards Board requires that leases be classified as either capital or operating leases. Appendix B of Statement of Financial Accounting Standards No.13, Basis for Conclusions states that (page 28):

> ... a lease that transfers substantially all of the benefits and risks incident to the ownership of property should be accounted for as the acquisition of an asset and the incurrence of an obligation by the lessee and as a sale or financing by the lessor. All other leases should be accounted for as operating leases. In a lease that transfers substantially all of the benefits and risks of ownership, the economic effect on the parties is similar, in many respects, to that of an installment purchase.

Therefore, capital leases are similar to property ownership, whereas the nature of operating leases can be quite different. It may be argued that firms that inherently need more flexibility in their capital holdings may self select into leasing with relatively flexible terms rather than owning because leased capital may be more easily redeployed than owned capital and hence be more reversible (Eisfeldt and Rampini, 2009).[21] Slovin, Sushka, and Poloncheck (1990) report that firms that engage in sale-leaseback transactions experience positive abnormal returns following their announcement. This result is consistent with the view that leasing provides more flexibility than ownership; therefore, firms that engage in sale-leasebacks effectively become less risky and hence experience a positive price response.[22] Besides this argument, some of the operating leases can still look like ownership in the short run and can be even more inflexible than ownership if the lease contract puts a lot of restrictions on how the asset can de deployed or utilized. Therefore, operating leases is a potential concern and might be contaminating the results.

I present the results for a subset of firms that do not have a significant amount of operating leases. In this subsample, I normalize the rental expense from Compustat Industrial Annual (which includes only rental payments for operating leases) with the gross PPE, and exclude firms that have more than 5% normalized rental expense from the sample. It is not practical to exclude all the firms that have positive rental expense; in the data, almost all firms have some rental expense in a given period. Even excluding firms with more than 5% normalized rental expense leads to more than 60% loss in my sample size, yet I still have on average about 170 firms in each quintile portfolio.[23] Table 8 presents the descriptive statistics on the excess returns and the industry adjusted returns for the RER sorted portfolios of these non-lessee firms. The dispersion in the real estate ratios of these non-lessee firms is quite similar to the dispersion in real estate ratios of all firms. However, the dispersion in both the excess returns and the industry adjusted returns of value and equally weighted portfolios is much bigger. The returns of the high RER firms exceed the returns of the low RER firms (5-1) by about 3-5% per annum, and the difference in returns is statistically significant for all return measures. Most of the change is in the returns of the low RER portfolio: The returns of the non-lessee low RER firms is about 2% lower than the returns of the low RER firms from the whole sample. This can be interpreted as these non-lessee, low RER firms being less risky than the low RER firms

[21] Eisfeldt and Rampini (2009) test this hypothesis by looking at the likelihood of low cash flows for firms; they find that firms with a higher likelihood of low cash flow realizations lease more. I look at the volatility of firm returns and find evidence for the same hypothesis. I find that the sample of firms that do not have a significant amount of operating leases have less volatile returns than the sample of all firms (Table 8 and Table 5), implying that the firms that engage in lots of operating leases have much more volatile returns than the firms that do not have a significant amount of operating leases. In unreported results, I find that the volatility of stock returns is significantly positively related to the normalized rental expense and negatively related to the firm size.

[22] The authors interpret the positive market reaction to sale-leaseback transactions through the traditional finance view of leases as a result of an overall reduction in the present value of expected taxes.

[23] Excluding firms with more than 3% normalized rental expense, rather than 5% reported here, further improves the results. However, lower cutoff leads to a bigger loss in sample size.

that have a lot of operating leases. Similar observations are made for the volatility of returns. The volatility of returns for the non-lessee, high RER firms generally exceed the volatility of low RER firms. The RER sorted portfolios of non-lessee firms typically have lower volatility, where the biggest decreases in volatility are observed for the low RER portfolios.

Table 8: Descriptive Statistics for RER Sorted Portfolios (%, annualized)
Non-lessee Firms
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
RER	-0.20	-0.08	-0.02	0.06	0.24	0.44
N	168	169	169	169	168	
	Average Excess Returns					
r^e_{VW}	2.98	6.24	4.87	3.59	9.14	6.16
	(0.90)	(2.03)	(1.63)	(1.17)	(2.51)	(2.93)
σ^e_{VW}	19.34	17.96	17.44	17.93	21.22	12.26
r^e_{EW}	9.97	10.03	10.72	11.03	13.28	3.31
	(2.48)	(2.88)	(3.09)	(3.12)	(3.42)	(2.60)
σ^e_{EW}	23.40	20.30	20.21	20.61	22.65	7.42
	Average Industry Adjusted Returns					
r^{IA}_{VW}	-2.11	1.13	-0.03	-0.86	3.12	5.24
	(-1.98)	(1.90)	(-0.05)	(-1.62)	(2.91)	(3.41)
σ^{IA}_{VW}	6.21	3.45	4.13	3.10	6.27	8.95
r^{IA}_{EW}	-1.28	-0.69	0.21	-0.39	1.93	3.21
	(-1.64)	(-1.17)	(0.37)	(-0.65)	(2.57)	(2.64)
σ^{IA}_{EW}	4.56	3.45	3.26	3.53	4.39	7.09

Note: The table presents the descriptive statistics of non-lessee firms, sorted on RER. I exclude firms with more than 5% normalized rental expense (rental expense / PPE) from my sample. For RER, equal-weighted averages are first taken over all firms in that portfolio, then over years. RER is defined as $[(\text{buil.+cap. leases})\,/PPE]_{firm}$ $-[(\text{buil.+cap. leases})\,/PPE]_{industry}$. N is the average number of firms in each portfolio. r^e_{VW} is value-weighted monthly average excess returns (excess of risk free rate), r^e_{EW} is equal-weighted monthly average excess returns, annualized; averages are taken over time (%). r^{IA}_{VW} is value-weighted monthly average industry adjusted returns (in excess of industry returns, where industry returns are calculated by value weighting the firms in that industry), r^{IA}_{EW} is equal-weighted monthly average industry adjusted returns (in excess of industry returns, where industry returns are calculated by equally weighting the firms in that industry), annualized; averages are taken over time (%). σ^e_{VW}, σ^e_{EW}, σ^{IA}_{VW}, σ^{IA}_{EW} are the corresponding standard deviations. All returns are measured in the year following the portfolio formation and annualized (%); returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the average returns by their time series standard errors and presented in parentheses.

Tables 9 and 10 present the alphas and betas of RER sorted portfolios of non-lessee firms using the four-factor model. Like the excess returns, risk adjusted returns (alphas) increase monotonically as the RER increases. The portfolios that are formed by going long in the high RER portfolios and short in low RER portfolios (5-1) have alphas around 4-5% over the 1971-2005 period (both value and equally weighted). Excluding the firms that lease heavily from my sample increases the alphas of the 5-1 portfolios by more than 2%.

Excluding firms that lease heavily leads to a major improvement in the results for equally weighted portfolios. The explanation for this improvement comes from the nature of the lessee firms: The firms that lease heavily tend to be smaller firms. Even though their returns do not constitute a big part of the value weighted portfolio returns, they are well represented in the equally weighted portfolios; hence, they contaminate equally weighted portfolio results more than the results for the value weighted portfolios. Therefore, removing heavily leasing firms from the sample leads to a stark improvement in equally weighted portfolio results.

Table 9: Alphas and Betas of Portfolios Sorted on RER, Non-lessee Firms
Dependent Variable: Excess Returns
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
			Value Weighted Portfolios			
$alpha$	-3.34	0.83	1.48	0.31	4.89	8.22
	(-2.66)	(0.52)	(1.08)	(0.22)	(2.85)	(3.97)
MKT	1.03	0.99	0.92	0.95	1.05	0.02
	(35.68)	(30.17)	(29.75)	(33.68)	(24.25)	(0.37)
SMB	0.25	-0.02	-0.05	-0.06	0.13	-0.12
	(5.01)	(-0.52)	(-1.23)	(-1.36)	(1.81)	(-1.13)
HML	-0.12	-0.01	-0.22	-0.23	-0.32	-0.20
	(-2.43)	(-0.25)	(-5.00)	(-4.78)	(-4.47)	(-2.48)
MOM	0.04	-0.02	-0.06	-0.08	-0.03	-0.07
	(1.18)	(-0.54)	(-1.55)	(-2.38)	(-0.62)	(-1.20)
			Equal Weighted Portfolios			
$alpha$	3.94	2.60	3.56	3.83	5.80	1.85
	(1.96)	(2.26)	(3.28)	(3.25)	(3.96)	(1.22)
MKT	1.01	1.00	1.00	1.00	0.99	-0.02
	(29.50)	(33.35)	(37.16)	(36.39)	(32.03)	(-0.79)
SMB	0.98	0.81	0.80	0.84	1.02	0.04
	(18.72)	(14.53)	(18.03)	(17.01)	(22.06)	(1.28)
HML	0.08	0.29	0.27	0.25	0.17	0.09
	(1.12)	(5.80)	(6.13)	(5.65)	(3.09)	(2.02)
MOM	-0.23	-0.17	-0.18	-0.18	-0.14	0.09
	(-2.98)	(-4.80)	(-5.84)	(-5.10)	(-3.28)	(2.00)

Note: The table presents the regressions of value and equal weighted excess portfolio returns of non-lessee firms on FF and momentum factor returns. The portfolios are sorted on RER. Alphas are annualized (%). Returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the slope coefficient by its time series standard error and presented in parentheses.

Table 10: Alphas and Betas of Portfolios Sorted on RER, Non-lessee Firms
Dependent Variable: Industry Adjusted Returns
July 1971 - June 2005

RER quintile	low	2	3	4	high	5-1
	Value Weighted Portfolios					
$alpha$	-3.04	1.01	0.67	-0.60	2.17	5.21
	(-3.11)	(1.50)	(0.89)	(-1.07)	(2.11)	(3.45)
MKT	0.05	-0.01	-0.02	0.00	0.04	0.00
	(2.24)	(-0.45)	(-1.62)	(0.09)	(1.85)	(-0.11)
SMB	0.20	-0.03	-0.01	-0.06	0.11	-0.09
	(5.58)	(-1.93)	(-0.33)	(-2.81)	(3.43)	(-1.69)
HML	-0.01	0.04	-0.03	-0.01	-0.03	-0.02
	(-0.14)	(2.06)	(-1.04)	(-0.59)	(-0.74)	(-0.41)
MOM	0.02	0.00	-0.04	-0.01	0.06	0.04
	(0.89)	(-0.15)	(-1.54)	(-0.39)	(2.83)	(0.89)
	Equal Weighted Portfolios					
$alpha$	-0.75	-0.77	0.28	-0.33	1.36	2.12
	(-0.83)	(-1.32)	(0.47)	(-0.50)	(1.83)	(1.53)
MKT	0.02	0.00	0.00	0.00	-0.02	-0.03
	(1.00)	(-0.35)	(0.18)	(0.10)	(-1.25)	(-1.41)
SMB	0.07	-0.06	-0.07	-0.05	0.11	0.05
	(2.71)	(-2.38)	(-4.60)	(-2.92)	(5.88)	(1.53)
HML	-0.05	0.03	0.00	0.01	0.00	0.05
	(-1.71)	(1.55)	(0.21)	(0.69)	(0.04)	(1.25)
MOM	-0.04	0.00	0.00	0.00	0.04	0.08
	(-1.50)	(0.30)	(0.31)	(-0.20)	(2.86)	(2.23)

Note: The table presents the regressions of value and equal weighted industry adjusted portfolio returns of non-lessee firms on FF and momentum factor returns. The portfolios are sorted on RER. Alphas are annualized (%). Returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the slope coefficient by its time series standard error and presented in parentheses.

6.2.2 Refinement: Leverage

In the equilibrium model considered in this paper, all firms are equity financed. The model overlooks the financing decisions of firms. The firms with lower flexibility, i.e. the high real estate firms, are riskier. However, it is plausible that buildings are better forms of collateral than equipment. Hence, it could be expected that firms with many buildings are more levered than firms with fewer buildings. To the extent that leverage drives expected returns, higher returns of the higher RER quintiles could be explained by their higher leverage, and not by their capital composition.

In order to address this concern, I perform a double-sorting of firms into RER and long term debt ratio, $LTDR$ portfolios. Table 11 reports the descriptive statistics for RER and $LTDR$

sorted portfolios. The firms are simultaneously sorted in both dimensions, and distributed into portfolios. There is significant dispersion in leverage ratios of firms; average $LTDR$ ranges from 2% for the low leverage portfolios to 47% for the high leverage portfolios. Checking the distribution of the firms into portfolios reveal that there are slightly higher number of firms along the "diagonal", however, we end up with a fair number of firms in each portfolio. Table 12 reports the average excess and industry adjusted returns for the portfolios sorted on leverage and RER. Controlling for leverage, average returns typically rise as the RER goes up. For low and high leverage portfolios, the spreads between the highest and lowest RER portfolios are positive and statistically significant. Therefore leverage does not seem to be the driving force behind the spreads between the high and low RER portfolios. Likewise, controlling for RER, average returns tend to rise as the leverage goes up. However, the "medium leverage" portfolios break the return monotonicity in both dimensions. The results for risk adjusted returns (alphas) are qualitatively similar (results are not reported for brevity).

Table 11: Descriptive Statistics for RER and Leverage Sorted Portfolios
(%, annualized)
July 1971 - June 2005

	RER quintile					
Leverage	low	2	3	4	high	
			N			$LTDR$
low	196	159	128	122	127	0.02
medium	125	142	153	158	155	0.14
high	112	137	158	159	155	0.47
RER	-0.22	-0.09	-0.01	0.07	0.25	

Note: The table presents the average number of firms, RER and $LTDR$ for portfolios that are simultaneously sorted on RER and $LTDR$. For RER and $LTDR$, equal-weighted averages are first taken over all firms in that portfolio, then over years. $LTDR$ is defined as $[LTD/(LTD+MV_{equity})]$. RER is defined as $[(\text{buil.+cap. leases})/PPE]_{firm} - [(\text{buil.+cap. leases})/PPE]_{industry}$. N is the average number of firms in each portfolio.

Table 12: Average Returns for RER and Leverage Sorted Portfolios
(%, annualized)
July 1971 - June 2005

Leverage		RER quintile low	2	3	4	high	5-1
		Average Excess Returns					
low	r^e_{VW}	1.40	4.24	5.86	4.68	9.33	7.92
		(0.30)	(0.94)	(1.38)	(1.22)	(2.20)	(2.49)
	σ^e_{VW}	27.10	26.37	24.79	22.28	24.70	18.57
medium	r^e_{VW}	7.61	7.37	5.84	5.68	7.10	-0.51
		(2.17)	(2.59)	(1.89)	(1.77)	(1.99)	(-0.24)
	σ^e_{VW}	20.50	16.62	17.98	18.67	20.84	12.54
high	r^e_{VW}	4.58	5.75	8.26	8.95	10.04	5.46
		(1.23)	(1.63)	(2.39)	(2.47)	(2.93)	(2.53)
	σ^e_{VW}	21.61	20.55	20.15	21.16	19.96	12.59
		Average Industry Adjusted Returns, r^{IA}_{VW}					
low	r^e_{VW}	-3.87	-1.98	0.62	0.03	3.49	7.36
		(-1.87)	(-1.21)	(0.43)	(0.03)	(2.10)	(2.63)
	σ^e_{VW}	12.06	9.59	8.48	7.31	9.70	16.31
medium	r^e_{VW}	1.00	0.61	-0.16	0.11	2.37	1.37
		(0.67)	(0.69)	(-0.18)	(0.13)	(1.77)	(0.73)
	σ^e_{VW}	8.72	5.14	5.01	4.89	7.83	10.93
high	r^e_{VW}	-0.83	0.36	1.52	1.27	2.30	3.13
		(-0.55)	(0.27)	(1.21)	(0.87)	(2.05)	(1.84)
	σ^e_{VW}	8.77	7.81	7.31	8.54	6.54	9.92

Note: The table presents excess returns and the industry adjusted returns for RER and $LTDR$ sorted portfolios. r^e_{VW} is value-weighted monthly average excess returns (excess of risk free rate), annualized; averages are taken over time (%). r^{IA}_{VW} is value-weighted monthly average industry adjusted returns (in excess of industry returns, where industry returns are calculated by value weighting the firms in that industry), annualized; averages are taken over time (%). σ^e_{VW} and σ^{IA}_{VW} are the corresponding standard deviations. All returns are measured in the year following the portfolio formation and annualized (%); returns are measured from July 1971 to June 2005. $t-$statistics are calculated by dividing the average returns by their time series standard errors and presented in parentheses.

Overall, the returns of the firms sorted based on their RER reveals an interesting pattern. I find that the firms with higher RER indeed earn higher returns after adjusting for common risk factors, suggesting that the owners of these firms are compensated for their real estate risk exposure. This empirical result is consistent with the predictions of the model economy, that the firms with a high share of real estate are riskier than the low real estate firms, hence have higher expected returns than that of the lower real estate firms.

6.3 Conditional Betas of RER Sorted Portfolios

The previous section investigated whether a firm's expected return is related to its capital composition by sorting and grouping the firms according to their real estate ratio (RER). Both excess returns and industry adjusted returns of portfolios increase monotonically with the real estate holdings of firms. As suggested by the model economy, I interpret the higher returns of firms with higher real estate holdings as compensation for their real estate risk exposure. In this section, I present more direct empirical evidence for this story by estimating the conditional market betas of RER sorted portfolios using the same procedure as in the model calibration in Section 5. If risk is related to the real estate exposure of firms, the estimated betas should go up as the RER of the firms go up. Furthermore, the model economy suggests that the risk exposure of the higher RER firms is countercyclical; therefore the conditional betas should be higher in contractions compared to expansions.

Table 13 presents the average conditional betas estimated for the RER sorted portfolios using equation 16 from Section 5. The conditioning variable is the empirical counterpart of the scaling variable used with the simulated data, growth in total factor productivity (TFP).[24] Betas are reported for different states of the world. Similar to Section 5, I define states of the world based on the conditioning variable. I classify periods with the highest 10% observations of the TFP growth as "peak" periods; the remaining periods with above median productivity growth as "expansion" periods; periods with below median productivity growth, except for the lowest 10%, as "contraction" periods; and periods with the lowest 10% TFP growth as "trough" periods. I report the average conditional betas for the RER sorted portfolios for these different states.

Table 13: Average Conditional Betas for RER Sorted Portfolios

July 1971 - June 2005

	RER quintile						
	low	2	3	4	*high*	$5-1$	$5-2$
All Firms							
All States	1.21	1.02	1.01	1.07	1.18	-0.02	0.16
Peak	1.20	1.03	1.00	1.07	1.16	-0.04	0.13
Expansion	1.20	1.02	1.01	1.07	1.18	-0.03	0.15
Contraction	1.21	1.02	1.02	1.07	1.19	-0.02	1.17
Trough	1.21	1.00	1.03	1.06	1.21	0.00	0.21
Non-lessee Firms							
All States	1.11	0.99	0.99	1.02	1.16	0.05	0.17
Peak	1.11	0.99	1.01	1.05	1.10	-0.02	0.11
Expansion	1.11	0.99	0.99	1.03	1.14	0.03	0.15
Contraction	1.11	1.00	0.98	1.01	1.18	0.07	0.19
Trough	1.10	1.00	0.96	0.98	1.24	0.14	0.24

[24] I demean the conditioning variable in my empirical analysis. In unreported results, I find similar conditional betas using alternative scaling variables (such as the predictive variables studied by Goyal and Welch (2008)) that did not necessarily have counterparts in the model economy.

Note: The table reports the empirical average conditional market betas for the portfolios sorted on RER.The top panel reports the betas for portfolios formed by sorting all firms in the sample, and the lower panel reports the betas for portfolios formed by sorting only non-lessee firms as defined in Section 6.2.1. Averages are reported for all times and for different states of the world. States are defined by sorting on the growth in TFP. Periods with the highest 10% observations of the productivity growth are classified as "peak" periods; the remaining periods with above median productivity growth as "expansion" periods; periods with below median productivity growth, except for the lowest 10%, as "contraction" periods; and periods with the lowest 10% TFP growth as "trough" periods. Returns are measured from July 1971 to June 2005.

I find that the betas of firms in the higher RER quintiles exceed those of firms in the lower RER quintiles, except for the firms in the lowest RER quintile. Excluding the firms in the lowest RER quintile, the betas typically go up monotonically with RER and attain their highest values for the highest RER quintile. The betas of the lowest RER quintile pose less of an anomaly when the sample of firms is restricted to the non-lessee firms. For this sample of firms that do not lease heavily, even though the betas of firms in the lowest RER quintile are somewhat higher than the betas in the subsequent quintiles, they are much lower in magnitude (compared to the beta estimates for the same quintile for the sample of all firms) and considerably lower than the betas of firms with the highest real estate ratios. Because of the issues raised in Section 6.2.1 regarding the nature of firms in the lowest RER quintile, it would be wise to interpret the beta estimates for this group of firms with some caution. Therefore, in addition to the typical $5-1$ portfolio, I also report beta estimates for the $5-2$ portfolio. The average beta for the $5-2$ portfolio is 0.16 and it is highly statistically significant with a $t-$statistic of 5.27. I interpret the higher beta estimates for the higher RER portfolios as an empirical confirmation that higher RER firms are indeed riskier than the lower RER firms. Furthermore, the betas of both the $5-1$ and the $5-2$ portfolios are countercyclical, which are caused by the countercyclical movements in the betas of the highest RER portfolio. The average conditional beta of the $5-2$ portfolio increases from 0.13 to 0.21 (from 0.11 to 0.24 for the non-lessee firms) as the economy moves from the peak times to the troughs.[25] During recessions, firms with a high ratio of real estate in their capital are hit particularly hard, making them riskier than the firms with low RERs.

[25] Defining the state of the economy with NBER business cycle dates produce similar results. The average conditional beta for the $5-2$ portfolio is 0.16 during NBER expansions and 0.19 during NBER contractions (0.16 and 0.20 for the non-lessee firms). The disadvantage of using the NBER dates is that NBER only classifies dates as expansion or contraction, which does not allow more than two states of the world.

7 Conclusion

I introduce a general equilibrium production model where the firms use two factors, real estate capital and other capital, and investment is subject to asymmetric adjustment costs, where cutting the capital stock is costlier than expanding it. Slow depreciation of real estate capital makes real estate investment riskier than investment in other capital. Due to costly reversibility, the firm will find it difficult to reduce its real estate holdings when it would like to do so, whereas the other capital depreciates and hence decreases faster. Therefore, recessions hurt firms with high real estate holdings particularly badly. In equilibrium, investors demand a premium to hold these firms. This prediction is also empirically supported. Using a portfolio based approach, I find that the returns of firms with a high share of real estate capital compared to other firms in the same industry exceed that for low real estate firms by 3-6% annually, adjusted for exposures to the market return, size, value, and momentum factors. Moreover, conditional beta estimates reveal that high real estate firms indeed have higher market betas, and the spread between the betas of high and low real estate firms is countercyclical.

The capital breakdown I consider is limited to real estate and other physical capital. Nevertheless, this breakdown excludes at least one big class of capital, which is referred to as intangible or organizational capital. Accounting for intangible capital is inherently difficult; there is neither a consensus on how intangible capital is defined nor on how it is measured. Its definition involves a variety of concepts, such as organizational culture, copyrights, and research and development. Further research integrating intangible capital into the firm's capital composition would provide a more realistic and comprehensive view of the firm than what this simple model portrays.

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Appendix A: Computational Solution

I solve the Euler equations (Eq.7-8) using a modified version of the parameterized expectations algorithm (PEA) by Marcet (1988). In order to use the PEA, the system should be "invertible", i.e., once the parameterized expectation is substituted in the equilibrium condition, one should be able to construct the policy function. The Euler equations that will be solved (Eq.7-8) are not "invertible" in the sense that the policy functions, $[K_i', H_i']$, cannot be retrieved by substituting the parameterized expectations in the right hand side. A value for C can be found from any one of the many Euler equations, but there is no way to compute individual policy functions. In order to make the system invertible, following Marcet and Lorenzoni (1998), I slightly modify the Euler equations by premultiplying both sides of the Euler equations by K_i' and H_i', respectively. Since capital levels are never zero in equilibrium, the new equations are satisfied if and only if the original Euler equations are satisfied. The modified Euler equations are as follows, where the primes denote next period's values[26]:

$$u_C K_i' = \int\int \beta u_{C'} \frac{F_{K_i'} + (1-\delta)q_i^{k\prime} + \frac{1}{2}\eta^k \left(\frac{I_i^{k\prime}}{K_i'}\right)^2}{q_i^{k\prime}} K_i' p_{z_i}(z_i'|z_i) p_a(a'|a) d_{z_i} d_a$$

$$u_C H_i' = \int\int \beta u_{C'} \frac{F_{H_i'} + (1-\mu)q_i^{h\prime} + \frac{1}{2}\eta^h \left(\frac{I_i^{h\prime}}{H_i'}\right)^2}{q_i^{h\prime}} H_i' p_{z_i}(z_i'|z_i) p_a(a'|a) d_{z_i} d_a$$

where $u_{C'} = u_C(C')$, $F_{K_i'} = F_K(K_i')$, $F_{H_i'} = F_H(H_i')$.

The solution is a function $\widehat{e}_\psi(K_i, H_i, z_i, S, a)$, with a finite set of parameters, ψ :

$$\exp\left(\widehat{e}_{\psi i}\right) \approx \left| \begin{array}{c} \int\int \beta u_{C'} \frac{F_{K_i'} + (1-\delta)q_i^{k\prime} + \frac{1}{2}\eta^k \left(\frac{I_i^{k\prime}}{K_i'}\right)^2}{q_i^{k\prime}} K_i' p_{z_i}(z_i'|z_i) p_a(a'|a) d_{z_i} d_a \\ \int\int \beta u_{C'} \frac{F_{H_i'} + (1-\mu)q_i^{h\prime} + \frac{1}{2}\eta^h \left(\frac{I_i^{h\prime}}{H_i'}\right)^2}{q_i^{h\prime}} H_i' p_{z_i}(z_i'|z_i) p_a(a'|a) d_{z_i} d_a \end{array} \right|' . \tag{17}$$

The relations linking the policy functions, $\hat{s}_\psi'(K_i, H_i, z_i, S, a) = [\widehat{K}_{i_\psi}', \widehat{H}_{i_\psi}']$ and the consumption function, $\widehat{C}_\psi$ to $\widehat{e}_\psi(K_i, H_i, z_i, S, a)$ are as follows:

$$\hat{s}_\psi' = \exp\left(\widehat{e}_{i_\psi}\right) / u_{\widehat{C}_\psi}$$

$$\widehat{I}_{i_\psi}^k = \widehat{K}_{i_\psi}' - (1-\delta)K_i$$
$$\widehat{I}_{i_\psi}^h = \widehat{H}_{i_\psi}' - (1-\mu)H_i$$

$$\widehat{g}_{i_\psi}^k = \frac{1}{2}\eta^k \frac{\widehat{I}_{i_\psi}^k}{K_i} \widehat{I}_{i_\psi}^k$$
$$\widehat{g}_{i_\psi}^h = \frac{1}{2}\eta^h \frac{\widehat{I}_{i_\psi}^h}{H_i} \widehat{I}_{i_\psi}^h$$

[26] When the time subscript is dropped, primes denote next period's values.

$$\widehat{C}_{\psi} = \sum_i \left(Y_i + K_i + H_i - \widehat{K}'_{i_\psi} - \widehat{H}'_{i_\psi} - \widehat{g}^k_{i_\psi} - \widehat{g}^h_{i_\psi} \right)$$

The procedure to estimate the parameter set, ψ, starts with generating series for a and $z_i, i = 1, ..., n$, of length T, where n is the number of firms in the economy, and coming up with an initial guess for ψ, ψ_0. The standard PEA algorithm (Marcet, 1988, Den Haan and Marcet, 1990) has two steps. In the first step, the economy is simulated using the series for productivity, initial capital levels, and initial guess, ψ_0, and $\{C_t(\psi_0), K_{it}(\psi_0), H_{it}(\psi_0), i = 1, ..., n\}_{t=1}^{T}$ is generated (simulation step). In the second step, a non-linear least squares regression of $\left\{ \beta u_{C_t} \frac{F_{K_{it}} + (1-\delta) q^k_{it} + \frac{1}{2}\eta^k \left(\frac{I^k_{it}}{K_{it}} \right)^2}{q^k_{it}} K_{it} \right\}_{t=1}^{T}$ and $\left\{ \beta u_{C_t} \frac{F_{H_{it}} + (1-\mu) q^h_{it} + \frac{1}{2}\eta^h \left(\frac{I^h_{it}}{H_{it}} \right)^2}{q^h_{it}} H_{it} \right\}_{t=1}^{T}$ is run on the power function of e and ψ_1 is estimated (regression step). If ψ_1 is sufficiently close to ψ_0, the program stops; otherwise, ψ_0 is updated and the two steps are repeated. I make several changes in the implemantation of the algorithm. The standard PEA concentrates observations on points of high probability. Marcet and Marimon (1992) observed that greater dispersion in the state variables is desirable while studying the far-from-steady-state properties of a model. Christiano and Fisher (2000) suggest that this observation still applies when the object of interest is the steady state properties of the distribution, since high variance in explanatory variables implies greater precision in regression estimates. Marcet and Marimon (1992) overcome this problem by exogenously oversampling from the tails of the state variable distribution in their simulations. Christiano and Fisher (2000) suggest an alternative to the standard PEA, the Chebyshev PEA. Chebyshev PEA considers a fixed distribution of state variables in the simulation step, which is more widely dispersed than the ones in the standard PEA. I modify the standard algorithm in the following way: In order to ensure that there is sufficient dispersion in aggregate productivity, I find $N_a \times 5$ periods from the simulated productivity series a that minimizes $\frac{(a - \overline{a}_j)^2}{\overline{a}_j^2}$, where $\overline{a} = \exp([-2\sigma_a \quad -\sigma_a \quad 0 \quad \sigma_a \quad 2\sigma_a])$. After simulating the economy for T periods as in the first step explained above, I evaluate the conditional expectations in Eq. 17 at these $N_a \times 5$ periods using Monte Carlo simulation[27]. Next, in order to ensure sufficient dispersion in firm level productivity, I find $N_z \times 5$ firms in each of the $N_a \times 5$ periods that minimizes $\frac{(z_i - \overline{z}_j)^2}{\overline{z}_j^2}$, where $\overline{z} = \exp([-2\sigma_{z_i} \quad -\sigma_{z_i} \quad 0 \quad \sigma_{z_i} \quad 2\sigma_{z_i}])$. So, I end up with $N_a \times N_z \times 5 \times 5$ [time, firm] pairs that I will use in the regression stage.[28] Finally, I run a non-linear least squares regression of the conditional expectations in Eq. 17 evaluated for these $N_a \times N_z \times 5 \times 5$ [time, firm] pairs on the power function of $\widehat{e}_{\psi}(K_i, H_i, z_i, S, a)$.

The functional form I use for e is a second order polynomial of the aggregate productivity, first two moments of the capital distribution and firm level state variables (firm's capital holdings and productivity).

[27] Unlike the model presented in Christiano and Fisher (2000), the distribution of capital over firms is not trivial in this model. Therefore, I still have to simulate the economy for a long time to get a reasonable distribution of capital, even though I do not evaluate the conditional expectations at every period.

[28] Like the simulated series for a and z_i, I fix these [time, firm] pairs once and use the same set during each iteration.