Uncertainty, Delegation and Incentives

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Abstract

How does imperfect contractibility of preferences influence the governance of a contractual relationship? We analyze a two-party decision-making problem where the optimal decision is unknown at the time of contracting. In consequence, instead of contracting on the decision directly, the parties need to design a contract that will induce good decision-making in the future. We examine how environmental uncertainty, quality of available performance measures and interim access to information influence the joint determination of the allocation of authority, use of performance pay and direct controls. We use the results from the model to cast light on (i) the conflicting empirical evidence on the risk-incentives tradeoff found in work on executive compensation and franchising, (ii) complementarities in organizational design and (iii) the determinants of the choice to delegate.

Keywords: decision-making, delegation, uncertainty, incentives

JEL codes: D82, J33, M52

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1 Introduction

Many contractual relationship have two features in common. First, at the time of contracting, the parties face uncertainty over the future decisions that will maximize the value of the relationship. Second, additional information about the consequences of different actions is learned during the execution of the contract. As a result, instead of attempting to specify the complete plan ex ante or relying on constant renegotiation, many contracts simply allocate authority over different decisions among the participants, and then use contractual controls to guide the exercising of that authority. The canonical example of such contracts is the employment relationship, but most contracts exhibit some degree of open-endedness with respect to the exact actions that will be taken during the execution of the contract.

This paper examines what factors determine the allocation of authority in such settings and how the contract can be used to induce appropriate use of that authority. We construct a model where a single decision that influences the payoffs of two parties needs to be made, but the optimal decision is unknown at the time of contracting. While additional information is learned by both parties before the decision needs to be made, the opportunities for renegotiation are limited. As a result, the parties need to design a contract, consisting of (i) an allocation of the decision right and (ii) a compensation package (incentive contract), that maximizes the expected value of the relationship.

We assume that the preferences of the two parties are not directly contractible but there exist performance measures that (imperfectly) track those preferences. However, those performance measures can be misaligned in a way that the decisions that maximize the expected value of the performance measures can be different from the decisions that would maximize total surplus. This misalignment implies that perfect preference alignment is impossible even if the parties are risk-neutral and have deep pockets (Baker, 1992). Instead, whenever the performance measures are used, the contract needs to account for the risk of gaming of the performance measures to the detriment of overall surplus, or "The Folly of Rewarding for A While Hoping for B" (Steven Kerr, 1975). Some extreme examples of such gaming include teachers handing out test questions beforehand when compensated based on students' performance in standardized tests, workers in an archaeological dig breaking down bones when compensated based on the number of bone fragments found and Sears auto mechanics performing unnecessary repairs to meet their monthly quotas.

In addition to the performance measures, we assume that the contract can also specify a default decision and any penalties that the decision-maker needs to pay to (or receive from) the other party when deviating from this default decision. This component of the contract can be interpreted either narrowly (as a formal adjustment clause) or broadly (as the degree of job flexibility grated to the decision-maker). We first derive the optimal contract, consisting of (i) the identity of the decision-maker, (ii) the weights placed on available performance measures, (iii) the default decision and (iv) the penalty scheme, under the assumption of perfect interim information and risk-neutrality. Second, we will discuss briefly how the results change when we introduce asymmetric information (specific knowledge), information acquisition and risk-aversion into the model.

We can break the results into (i) the characteristics of the contract conditional on the identity of the decision-maker (employment versus delegation) and (ii) the choice to delegate. Conditional on the identity of the decision-maker, the main characteristics of the optimal performance contract are as follows. First, when the decision-maker is risk-neutral, the incentive strength is increasing in the volatility of the environment and decreasing in the noisiness of the performance measures. Second, incentive strength and job flexibility are complements. Third, if the decision-maker is risk-averse, then the relationship between volatility and incentive strength can be non-monotone. The intuition behind these results is as follows. The decision-maker can be controlled either through performance pay (incentives) or direct controls. The value of incentives derives from their ability to induce the agent to respond to information that he otherwise would not respond to. The value of such responsiveness is increasing in the volatility of the environment, leading to a positive relationship between incentive strength and volatility. Relatedly, job flexibility is increasing in volatility to further support such responsiveness. Conversely, when the performance measure becomes noisier, it becomes less useful in guiding the agent's behavior and, as a result, relatively more weight will be placed on direct controls. Finally, risk-aversion provides an additional cost of providing incentives, which counters the increasing productive value of incentives as the volatility of the environment goes up.

The second set of results illustrates when we should observe delegation. In short, the decision-maker should be the party who makes relatively better decisions, given the optimal performance contract. In other words, the decision-maker should be the party (i) whose payoff is more sensitive to the decision, (ii) who faces a more volatile environment, (iii) whose preferences are harder to contract on and (iv) who has better information about the realized states of the world. These results thus qualify the common intuition that increased uncertainty about the environment leads to increased delegation of authority. In particular, increased volatility faced by the principal will lead to *less* delegation unless, first, the increased volatility is accompanied by an increasing informational advantage held by the agent and, second, sufficiently accurate performance measures exist that can be used to guide the behavior of the agent.[1]

These predictions of the model regarding the co-movements of incentive strength, job flexibility and delegation with the volatility of the environment, the quality of information and the quality of performance measures are broadly consistent with a number of empirical regularities found in different literatures. To illustrate, consider the empirical literature examining the risk-incentives tradeoff. The work on executive compensation has found ambiguous results, with some studies finding positive, some negative and still others an insignificant relationship between risk and incentives (as summarized in Prendergast, 2002). At the same time, the literature on franchising has found a consistently *positive* relationship between risk and the choice to franchise (as summarized in Lafontaine and Slade, 1997). While neither the literature on executive compensation nor the literature on franchising has found consistent evidence of a negative trade-off between risk and incentives, as predicted by classic models of moral hazard, they are also in apparent conflict with each other. However, by making the distinction between the determination of incentives conditional on delegation (compensation of CEOs) and the change in compensation structure following the decision to delegate (decision to franchise), the apparent conflict disappears, while continuing to provide an explanation for the lack of evidence for a negative tradeoff between risk and incentives.[2]

The remainder of the paper is organized as follows. Section 2 reviews the related theoretical literature. Section 3 describes the model. Section 4 presents the solution

[1] While in many situations, such as franchising, we would expect environmental volatility and the amount of specific knowledge to be highly correlated, moving the focus from volatility to specific knowledge (as emphasized by Jensen and Meckling, 1992) also helps to explain why in some cases increased uncertainty leads the principal to retain *more* control, such as the case of offshore drilling analyzed in Corts and Singh (2004).

[2] This approach complements the more common view of franchising as a means of providing high-powered incentives. Here, franchising is associated with delegation of authority and high-powered incentives are the consequence of the increased value of incentive alignment following an increase in the discretion granted to the agent.

under full information and risk-neutrality, and outlines the impact of asymmetric information, active information acquisition and risk-aversion. Section 5 reviews the related empirical evidence and section 6 concludes.

2 Related literature

The present model is closely related to two strands of literature: the literature on hidden action and the literature on authority and delegation. The link to hidden action follows from the observation that while the model is framed in terms of decision-making, the incentive provision problem itself is isomorphic to a hidden action model with uncertainty over both costs and benefits of effort. In consequence, the results of the model with respect to the strength of incentives are related to a number of papers analyzing the typical effort-provision problem.

In particular, random productivity of effort has been forwarded as an explanation for the ambiguous relationship between risk and incentives by a number of authors. The key distinction between volatility and noise has been made in the case of risk-aversion by Zabojnik (1996) and Baker and Jorgensen (2003), and in the case of risk-neutrality but limited liability by Raith (2004). While in our model the noise is related to misaligned performance measures and so to Baker (1992), the results obtained regarding the determination of incentives (conditional on delegation) are directly analogous to these papers. However, this strand of literature has paid less attention to other controls that might be available through job design and the allocation of authority, even if some attention has been given to action-contingent transfers, in particular in Raith (2004). Considerations of job design can be found in, for example, Holmström and Milgrom (1991), but the role of environmental volatility is not analyzed.

While models of hidden action have not dealt with matters of authority and delegation in any extensive way, the literature on authority and delegation has, in turn, paid only limited attention to the role of incentive contracts, instead typically assuming an exogenously given degree of incentive conflict. Our main contribution to this literature is the endogenization of the degree of incentive conflict through the use of incentive contracts and thereby identifying imperfect contractibility of preferences as an additional determinant for the allocation of authority.

In our analysis of the choice to delegate, the logic of the model resembles most closely Prendergast (2002). The direct overlap of the underlying forces is, however, limited to the role of specific knowledge. In our model, the relative sensitivity of the payoffs to the decision, the volatility of the environment and the level of noise in the performance measures provide independent determinants for the allocation of authority. Also, we do not need to make the assumption of exogenous cost of measuring inputs and outputs (where measuring inputs is cheaper than measuring outputs) that is made in Prendergast (2002). Further, as discussed above, the optimal contract can exhibit significant heterogeneity even conditional on the identity of the decision-maker, a feature not present in Prendergast (2002). In related papers, Bester (2002, 2005) also analyzes the allocation of authority over project choice in various settings.

A number of other papers are more distantly related. First, Aghion and Tirole (1997) show how delegation improves the incentives of the agent to acquire information. The model, however, is silent on monetary compensation and assumes an asymmetric and exogenous degree of congruence between the preferences of the agent and the principal. Our model shows how the accuracy of available performance measures can provide microfoundations for such asymmetric degree of congruence. Further, the incentive effect of delegation is somewhat more nuanced in our setting. In particular, the model shows that when performance measurement is noisy, monetary incentives (even when available) cannot always be used as a substitute for the motivating role of delegation.

Second, Dessein (2002), Alonso, Dessein and Matouschek (2008) and Rantakari (2008a), among others, analyze the allocation of authority when the decision-making problem is linked with problems of strategic communication, but again rely on exogenous incentive alignment. Rantakari (2008b) relaxes the assumption of exogenous incentive alignment and analyzes the joint problem of generating, communicating and using information. In contrast, to focus on the contracting problem in a simple way, the present paper abstracts away from issues of communication and takes the information structure as given. Finally, Zabojnik (2002) links the decision problem with an implementation problem to analyze how the need to rely on the agent for implementation effort impacts the optimal allocation of authority.

3 The Model

We consider a setting where two parties enter a contractual relationship which involves making a decision d, optimal choice of which is uncertain at the time of contracting. Our goal is to study the joint determination of the allocation of the decision right, the degree of incentive provision and any potential constraints put on the decision itself, given the underlying environment. While the model is fully symmetric, in our discussion we will frame it as a principal-agent relationship.

Preferences: The preferences of the principal and the agent are given by

$$U_P = K_P - \gamma(\theta_P - d)^2 \qquad \text{and} \qquad U_A = K_A - (\theta_A - d)^2,$$

$$\text{where} \begin{pmatrix} \theta_P \\ \theta_A \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_P \\ \mu_A \end{pmatrix}, \begin{pmatrix} \sigma_P^2 & \sigma_{PA} \\ \sigma_{PA} & \sigma_A^2 \end{pmatrix} \right).$$

The relative importance of the decision to the principal is measured by γ, with $\gamma \in (0, \infty)$. K_P and K_A measure the maximum returns (or minimum costs) to the principal and the agent, respectively, and θ_P and θ_A index the decision that would realize these maximum private returns. We will refer to θ_P and θ_A as the states of the principal and the agent. At the time of contracting, the distributions of the states are common knowledge, but their exact realizations are not.

The preferences reflect the nature of the underlying relationship and the related payoffs the parties would receive absent any performance- or decision-contingent transfers. Generally, one can view the principal's preferences to reflect the value generated while the agent's preferences reflect the cost incurred. For example, in a procurement setting, U_P would reflect the realized value of the project to the principal, with θ_P indexing the value-maximizing decision, while U_A would reflect the realized cost of the project to the agent, with θ_A indexing the cost-minimizing decision. In a franchising setting, U_P would reflect the value generated by the new outlet while U_A would reflect the costs incurred by the franchisee in managing that outlet, with θ_P and θ_A again indexing value-maximizing and cost-minimizing decisions, respectively. Finally, it is worth noting that while we frame the problem in terms of decision-making, the setting is isomorphic to a standard effort provision problem, with the exception that both the value and cost of effort are uncertain at the time of contracting. The principal receives the benefits of effort while the agent bears the cost of effort, absent additional transfers.

Performance measures: We assume that the preferences of the agent and the principal are not directly contractible. However, there exist two (imperfect) performance measures, given by

$$X_P(s_P, d) = -(s_P - d)^2 + \varepsilon_{X_P} \qquad \text{and} \qquad X_A(s_A, d) = -(s_A - d)^2 + \varepsilon_{X_A},$$

where $s_i = \theta_i + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma^2_{\varepsilon_i})$. The key behind the analysis is the presence of the shock ε_i, which implies that the realized s_i is never perfectly aligned with θ_i. The primary consequence of this ex post misalignment is that decisions that maximize the value of the performance measures are different from the decisions that would maximize joint surplus.

One can view the performance measures to reflect the easy-to-quantify part of the parties preferences. For example, in the franchising setting, X_P could reflect the revenue generated by the outlet while X_A could reflect the monetary costs incurred by the franchisee, with s_P and s_A reflecting the revenue-maximizing and cost-minimizing decisions, respectively. As a result, s_P (and so X_P) is missing the impact of the decision on other outlets and the overall value of the franchise, while s_A (and so X_A) is missing the non-monetary costs incurred by the franchisee when operating the outlet.[3]

Departing from most of the existing literature, we assume that, in addition to the output-based measures, decision-contingent transfers can be made based on

$$X_d(\overline{d}, d) = -\left(\overline{d} - d\right)^2,$$

where $\overline{d}$ is a default decision to be determined by the contract. While the direct interpretation of this component is formal decision-contingent transfers, we prefer the broader interpretation of job flexibility granted to the decision-maker. The basic idea is that job design can be used to make some actions more costly than others.

Given the contractible variables, the decision-maker is then offered a linear contract of the form

[3]Note the conceptual distinction between s_i and X_i. If s_i is the project that maximizes the revenue stream, X_i is the revenue stream that is actually realized. Thus, we assume that X_i is always verifiable, while s_i might not be.

$$T(d) = \sum_i \alpha_i X_i + B.$$

As we will show in section 4, given the structure of s_i and the possibility of making transfers based on $\left|\overline{d} - d\right|$, this linear contract is optimal among all incentive contracts of the form $T(s_A, s_P, d, \overline{d})$, which makes the question of whether s_i are verifiable irrelevant for this analysis. When decision-contingent transfers are not available (or they involve waste), linear incentives will no longer be optimal and the verifiability of s_i starts to matter. However, with the exception of the role of correlation of preferences between the agent and the principal, the comparative statics remain qualitatively similar.[4]

Timing and information: The timing of events is standard for this strand of literature. At $t = 0$, the principal offers a contract that consists of a default decision $\overline{d}$, weights $\boldsymbol{\alpha}$ placed on the performance measures and decision-contingent transfers, a fixed transfer B and the identity of the decision-maker $g \in \{P, A\}$. We assume that both parties have deep pockets and so the optimization problem becomes one of maximizing the ex ante total surplus. Substituting out B, we can write this optimization problem as

$$\min_{g,\boldsymbol{\alpha},\overline{d}} E\left(\gamma(\theta_P - d^*)^2 + (\theta_A - d^*)^2\right)$$

$$\text{s.t.} \qquad d^* \in \arg\max_d E\left(U_g(.) + T_g(.)\,|\boldsymbol{\omega}_g\right),$$

where $\boldsymbol{\omega}$ indexes the information held by the decision-maker in the decision-making stage. At $t = 1$, information about $(\theta_A, s_A, \theta_P, s_P)$ is realized. In our main analysis we will focus on the case where both parties learn all information but assume that renegotiation at this stage is not possible. At $t = 2$, the decision is made by the actor controlling the decision right, as induced by the incentive contract and the realization of $(\theta_A, s_A, \theta_P, s_P)$.

[4] These extensions are briefly discussed in Appendix A.3

3.1 Underlying assumptions

The analysis relies on two key assumptions. First, the parties cannot design ex ante a mechanism that would specify both message-contingent decisions and transfers. If that were possible, the first-best decisions could be trivially achieved and the allocation of authority, together with performance measurement, would become irrelevant. We make two practical arguments for ruling out such mechanisms. First, achieving enforcement of such contracts could require disclosure of information that the parties prefer to keep within the relationship. Second, the contract could be so complex that while the parties to the contract can understand it, the courts would make mistakes in enforcing it. For example, assume that the while the agent and the principal would send messages θ_A and θ_P as induced by the mechanism, the implementer would receive $s_i = \theta_i + \varepsilon_i$ and make the decision based on (s_A, s_P). Such a mechanism would be dominated by decision-making by both the agent and the principal under the optimal incentive contract.[5]

Second, we assume that while decision-contingent transfers are available, the parties are unable to sign a contract on a decision after the private information is realized.[6] There are three partial resolutions to this conflict. First, the opportunities can be "fleeting," so that there is simply not enough time to write the contract before the opportunity is gone. Second, the decision problem can be repeated, so that the one-time design of the incentive contract joined with decision authority is significantly cheaper than repeated formal contracting. This idea is related to the simple observation that even under symmetric information, costless Nash bargaining is unlikely to take place. In practice, such negotiations cost both time and money as the parties are trying to capture a larger share of the available surplus. Third, if s_i are verifiable, it is possible, under a set of (admittedly restrictive) assumptions, to use X_i together with s_i to generate contracts that behave as if the agent were compensated based on $\left|d - \overline{d}\right|$, even if decisions are not directly verifiable.

[5] Message-contingent transfers alone are irrelevant (as all information is readily available) and message-contingent allocations of the decision right itself could help only if that allocation could be based on $|\theta_A - s_A|$ and $|\theta_P - s_P|$, which again brings us back to the complications of actually implementing such a contract.

[6] Note that from a practical perspective, the ability to make decision-contingent transfers need not conflict with the inability to design a full mechanism ex ante. As argued, verifying that any given decision was made (needed for decision-contingent transfers) is often an order of magnitude easier than verifying whether the "right" decision was made (needed for a first-best solution).

Finally, we don't want to push the concept of decision-contingent transfers too far. The primary reason for introducing them is to capture in a simple way some of the ideas related to job design without needing to deal with non-linear contracts (while admittedly allowing some internal inconsistency). The comparative statics with respect to the rest of the components remain qualitatively similar if we rule out decision-contingent transfers, while the comparative statics with respect to use of these decision-contingent transfers are qualitatively similar to the behavior of delegation sets (analyzed in, for example, Holmström, 1984 and Alonso and Matouschek, 2006) and the set of allowed outside activities (Holmström and Milgrom, 1991), two alternative methods of introducing job design to the decision problem.

4 Analysis

We begin our analysis by discussing the optimal contract and show how it can be implemented in the present environment with the simple linear contract discussed above. We then present the solution to the optimal contract under the assumption of zero correlation of preferences, as this is sufficient to understanding the key determinants in the relationship between volatility of the environment and the strength of incentives. Finally, we discuss what additional insights can be gained from the general solution and from extensions to imperfect information, information acquisition and risk-aversion. The analysis of active information acquisition, which is relatively involved, is available as a separate note from the author. A more detailed exposition of the other extensions is presented in Appendix A.

4.1 Optimal Contracts

The key to understanding the structure of the optimal incentive contract is to recognize that the purpose of the contract is to control the responsiveness of the decision-maker to information about $(\theta_A, s_A, \theta_P, s_P)$. If the preferences of the two parties were directly contractible, then the first-best decision would be given by

$$\min_d \gamma\left(\theta_P - d\right)^2 + \left(\theta_A - d\right)^2 \rightarrow d^{FB} = \tfrac{\gamma\theta_P + \theta_A}{1+\gamma}.$$

The agency problem in the model is generated by the fact that θ_A and θ_P are not contractible. Instead, while the decision-maker i will fully internalize his own state θ_i, he can be induced to care only about the realization of s_j, which, because of the presence of ε_j, will never exactly match the preferences of the non-deciding party θ_j.

Consider the case where the agent controls the decision right. Now, s_A is clearly redundant since the agent already knows and fully internalizes θ_A. Similarly, since θ_P is non-contractible, the agent can be induced to care only about s_P. Since s_P is perfectly observed by the agent, θ_P is redundant as well from his or her perspective. Thus, the only information that can be brought to bear on the agent's decision is (θ_A, s_P). Consider now the following problem: given information about (θ_A, s_P), what decision will maximize the expected total surplus? That is, given the information that can be used to bear on the decision, what is the optimal way to use that information. This second-best decision is given by

$$d^{SB} = \frac{\gamma E(\theta_P | s_P, \theta_A) + \theta_A}{1+\gamma}.$$

By Bayesian updating, $E(\theta_P | s_P, \theta_A) = \beta_0 \mu_P + \beta_1 s_P + \beta_2 \theta_A$, where the coefficients are determined by the informativeness of (s_P, θ_A). Thus, the second-best decision will be linear in the information of the agent:

$$d^{SB} = \delta_0 \mu_P + \delta_1 s_P + \delta_2 \theta_A.$$

We next show that an appropriate contract can induce this second-best decision. Given the linear contract $T(.) = B - \alpha_P (s_P - d)^2 - \alpha_A (s_A - d)^2 - \alpha_D \left(\overline{d} - d\right)^2$ and individual utility $U(.) = K_A - (\theta_A - d)^2$, the agent's first-order condition can be written as

$$d^A = \eta_0 \overline{d} + \eta_1 s_P + \eta_2 s_A + \eta_3 \theta_A.$$

Since s_A contains no additional information (over θ_A) about θ_P, $\eta_2 = 0$. The solution of $\left(\alpha_P, \alpha_D, \overline{d}\right)$ then follows from a matching of the coefficients to *induce the agent to respond to incoming information as a decision-maker who makes optimal decisions conditional on information only about* (s_P, θ_A). Such a solution exists and thus the linear form of the contract is optimal.[7]

[7] A corollary of this result, to which we alluded in section 3.1, is that the linear delegation contract

4.2 Solution under zero correlation

4.2.1 Delegation

Consider first the case where the correlation ρ between θ_A and θ_P is zero. Since θ_A contains no information about θ_P,

$$E\left(\theta_P|s_P,\theta_A\right)=\frac{\sigma_{\varepsilon_P}^2}{\sigma_P^2+\sigma_{\varepsilon_P}^2}\mu_P+\frac{\sigma_P^2}{\sigma_P^2+\sigma_{\varepsilon_P}^2}s_P,$$

which implies that the second-best decision will solve

$$d^{SB}=\frac{1}{1+\gamma}\left(\gamma E\left(\theta_P|s_P,\theta_A\right)+\theta_A\right)=\frac{1}{1+\gamma}\left(\gamma\left(\frac{\sigma_{\varepsilon_P}^2}{\sigma_P^2+\sigma_{\varepsilon_P}^2}\mu_P+\frac{\sigma_P^2}{\sigma_P^2+\sigma_{\varepsilon_P}^2}s_P\right)+\theta_A\right).$$

At the same time, the agent's decision for any $\left(\alpha_P,\alpha_D,\overline{d}\right)$ is given by

$$d^A=\frac{\alpha_P s_P+\alpha_D\overline{d}+\theta_A}{1+\alpha_P+\alpha_D}.$$

Matching the coefficients yields the following proposition:

Proposition 1 *The optimal incentive contract under delegation and $\rho=0$ is given by*

$$\alpha_P=\frac{\sigma_P^2\gamma}{\sigma_P^2+\sigma_{\varepsilon_P}^2},\qquad \alpha_d=\frac{\sigma_{\varepsilon_P}^2\gamma}{\sigma_P^2+\sigma_{\varepsilon_P}^2}\qquad \textit{and}\qquad \overline{d}=\mu_P.$$

Proof. Special case of proposition 4 ■

The strength of incentives is given by α_P. We can see that the higher the volatility of the principal's state, the more important it is that the agent's decisions track closely the principal's preferences. Similarly, the noisier the performance measure, the less closely the principal's state can be tracked and to avoid gaming of the contract by the agent (the "get what you pay for" or GWYPF problem), the less weight is placed on the performance measure. In consequence, the overall incentive strength is driven

based on (s_P,θ_A) always dominates an ex ante contract based on (s_P,s_A), even if such a contingent ex ante contract was feasible.

by the volatility-to-noise ratio $\sigma_P^2/\sigma_{\varepsilon_P}^2$, with an adjustment for the difference in the relative importance of the decision, γ.

The cost of deviating from the default decision $\overline{d}$ (and so the inverse of job flexibility) is given by α_d. It is also driven by the volatility-to-noise ratio but exactly in the opposite direction, with constraints increasing in $\sigma_{\varepsilon_P}^2$ and decreasing in σ_P^2. The intuition is straightforward. The agent can be controlled either through indirect incentives or direct controls. Other things constant, when the volatility of the environment increases, incentive alignment becomes more valuable as it induces the agent to make more use of his information. This effect is then complemented through reducing direct controls to increase the flexibility of the agent. In consequence, incentive strength and job flexibility are complements, as in Holmström and Milgrom (1991, 1994). Similarly, they both decrease in the noisiness of the performance measure. However, unlike Holmström and Milgrom (1991, 1994) which are silent on the matter, they also co-move *positively* with the volatility of the environment. Further, the intuition behind the co-movement is somewhat different, with the co-movement in Holmström and Milgrom (1991, 1994) arising from multi-tasking problems and here from the value of flexibility in responding to uncertain events.

Finally, the default decision $\overline{d}$ is biased in favor of the principal. The intuition for this result is that since $\alpha_P < \gamma$, the agent is always relatively less responsive to information about the principal's preferences. To compensate for this decisional bias favoring the agent, the default decision is biased in favor of the principal. Note also that the optimal incentive strength and degree of job flexibility are independent of σ_A^2, the volatility of the agent's preferred decision. This result follows because the agent already fully internalizes any variance in his own needs. It is only the preferences of the principal that the agent needs to be induced to internalize.

4.2.2 Employment

The employment solution follows symmetrically and is given by the following proposition:

Proposition 2 *The optimal incentive contract under employment and $\rho = 0$ is given by*

$$\alpha_A = \frac{\sigma_A^2}{\sigma_A^2+\sigma_{\varepsilon_A}^2}, \qquad \alpha_d = \frac{\sigma_{\varepsilon_A}^2}{\sigma_A^2+\sigma_{\varepsilon_A}^2} \qquad \text{and} \qquad \overline{d} = \mu_A.$$

Proof. Special case of proposition 4 ■

The symmetry of the solution follows immediately from the observation that the employer is no more benevolent than the employee. Just like the employee, given his wage, would like to work as little as possible and choose projects requiring minimal input by him, the employer, given the decision right over some aspect of the employee's behavior, needs to be controlled against abusing the employee.

4.2.3 Choice of Contract

Substituting the equilibrium decision in the total expected loss and comparing the solutions under employment and delegation yields the following proposition:

Proposition 3 *The equilibrium expected losses under (D)elegation and (E)mployment are given by*

$$EL^D = EL^{FB} + \frac{\gamma^2}{(1+\gamma)}\frac{\sigma_P^2\sigma_{\varepsilon_P}^2}{\left(\sigma_P^2+\sigma_{\varepsilon_P}^2\right)} \qquad \text{and} \qquad EL^E = EL^{FB} + \frac{1}{(1+\gamma)}\frac{\sigma_A^2\sigma_{\varepsilon_A}^2}{\left(\sigma_A^2+\sigma_{\varepsilon_A}^2\right)}.$$

Thus, delegation is preferred iff $\gamma^2 \leq \frac{\sigma_A^2\sigma_{\varepsilon_A}^2\left[\sigma_P^2+\sigma_{\varepsilon_P}^2\right]}{\sigma_P^2\sigma_{\varepsilon_P}^2\left[\sigma_A^2+\sigma_{\varepsilon_A}^2\right]}$.

That is, the likelihood of delegation is increasing in $\sigma_{\varepsilon_A}^2, \sigma_A^2$ *and decreasing in* $\sigma_P^2, \sigma_{\varepsilon_P}^2$ *and* γ.

Proof. Special case of proposition 4 ■

As discussed in section 4.1, the purpose of the incentive contract is to induce best possible use of the information that the decision-maker can be induced to use. The quality of resulting decisions, in turn, depends on the volatility-to-noise ratio $\sigma_i^2/\sigma_{\varepsilon_i}^2$ of the state that the decision-maker is not directly internalizing, with an adjustment for the relative sensitivity of the payoff of each party to the decision. In consequence, delegation becomes more likely whenever the agent's preferences become harder to contract on $\left(\sigma^2_{\varepsilon_A} \uparrow\right)$ or the volatility his state increases $(\sigma^2_A \uparrow)$, while the converse happens when either the principal's preferences become harder to contract on or the principal's state becomes more volatile. Further, delegation becomes less likely when the principal's payoff becomes more sensitive to the decision (γ).

To summarize, increased volatility of the principal's payoff increases the incentive strength and job flexibility granted to the agent conditional on delegation, but it also makes delegation *less* likely. In consequence, the results so far would appear to go partially against the accumulating empirical evidence that delegation is associated with greater uncertainty. However, what the results so far highlight is that uncertainty alone is not sufficient in supporting delegation, an example of which is given below.

An example: The basic logic of the analysis so far is well illustrated with offshore drilling contracts, discussed in Corts and Singh (2004). Having acquired a lease on an offshore tract, oil and gas exploration and production (E&P) companies contract with independent drilling companies to drill wells on the tract. The wells come in two forms: exploratory and development wells. Exploratory wells are used to gauge the type and value of fuels present within a tract. Development wells are used to extract these reserves as efficiently as possible. The contracts governing the relationship also come primarily in two forms: day-rate and turnkey contracts, corresponding roughly with cost-plus and fixed-price contracts, respectively. The regularities of interest for the present analysis are that (i) day-rate contracts are more prominent in governing the drilling of development wells, (ii) day-rate contracts are associated with an E&P representative being present on the rig during the drilling and retaining a number of decision rights, while (iii) no representative is present under turnkey contracts and control of the project is retained by the drilling company.

The intuition for this pattern follows from the observation that while exploratory wells exhibit greater financial uncertainty as to the profitability of the tract, the drilling itself is fairly standard. In contrast, development wells exhibit significant uncertainty at the time of contracting *over the best way of extracting the resources.* In terms of the coefficients of the model, $\sigma^2_P(development) > \sigma^2_P(exploratory)$ even if $\sigma^2_{\varepsilon_{XP}}(development) > \sigma^2_{\varepsilon_{XP}}(exploratory)$. As discussed above, it is exactly high σ^2_P that leads the principal to retain control. Since the principal retains control, he needs to be made to internalize the costs of adjustment incurred by the drilling company (use of cost-plus terms). Further, the presence of the E&P representative on the rig can be interpreted as an attempt to limit informational asymmetries. Conversely, when σ_P^2 is low, a fixed-price contract is used to incentivize the drilling company to manage costs better, together with discretion over such decisions.

4.3 General Solution

The qualitative nature of the solution is remains unchanged when we allow for the preferences of the agent and the principal to be correlated. The results are summarized in the following proposition.

Proposition 4 *The parameters of the optimal incentive contract under delegation are given by*

$$\alpha_P = \frac{\gamma\left(1-\rho^2\right)\sigma_A^2\sigma_P^2}{(1-\rho^2)\sigma_A^2\sigma_P^2+\sigma_{\varepsilon_P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)},\ \ \alpha_d = \frac{\sigma_{\varepsilon_P}^2\gamma\left(\sigma_A^2-\rho\sigma_A\sigma_P\right)}{(1-\rho^2)\sigma_A^2\sigma_P^2+\sigma_{\varepsilon_P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)},\ \ \overline{d} = \frac{\mu_P\sigma_A-\mu_A\rho\sigma_P}{\sigma_A-\rho\sigma_P},$$

while the parameters of the optimal incentive contract under employment are given by

$$\alpha_A = \frac{\gamma\left(1-\rho^2\right)\sigma_A^2\sigma_P^2}{\gamma(1-\rho^2)\sigma_A^2\sigma_P^2+\sigma_{\varepsilon_A}^2\left(\gamma\sigma_P^2+\rho\sigma_A\sigma_P\right)},\ \ \alpha_d = \frac{\gamma\sigma_{\varepsilon_A}^2\left(\sigma_P^2-\rho\sigma_A\sigma_P\right)}{\gamma(1-\rho^2)\sigma_A^2\sigma_P^2+\sigma_{\varepsilon_A}^2\left(\gamma\sigma_P^2+\rho\sigma_A\sigma_P\right)},\ \ \overline{d} = \frac{\mu_A\sigma_P-\mu_P\rho\sigma_A}{\sigma_P-\rho\sigma_A}.$$

The resulting expected losses are then given by

$$EL^E = EL^{FB} + \frac{1}{(1+\gamma)}\frac{\sigma_A^2\sigma_{\varepsilon_A}^2(1-\rho^2)}{\left(\sigma_A^2(1-\rho^2)+\sigma_{\varepsilon_A}^2\right)} \qquad \text{and} \qquad EL^D = EL^{FB} + \frac{\gamma^2}{(1+\gamma)}\frac{\sigma_P^2\sigma_{\varepsilon_P}^2\left(1-\rho^2\right)}{\left(\sigma_P^2(1-\rho^2)+\sigma_{\varepsilon_P}^2\right)}.$$

Thus, delegation is preferred iff $\gamma^2 \leq \frac{\sigma_A^2\sigma_{\varepsilon_A}^2\left[\sigma_P^2\left(1-\rho^2\right)+\sigma_{\varepsilon_P}^2\right]}{\sigma_P^2\sigma_{\varepsilon_P}^2\left[\sigma_A^2(1-\rho^2)+\sigma_{\varepsilon_A}^2\right]}$.

That is, the likelihood of delegation is increasing in $\sigma_{\varepsilon_A}^2, \sigma_A^2$ *and decreasing in* $\sigma_P^2, \sigma_{\varepsilon_P}^2$ *and* γ. *The role of* ρ *depends on the variance/noise ratio. In particular, if* $\sigma_A^2/\sigma_{\varepsilon_A}^2 > \sigma_P^2/\sigma_{\varepsilon_P}^2$, *then the likelihood of delegation is increasing in* $|\rho|$.

Proof. See Appendix B ■

The primary impact of correlation (positive or negative) is to improve the quality of decision-making under both employment and delegation contracts and cause a shift away from the use of performance measures and towards direct controls. The reason for this substitution is that when $\rho \neq 0$, it becomes possible to use decision-contingent transfers to induce decision-maker i to use information about θ_j contained in θ_i, instead of relying on the noisy performance measure. When $|\rho| \to 1$, θ_i becomes perfectly informative about θ_j. As a result, first-best decisions can be induced purely through job design and pay-for-performance will converge to zero. Conversely, the need for pay-for-performance tends to be highest when $|\rho|$ is small, since in this case direct controls are least efficient in guiding the behavior of the agent. Thus, the equilibrium strength of incentives will be non-monotone in ρ. For the same reason, if ρ is large enough, it is possible that the decision-contingent transfers are negative and the decision-maker is actually paid to move away from the default decision.

It might appear counterintuitive that the strength of incentives can be decreasing in the expected size of the ex post incentive conflict. However, the intuition follows from the observation that if the parties know that ex post they are likely to be in opposing corners, this information can be used to guide decision-making directly, without needing to rely on the use of performance measures. It is only when the size of the conflict is uncertain that performance measures are most valuable. The fact that this non-monotonicity is related to decision-contingent transfers resonates well with the idea of learning to contract (Mayer and Argyres, 2004, for example). As the parties learn their environment better, direct controls can be used to substitute for pay-for-performance, independent of whether learning leads to $E(\rho) \downarrow$ or $E(\rho) \uparrow$. Even if the expected conflict increases, knowledge of the regularities involved in the conflict help to design the initial contract better. On the other hand, the parties might also learn to construct more accurate performance measures, which would imply that $\sigma^2_{\varepsilon_i} \downarrow$, in turn implying heavier use of better-constructed performance measures.

This ability to substitute away from performance measures also explains the limited role of ρ in determining the choice between delegation and employment. As $|\rho| \to 1$, both contracts achieve first-best. When $|\rho| < 1$, increases in $|\rho|$ improve relatively more the contract that would otherwise induce statistically worse decisions.

4.4 Extensions

The analysis so far has assumed that both the principal and the agent become fully informed before the decision needs to be made. However, in many situations an informational asymmetry develops between the agent and the principal during the execution of the relationship. Store managers have often better direct access to local information than the headquarters. An R&D lab becomes better informed about the viability of various research alternatives when the project is on its way. Similarly, while some information can become readily available over time, it is often also actively acquired. In this subsection, we will review some of the results that arise when we relax the assumptions of perfect and free information and risk-neutrality.

4.4.1 Incomplete and asymmetric information

In Appendix A.1, we show how the development of an informational asymmetry between the principal and the agent leads to more delegation. This result should come as no surprise since the ultimate purpose of the contract is to induce good decision-making. Whether decision-making is compromised because of imperfect information or imperfect performance measures makes very little difference in terms of the expected total loss. However, while the worsening of the quality of information held by the principal increases the likelihood of delegation, keeping the informational asymmetry constant, an increase in the volatility faced by the principal still decreases the likelihood of delegation. Thus, for an increase in uncertainty faced by the principal to lead to delegation, the informational asymmetry needs to be increasing in the volatility of the environment. In consequence, the correlation between volatility and delegation can sometimes be positive (the choice to franchise, for example) and sometimes negative (the choice of offshore drilling contracts, for example).

Finally, while informational asymmetries impact the allocation of authority, it has only a limited impact on the contractual terms conditional on the allocation of authority. This result follows because the parties naturally discount information that is imperfect and it is only the *relative* sensitivity to information that needs to be controlled. Thus, while imperfect information has some impact on the level of incentives, that impact depends on the particular information structure. In particular, worse information need not imply weaker incentives.

4.4.2 Information acquisition

While some information can become readily available over time, information is often also actively acquired. Consider a setting where information about the agent's preferences remains free but the parties need to undertake privately costly effort to learn about the principal's preferences. This setting provides an alternative explanation for the correlation between uncertainty and delegation: if the agent is in a better position to acquire information, then increased uncertainty can increase the probability of delegation even if all information learned by the agent can be perfectly transmitted to the principal before decision-making.

As uncertainty increases, the importance of utilizing the agent to acquire information becomes more valuable. As in Aghion and Tirole (1997), delegation of the decision right can be used to motivate the agent. Further, the results suggest why the motivational role of delegation can be hard to replicate with purely monetary incentives. First, absent any pay-for-performance, the agent places a negative value on any information about the principal's preferences. This result follows because any additional information will in expectation restrict the agent's ability to have his way. In consequence, to induce information acquisition by the agent, not only does pay-for-performance need to be positive, it needs to be sufficiently strong as to overcome this inherent negative value. Second, even if the agent can be motivated to acquire information through sufficiently strong monetary incentives, such compensation in turn generates poor decision-making. The reason for this is that the principal now has strong incentives to game the compensation structure and to make decisions that will limit the amount he has to pay to the agent. In short, providing the agent with incentives to acquire information simultaneously generates incentives for the principal to make poor decisions. In consequence, delegation arises as a motivational tool that is qualitatively different from (and which can be superior to) monetary compensation.

4.4.3 Risk aversion

Appendix A.2 analyzes the determination of incentive strength when the agent is risk-averse. The results show that while an increase in the noise in the performance measure leads to the familiar negative trade-off between noise and incentives (as in the case of risk-neutral agent analyzed above), the relationship between volatility and incentives is now inherently non-monotone and dependent on the amount of noise. In particular, the noisier the performance measure, the lower the incentive strength and the higher the likelihood that an increase in the volatility of the environment will lead to an increase in incentives. The intuition behind this result is straightforward. When the volatility of the environment increases, there are now two opposing forces. First, the value of responsiveness increases, as above, supporting stronger incentives. Second, risk-aversion makes incentives more costly to provide, supporting weaker incentives. Depending on the parameters, either one can dominate and the relationship is inherently ambiguous. Further, if the performance measure is sufficiently noisy (and in consequence the level of incentives is low), then the 'value' aspect of incentives will dominate the 'risk' aspect of incentives at the margin and the predicted relationship between volatility and incentives is positive.

We have not examined the impact of risk aversion on the actual choice to delegate, but the results appear ex ante ambiguous because of two opposing forces. First, given an incentive structure, delegation can be used to allow the agent to have more control over his environment, thus supporting a positive relationship between risk-aversion and delegation. Second, not delegating can limit the need for performance pay in the first place, thus limiting the amount of risk faced by the agent and so supporting a negative relationship between risk-aversion and delegation. Casual empiricism would suggest that the latter effect often dominates.

5 Empirical Observations

The logic and the results of the model shed light on a number of empirical findings. We will first discuss empirical work on managerial incentives, revisiting the "tenuous trade-off between risk and incentives" and discussing some of the recent advances that have either made the distinction between volatility and noise or examined the co-determination of different variables. Second, we will discuss some empirical regularities found in the literature on franchising. In the case of franchising, while the discussed regularities (with the exception of the positive relationship between risk and incentives) can be explained also with a hidden action framework, the additional contribution of our model is in providing an explicit role for delegation.

The lessons from this review are two-fold. First, a number of empirical regularities can be fruitfully viewed under the common lense of managing (mal)adaptation problems. Second, the behavior of agents is typically managed through multiple control instruments. Failure to recognize and control for the simultaneous determination of these instruments runs the risk of false associations and conclusions. We hope that our explicit categorization of various control instruments (delegation, job flexibility and incentives) and environmental features (volatility of the environment, informational asymmetries, the quality of performance measures and the level of non-controllable risk) will help to guide future empirical work and further our understanding of the determination of different incentive systems.

5.1 Inside Firms - Managerial incentives and delegation

As reviewed in Prendergast (2002), the empirical evidence on the trade-off between risk and incentives in executive compensation is mixed, with some studies finding a significantly positive, some studies finding a significantly negative, and still some studies finding an insignificant relationship between risk and incentives, where risk is typically measured by some transformation of the variance of stock returns. Given that the allocation of decision rights is relatively homogeneous among CEOs (whose incentive compensation has traditionally been used as the basis for examining the relationship between risk and incentives), we believe that this setting is most appropriately analyzed as the determination of incentives conditional on delegation having already taken place.

If we consider risk-neutrality to be the appropriate representation of the problem, then the model suggests that the conflicting results are a result of the empirical studies failing to distinguish between the two types of uncertainty (volatility and noise), leading to a potential misspecification problem in the existing empirical work. Recall that the optimal strength of incentives conditional on delegation was given by

$$\alpha_P = \frac{\sigma_P^2 \gamma}{\sigma_P^2 + \sigma_{\varepsilon_P}^2}.$$

We see that the optimal strength of incentives is increasing in the volatility of the state itself and decreasing in the noise contained in the performance measure. Further, if the agent is risk-averse, α_P can be non-monotone in σ_P^2 and $\sigma_{\varepsilon_{XP}}^2$ and σ_P^2 will interact in the determination of the equilibrium strength of incentives, with high $\sigma^2_{\varepsilon_{XP}}$ increasing the likelihood that α_P is increasing in σ^2_P. Thus, not only should empirical work account for the distinction between volatility and noise, but also account for the possibility that the relationship between volatility and incentive strength can be inherently non-monotone and dependent on the amount of noise.

5.1.1 Recent advances

(i) Volatility and noise Shi (2006), motivated by an information-acquisition perspective, is to our knowledge the only empirical paper that makes the distinction between volatility and noise that is conceptually equivalent to the present model. He decomposes the variance of a firm's stock return to its market-, industry- and firm-specific components and finds that the strength of incentives is decreasing in market-level risk while increasing in industry- and firm-specific risk. While this decomposition is clearly an imperfect proxy for the theoretical constructs themselves, the evidence is highly suggestive that forces beyond risk-aversion and noisy performance measures are at play in the determination of managerial incentives.[8]

To our knowledge, no paper has attempted to examine the potential non-linearity in the relationship between volatility and incentives or the interaction between the impacts of volatility and noise. Also, a common feature in much of the literature on executive compensation is the use of various firm-level characteristics such as book-to-market ratio, R&D and advertising intensities and industry classifications simply as additive control instruments. In contrast, our analysis suggests that such variables can contain valuable information regarding the underlying environment and the importance of managerial control. In consequence they can influence not only the overall sensitivity of compensation but also potentially how the return uncertainty or other characteristics impact the strength of incentives. Finding variables that are better associated with the theoretical constructs of both volatility and managerial control

[8] These results stand in some contrast to Jin (2002) and Garvey and Milbourn (2003) who also decompose the return risk to idiosyncratic and market components, but from a relative performance evaluation perspective, and find that compensation is not related to the systematic (market) part while decreasing in the firm (idiosyncratic) risk. The decompositions used, however, are different, and Shi (2006) places more weight on industry-level risk and recognizes the potential endogeneity of the level of volatility.

than firm- and industry-level stock return volatility should prove valuable in furthering our understanding of the link between the environment faced by a firm and its chosen managerial compensation policy. For example, Finkelstein and Boyd (1998) provide evidence that the *level* of managerial compensation is tied to the degree to which the manager is able to influence the performance of the firm, as measured by various variables along the lines mentioned above, and Shi (2006) explicitly discusses how pay-for-performance sensitivity is lower in capital-intensive industries.

(ii) Delegation and incentives While the set of tasks that a CEO performs is relatively homogenous, with the above qualifier that the environment and, relatedly, the extent to which the CEO is able to control it, can vary across industries, the task set of other managers and workers can exhibit significant variation. This variation in the task set further clouds the observed relationship between risk and incentives.

A number of recent papers have examined how the strength of incentives varies by executive rank and how the degree of delegation within a managerial group is influenced by uncertainty and other environmental variables. Aggarwal and Samwick (2003) provide empirical evidence on how the top management incentives vary by responsibility. Grouping executives in four categories according to their responsibilities, they show that the pay-for-performance sensitivity is significantly higher for CEOs than for divisional managers, with the strength of incentives decreasing in all categories with return volatility. These results are complemented by Barron and Waddell (2003), who rank executives within a firm according to their pay level, and provide evidence that the highest-paid executives also face the strongest incentives. However, while these results clearly illustrate that the incentives provided depend on the responsibilities of the agent, they do not shed light on how the division of responsibilities is in turn influenced by the environment.

The positive association of delegation and incentives is more appropriately analyzed by studies that focus on a single category of managers. Nagar (2002) analyzes the degree of authority and incentive compensation of retail branch managers and finds a positive association between the two. Wulf (2006), using data from a proprietary compensation survey of Hewitt Associates on the compensation of division managers, also finds that the compensation of division managers with broader authority is more sensitive to global performance measures, while conditional on the degree of authority, the trade-off between risk and incentives is negative. Similar findings are contained in, among others, Abernethy et al (2004) who use self-collected questionnaire data on division managers, Foss and Laursen (2005) who use survey data from Danish firm, DeVaro and Kurtulus (2006) who use the British Workpace Employee Relations Survey and Demers, Shackell and Widener (2002) who combine survey and archival data on B2C Internet companies.

By focusing on a more homogeous group of managers, these studies are also able to shed some light on the determinants of delegation and incentives. However, while these studies do identify the importance of delegation, they tend to pool all risk under one heading (or not deal with risk at all). Also, by testing different theories, none of the studies are able to provide a fully consistent analysis of the relationships suggested by our model, and because of their highly varying approaches and sets of controls, neither are they fully comparable with each other.[9] Thus, instead of attempting a comprehensive review of the findings and shortcomings in this literature, we will simply report here some of the found associations as they relate to our framework.

DeVaro and Kurtulus (2006) find that there is a negative relationship between risk and incentives (a relationship which is amplified when controlling for delegation), a positive relationship between risk and delegation and a positive relationship between performance pay and delegation, thus providing the most consistent piece of evidence in accordance with our model. With respect to the role of specific knowledge, Abernethy et al (2004) measure the degree of informational asymmetry (specific knowledge) through a combination of survey questions related to perceived informational asymmetry between the manager and his or her superior, the rate of growth in the industry and the size of the firm. They find that while delegation is positively related to the degree of informational asymmetry, the use of divisional summary measures as the basis for compensation is independent of the informational asymmetry once delegation is accounted for. This observation resonates well with our result that while increasing the informational asymmetry increases the likelihood of delegation, conditional on delegation, the agent's incentives are determined by the relative quality of the performance measure and the absolute quality of the agent's information, instead of the degree of informational asymmetry. In similar spirit, Nagar (2002) finds that the degree of delegation of responsibilities to branch level is higher in high-growth,

[9] Also, some related studies focus only on the determinants of delegation and others focus only on the composition of compensation, neglecting the interaction between the two.

volatile and innovative banks (and that branch managers with more authority receive more incentive-based pay).

Baiman et al. (1995) proxy the existence of a knowledge gap between the business unit and the parent company by whether they operate in the same 2-digit SIC code. They find that fewer tasks are allocated to the business unit manager when the principal has more expertise or when the unit is relatively larger. These results are also consistent with our model, which suggests that the principal retains the right to decide whenever (i) he has better information and (ii) the decision is more important to him. Colombo and DelMastro (2004) analyze plant-level data and proxy the degree of informational asymmetry through whether the plant has installed a local area network and/or has an on-line connection with the corporate headquarters. In contrast to Baiman et al (1995), they find that size is *positively* related to delegation. Their argument is that size is positively related with the complexity of the operations and thus with the degree of informational asymmetry. They also find that delegation is positively related to the adoption of advanced communication technologies and that the interaction between communication technology and size is negative. This result can be interpreted as follows: the installment of advanced communication technology improves both performance measurement (increased delegation) and flow of information to the principal (decreased delegation). When the plant is large, the information flow effect dominates and vice versa. In addition, they show that delegation of capital-related decisions is less likely than the delegation of labor-related decisions and that the overall level of delegation is decreasing in the capital-intensity of the plant, a result that can be related to the relative importance of the decisions to the two parties. Finally, Christie et al (2003) measure the degree of specialized knowledge by classifying industries based on personal expertise and complement this categorization with more objective measures, such as growth options (market-to-book ratio), uncertainty (estimated volatility of the unit's returns[10]) and size of the firm[11] (measured by annual sales). They find that delegation is positively related to the measures of informational asymmetry. However, neither Colombo and DelMastro (2004) nor Christie et al (2003) examine the determination of incentives.

[10]Because the volatility of divisional returns on equity are not observable, they are estimated by using the volatility of the firm's return on equity together with correlations among among the firm's industries.

[11]Where the argument is that increasing firm size increases delegation because of the limited processing power of the CEO.

To our knowledge, the only paper that explicitly focuses on the role of performance measurement is Moers (2006), who finds that the likelihood of delegation is increasing in the quality of performance measures, as predicted by our model. However, a few other papers contain indirect evidence in support of this conjecture. Colombo and DelMastro (2004), while not explicitly examining the determinants of compensation, show that delegation is positively associated with the adaptation of more advanced performance measurement systems. Abernethy et al (2004) find some evidence that the weight placed in compensation on divisional summary measures is positively related to their sensitivity and precision, but do not explicitly examine the potential link to delegation. Finally, if one conjectures that the quality of performance measures is decreasing in the degree of interdependencies among divisions, then the findings that delegation and/or division-level incentive strength are decreasing in the degree of interdependencies is also supportive of this effect (Bushman et al., 1995, Christie et al., 2003, Colombo and DelMastro, 2004, among others).

To summarize, the evidence to date is broadly consistent with our framework. First, the positive association between delegation and incentives is found consistently. Second, the evidence suggests that the weight placed on performance measures is increasing in their quality and that improved quality of performance measurement supports delegation. Third, the likelihood of delegation is increasing in the informational asymmetry between the manager and his or her superiors (as evaluated by direct survey questions) and more objective variables that we intuitively associate with both uncertainty and the potential for the development of informational asymmetries. The primary future challenge lies in better disentangling the effects of informational asymmetry, volatility and noise, a set of theoretical constructs that tend to be inherently intertwined in many empirical contents.

5.2 Franchising and other contracts

5.2.1 The decision to franchise

Lafontaine and Slade (1997,1998) review the existing empirical work on the decision to franchise. They document a number of empirical regularities, of which the ones of most interest to us are: (i) risk is positively related to the choice to franchise, (ii) the importance of the agent's effort is positively related to the choice to franchise, (iii) high cost of monitoring output or sales is positively related to company operation, and (iv) high cost of monitoring effort directly is negatively related to company operation. With the exception of (i), variants of the traditional hidden action model are able to explain all the regularities when viewing franchising as a choice to provide high-powered incentives. The additional contribution of our model is that it provides an explicit role for the delegation of responsibilities to the agent, thus suggesting that franchising as an arrangement can be qualitatively different from simply providing high-powered monetary incentives.

(i) Risk is positively related to the use of high-powered incentives/decision to franchise: As discussed above, much of apparent risk is uncertainty that a well-informed party is able to react to, captured by the variance of states.[12] We showed that while increased volatility faced by the principal does not necessarily lead to delegation, if an informational asymmetry develops between the principal and the agent and/or if information needs to be actively collected, then delegation of decision-making can be used to motivate information acquisition and/or to encourage responsiveness to that information. Thus, to the extent that the various measures, such as variation in detrended sales per outlet and the fraction of outlets discontinued within a given period of time, currently used to capture risk (noise) actually capture volatility that the agent is in a better position to respond to, then increased uncertainty should indeed lead to franchising instead of company-ownership.[13]

(ii) The importance of the agent's effort is positively related to the decision to franchise: While it is an old prediction that the incentives of the agent are increasing in the importance of his effort, this explanation still leaves unanswered the question of why use franchising instead of higher bonuses under company ownership

[12] See Lafontaine and Bhattacharyya (1995) on the empirical challenges of measuring "risk."

[13] An interesting alternative is provided by Mazzeo (2004). He highlights the point that the work on the choice to franchise ignores the franchisee's choice to be a fully independent agent. He finds evidence that in the motel industry, the likelihood of chain affiliation (as opposed to independent operation) is increasing in the uncertainty of the environment, suggesting a potential insurance feature of chain affiliation.

to motivate that effort. Our model suggests that when effort needs to be directed at information acquisition, then delegation can be necessary to induce information acquisition by the agent. Further, some of the measures that have been used to measure the importance of the agent's effort, such as the capital/labor ratio, the amount of discretionary inputs and a "personalized service" dummy, appear to potentially correlate also with the value of the *responsiveness* of the agent.[14]

(iii) & (iv) A high cost of monitoring output or sales is positively related to company operation, while a high cost of monitoring effort directly is negatively related to company operation: Again, these regularities conform with the prediction of the moral hazard models with respect to the strength of incentives, while our model extends this logic to the choice to delegate. In particular, these two results together highlight the practical fact that who gets to decide depends not only on the information available to the participants but also on the degree of controls available. Recall that the decision to delegate was given by

$$\gamma^2 \leq \frac{\sigma_A^2 \sigma_{\varepsilon_A}^2 \left[\sigma_P^2\left(1-\rho^2\right)+\sigma_{\varepsilon_P}^2\right]}{\sigma_P^2 \sigma_{\varepsilon_P}^2 \left[\sigma_A^2(1-\rho^2)+\sigma_{\varepsilon_A}^2\right]}.$$

Now, the difficulty of monitoring output or sales can be associated with the difficulty of contracting on the principal's (franchisor's) preferences (high $\sigma_{\varepsilon_P}^2$), indeed supporting company operation. The cost of monitoring the agent's effort is logically equivalent but slightly more nuanced. First, if the action d is truly hidden, then the principal is *de facto* forced to delegate that decision to the agent, with the only means of controlling him being high-powered incentives. Second, which was the focus of our model, even if d is transferable between the agent and the principal, if measuring the impact of that decision on the agent is sufficiently hard, then delegation will take place.

5.2.2 Contractual Allocation of Decision Rights

While the empirical work on franchising has tended to approach the choice to franchise from the traditional moral hazard perspective (at the expense of considerations of the

[14]A complementary answer can be derived from multi-tasking considerations (see, for example, Holmström, 1999).

allocation of authority), the empirical work on other contracts has tended to take a property rights approach (at the neglect of most maladaptation problems during the execution of the contract). The notable exceptions are the papers by Arrunada et al (2001) and Zanarone (2005), who study the relationship between car manufacturers and dealerships. They show for the cases of Spain and Italy, respectively, how more control rights are allocated to manufacturers in networks where the risk of the dealers' moral hazard is higher and the cost of misbehavior on the manufacturer is higher, as measured by the size of the network (free-riding) and the unit price of cars (brand value) respectively. We can consider brand value to be reflected by γ, which measures how important appropriate decisions are to the principal, while the size of the network (and hence the size of potential spillovers) is weakly analogous to $\sigma^2_{\varepsilon_P}$, since an increase in potential spillovers makes the provision of sharp incentives harder. Increase in either variable makes the retainment of decision rights by the principal more likely.

6 Conclusion

We have analyzed the joint determination of an allocation of a decision right, incentive strength and job flexibility, as influenced by the underlying environment. First, conditional on the allocation of the decision right, we showed that the greater the ex ante uncertainty over the appropriate decision, the stronger the incentive strength and the greater the job flexibility granted to the decision-maker. Conversely, the noisier the performance measures used to align incentives, the lower the incentive strength and the lower the job flexibility granted to the decision-maker. These results resonate well with the basic logic that as the environment becomes more uncertain, responsiveness of the decision-maker becomes more valuable and, as a result, indirect incentive alignment through the use of performance measures is used as a substitute for direct controls. We suggested the distinction between volatility and noise as an explanation for the conflicting empirical evidence on the trade-off between risk and incentives found in the literature on executive compensation and complemented this distinction by noting the additional considerations that arise if the agent is risk-averse.

Second, we analyzed the choice to delegate, given the optimal performance contract. In the full-information setting, we illustrated how the decision right should be allocated to the individual whose preferences are harder to contract on, who faced higher volatility of the environment and to whom the decision was more important, following the simple logic that the decision right should be granted to the party who can be induced to make better decisions, given the optimal performance contract. In particular, when the uncertainty faced by the principal increased, the likelihood of delegation decreased, a result which we illustrated with the example of offshore drilling. In the extensions, we showed that, in contrast with the full-information case, increased volatility faced by the principal at the time of contracting could lead to increased delegation if (i) an informational asymmetry developed during the execution of the contract between the agent and the principal (specific knowledge) and/or (ii) it became more efficient to rely on information acquisition by the agent. We suggested these effects as an explanation for the positive relationship between uncertainty and the choice to franchise found in the empirical literature, thus linking franchising with increased delegation as an effect separate from providing high-powered incentives.

Moving beyond the risk-incentives trade-off, the predictions of the model were found to be broadly consistent with the emerging empirical literature examining the co-determination of authority and incentives inside firms and with other empirical determinants of the choice to franchise, suggesting that many organizational choices can be fruitfully viewed as attempts to manage (mal)adaptation problems, even if the picture is still far from finished. However, it is also clear that organizations do not deal only with adaptation problems, as evidenced by the large literatures on moral hazard and property rights. Examining the interrelationships among the different organizational challenges appears a particularly promising avenue for building a more comprehensive picture of the determinants of organizational design and the nature of contractual relationships.

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A Extensions

A.1 Imperfect information

In many practical situations we would expect a natural informational asymmetry to develop between the agent and the principal, with the principal having worse direct access to information about local conditions. In this section we provide an illustration how such an informational asymmetry alters the choice to delegate and the underlying contract.

First, note that imperfect information has only a small impact on the degree of job flexibility and incentive strength. This result follows because the decision-maker, when making decisions, already internalizes any imperfections in the information itself. The only purpose of the incentive contract is to control the *relative* responsiveness of the decision-maker to that information. Some effects are present on levels but not in comparative statics. To illustrate, consider a situation where the agent has imperfect information about both s_P and θ_P (signals υ_{P1} and υ_{P2}, respectively). Then, when making decisions, the agent will form beliefs through $E\left(s_P|\upsilon_{P1},\upsilon_{P2}\right)$. Now, if υ_{P2} receives no weight, then the solution parallels that in section 4. If υ_{P2} receives a positive weight, then the correlation with θ_P is increased and it is optimal to provide *stronger* incentives. Indeed, if the agent receives no direct information about s_P but observes θ_P, then $E\left(s_P|\theta_P\right)=\theta_P$ and we actually achieve first-best decision-making.

The more significant effect of imperfect information comes from the worsening of the quality of the decisions. As a simple example, consider a situation where the agent learns perfectly (s_P,θ_A) and the principal learns perfectly s_A but receives only a signal $\upsilon_P=\theta_P+\eta_P$ about the realization of his state, where $\eta_P\sim N(0,\sigma^2_{\eta_P})$. The imperfection can be seen as arising from a variety of sources, including limited direct access to information and delays in the transmission of information.

For simplicity, assume that $\rho=0$ and $\gamma=1$. Under these informational assumptions, the solution under delegation remains unchanged. Under employment, the decision of the principal is now given by

$$d^P=\tfrac{\alpha_A s_A+\alpha_D\bar{d}+E_P(\theta_P|\upsilon_P)}{1+\alpha_A+\alpha_D},$$

while the second-best decision is given by

$$d^{SB} = \frac{E_P(\theta_A|s_A)+E_P(\theta_P|v_P)}{2}.$$

Matching coefficients and solving for the expected loss yields the following proposition:

Proposition 5 *The parameters of the optimal incentive contract under employment are given by*

$$\alpha_A = \frac{\sigma_A^2}{\sigma_{\varepsilon_A}^2+\sigma_A^2} \qquad \alpha_D = \frac{\sigma_{\varepsilon_A}^2}{\sigma_{\varepsilon_A}^2+\sigma_A^2} \qquad \overline{d} = \mu_A.$$

The resulting expected loss is then given by

$$EL^E = EL^{FB} + \frac{1}{2}\left[\left(\frac{\sigma_A^2\sigma_{\varepsilon A}^2}{\sigma_A^2+\sigma_{\varepsilon A}^2}\right) + \left(\frac{\sigma_P^2\sigma_{\eta P}^2}{\sigma_P^2+\sigma_{\eta P}^2}\right)\right].$$

Thus, delegation is preferred over employment iff

$$\frac{\sigma_P^2(x_{\varepsilon p}-x_{\eta p})}{(1+x_{\varepsilon p})(1+x_{\eta p})} < \left(\frac{\sigma_A^2\sigma_{\varepsilon A}^2}{\sigma_A^2+\sigma_{\varepsilon A}^2}\right),$$

where $x_{\varepsilon p} = \frac{\sigma_{\varepsilon P}^2}{\sigma_P^2}$ *and* $x_{\eta p} = \frac{\sigma_{\eta P}^2}{\sigma_P^2}$.

Proof. See Appendix B ■

From the proposition we see that imperfect information is in its consequences equivalent to an imperfect performance measure. This should not be a surprise, since from the perspective of expected loss, it matters very little whether a given party makes poor decisions because he can be provided only with poor incentives to make decisions or whether the decisions are poor because the decision-maker has inaccurate information. As regards to the delegation choice, it is then immediate that if the agent's information is more informative than the principal's with respect to θ_P ($x_{\varepsilon p} < x_{\eta p}$), then delegation is always preferred. Thus, an informational advantage held by the agent increases the likelihood of delegation. However, note that if the relative quality of information is held constant, then it is still the case that an increase in volatility faced by the principal will decrease the likelihood of delegation. This result follows because an increase in σ_P^2 still increases the relative importance of tracking θ_P closely, and if $x_{\eta p} < x_{\varepsilon p}$, the principal is still in a better position to do so.

A.2 Risk-aversion

In this subsection we illustrate the inherent non-monotonicity in the relationship between volatility and incentives when the agent is risk-averse. Assume now that the agent has mean-variance preferences with a risk-aversion coefficient r. Further, assume that the decision has to be taken by the agent (as an effort choice), no decision-contingent transfers are available and that $\gamma = 1$. Further, assume that the performance measure is perfect up to an additive error term:

$$X_P = -\left(\theta_P - d^A\right)^2 + \varepsilon_{X_P}.$$

The agent is then offered a contract

$$T(.) = B - \alpha_P \left(\left(\theta_P - d^A\right)^2 + \varepsilon_{X_P}\right),$$

where the terms of the contract are chosen by the principal (after substituting out B) to:

$$\begin{aligned} &\min_{\alpha} E\left[\left(\theta_P - d^A\right)^2 + \left(\theta_A - d^A\right)^2\right] + rVar\left[\alpha_P\left(\theta_P - d^A\right)^2 + \alpha_P\varepsilon_{X_P} + \left(\theta_A - d^A\right)^2\right] \\ &s.t. \qquad d^A = \tfrac{\alpha_P\theta_P + \theta_A}{\alpha_P + 1}. \end{aligned}$$

Taking the first-order condition and rearranging gives

Proposition 6 *The optimal incentive strength is implicitly defined by:*

$$\tfrac{1}{(\alpha_P+1)^3}\left[(\alpha_P - 1)\left(\Delta\mu^2 + \sigma^2_{\Delta\theta}\right) + r\alpha_P\left(2\sigma^2_{\Delta\theta}\left(\sigma^2_{\Delta\theta} + 2\Delta\mu^2\right)\right)\right] + r\alpha_P\sigma^2_{\varepsilon_{X_P}} = 0,$$

where $\Delta\mu^2 = \left(\mu_A - \mu_P\right)^2$ *and* $\sigma^2_{\Delta\theta} = Var(\theta_A - \theta_P)$.

Thus,

$$\tfrac{\partial\alpha_P}{\partial\sigma^2_{\varepsilon_{X_P}}} < 0 \qquad \textit{and} \qquad sign\left(\tfrac{\partial\alpha_P}{\partial\sigma^2_{\Delta\theta}}\right) = sign\left(1 - \alpha_P\left(1 + 4r\left(\Delta\mu^2 + \sigma^2_{\Delta\theta}\right)\right)\right).$$

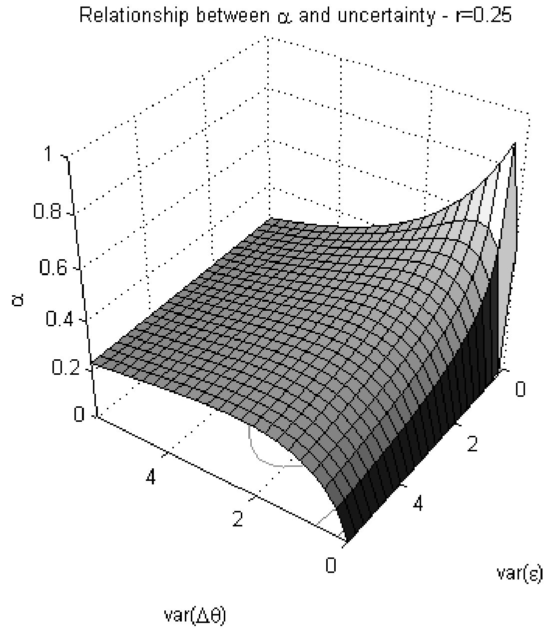

Figure 1: Relationship between incentive strength and uncertainty under risk-aversion

Proof. See Appendix B ■

Thus, as in all models of hidden action with risk-aversion, the strength of incentives is decreasing in the noisiness of the performance measure, $\sigma^2_{\varepsilon_{X_P}}$. This aspect of the problem is thus unchanged, which is relatively immediate since we have simply shifted the source of the agency problem from misaligned performance measures to risk-aversion.

What risk-aversion does change is the monotonicity result regarding the relationship between the volatility of the environment and incentives provided. The intuition for this result is clear. As $\sigma^2_{\Delta\theta}$ increases, the 'productive' value of incentives goes up as in the case of a risk-neutral agent. However, at the same time, as long as $\alpha_P > 0$, this also translates to a larger risk-cost of incentives. Which effect dominates depends on the equilibrium α_P. In particular, if α_P is small, then the increase in the value of incentives is likely to dominate, because low α_P implies low initial pass-through of volatility of the environment to volatility of pay (since the pass-through is proportional to α_P^2).

The observation that the sign of $\partial\alpha_P/\partial\sigma^2_{\Delta\theta}$ depends on α_P implies in turn that $\partial\alpha_P/\partial\sigma^2_{\Delta\theta}$ will depend on $\sigma^2_{\varepsilon_{X_P}}$. Note that in essence the equilibrium strength of incentives balances the productive value of incentives, determined by the value of inducing the agent to respond to the realized $(\theta_A - \theta_P)$, which in turn is related to $\sigma^2_{\Delta\theta}$, and the cost of providing those incentives, determined by the overall uncertainty $\left(\sigma^2_{\varepsilon_{X_P}}, \sigma^2_{\Delta\theta}\right)$. When $\sigma^2_{\varepsilon_{X_P}} >> \sigma^2_{\Delta\theta}$, then the risk-compensation component is dominated by $\sigma^2_{\varepsilon_{X_P}}$ and $\partial\alpha_P/\partial\sigma^2_{\Delta\theta} > 0$ because the productive value will dominate the marginal change. The converse happens when $\sigma^2_{\varepsilon_{X_P}} << \sigma^2_{\Delta\theta}$ and then $\partial\alpha_P/\partial\sigma^2_{\Delta\theta} < 0$. An example of this interaction between $\sigma^2_{\varepsilon_{X_P}}$ and $\sigma^2_{\Delta\theta}$ in determining the optimal linear α_P is given in figure 1. The comparative statics of $\partial\alpha_P/\partial\sigma^2_{\Delta\theta} \gtreqless 0$ and $\partial\alpha_P/\partial\sigma^2_{\varepsilon_{X_P}} < 0$ are qualitatively equal to those presented in Baker and Jorgensen (2003). This result should be no surprise since, while the structure of the models differs slightly, the structure of uncertainty is the same in both settings: $\sigma^2_{\Delta\theta} > 0$, $\sigma^2_{\varepsilon_{X_P}} > 0$ and $\sigma^2_{\varepsilon_P} = 0$.

A.3 Insufficient controls, job-design and second-best

The key behind the simplicity of our solution was the use of $\alpha_D \left(\overline{d} - d\right)^2$, which allowed us to control both the absolute and relative responsiveness of the agent to incoming information, which in turn allowed us to derive a simple optimal linear contract. If we are not able to contract on decisions directly, both linearity and the constrained optimality of the solution is usually lost. In this subsection we will make some observations on interpreting $\alpha_D \left(\overline{d} - d\right)^2$ as a job design parameter and consider the solution when no decision-contingent transfers are available.

A more flexible interpretation of $\alpha_D \left(\overline{d} - d\right)^2$ is the job flexibility granted to the agent. The general idea is that even if it might not be possible to make pure transfers based on the exact decisions made by the agent, it is often possible to make some actions easier to take than others. This interpretation is naturally constrained by our assumption of no waste. However, it is often possible to cause *destruction* of utility, for example with varying bureaucratic complexity. While the solution will no longer be linear due to typical screening arguments (at intermediate values of θ_A, when choosing the decision we need to acknowledge the fact that the marginal waste caused by that decision also has to be paid on all more extreme decisions, leading to convexity), it can still be used to provide some control of the decision-maker. Further, note that the *value* aspect of direct controls is unaffected whether their use is costly or not. Thus, while the shape of the optimal contract is affected, as long as the cost is independent of the underlying uncertainty, the basic intuitition behind the model continues to hold. In similar fashion, the coefficient bears a close resemblance to varying the set of allowed outside activities (Holmström and Milgrom, 1991), with both controlling the responsiveness of the agent to incentives (and information). The simplifying assumption here is that the overall surplus is not affected by the size of that set.

The same holds for the strength of incentives if we ruled out decision-contingent transfers altogether. Recall that the compensation contract optimally balances two sources of inefficiency: the gaming of the compensation contract caused by $\sigma_{\varepsilon_i}^2 > 0$ and biased decisions caused by $\alpha_P < 1$. When decisions can be contracted on directly, α_D and $\overline{d}$ can be used in a way that the expected severity of the two problems is independent of the realized states, resulting in linear optimal incentives given the formulation. If such transfers are no longer available, then the relative severity of the two problems *will* depend on the realized information. In particular, while the gaming problem is independent of the realized states (as it is driven by $\sigma_{\varepsilon_i}^2$), the expected value of incentive alignment from the perspective of bias is increasing in $(\theta_A - \theta_P)^2$. The reason for this follows from the assumption of strictly concave preferences, which implies that the further apart the preferred decisions are in expectation, the higher the marginal value of reducing the expected bias.

To illustrate, if we restricted our attention to linear incentives, then the optimal weight on the performance measure (absent decision-contingent transfers) would be

$$\alpha_P = \frac{E(\theta_P - \theta_A)^2}{E(\theta_P - \theta_A)^2 + (1+\gamma)\sigma_{\varepsilon_P}^2},$$

simply weighing the relative cost of the two sources of inefficiency: gaming and bias. Thus, the comparative statics on the strength of incentives remain the same as in the analysis of section 4, with the exception that the incentive strength is now increasing in the volatility of the preferences of both parties (since this increases the expected size of the incentive conflict) and decreasing in the correlation between preferences (since this decreases the expected size of the incentive conflict).

To the extent that X_i (and other contractible variables) are informative about the size of this conflict, the incentive strength can be conditioned on that information to get a better balance between the two.[15] Generally, if we let ω denote the contractible

[15] With the caveat that conditioning the weight on the realized X_i can attract additional gaming of the contract.

information, we could solve for the optimal non-linear contract of the form

$$T(.) = B + \alpha_P(\omega) X_P(.) + \alpha_A(\omega) X_A(.),$$

where the weights are determined so that conditional on the available information, the expected marginal cost of the two sources of inefficiency is equated. For example, in the case of (s_A, s_P) being verifiable, the delegation solution gives

$$\alpha_P(s_A, s_P) = \frac{E\left[(\theta_P-\theta_A)^2|s_A,s_P\right]}{E\left[(\theta_P-\theta_A)^2|s_A,s_P\right]+(1+\gamma)\sigma^2_{\varepsilon_P}} = \frac{\left(\Delta\mu_{|s_A,s_P}\right)^2+\sigma^2_{\Delta\theta|s_A,s_P}}{\left(\Delta\mu_{|s_A,s_P}\right)^2+\sigma^2_{\Delta\theta|s_A,s_P}+(1+\gamma)\sigma^2_{\varepsilon_P}}.$$

We still don't want to condition the compensation of the agent directly on X_A (or any other function based on s_A and d) to avoid attracting additional gaming, but the contract will use both s_A and s_P (which alone are non-manipulable) to condition α_P to reflect the expected size of the two sources of inefficiency. Otherwise, the comparative statics continue to hold pointwise for given (s_A, s_P) and thus with respect to $E(\alpha_P(s_A, s_P))$.

B Derivations

B.1 Proposition 4: Optimal Contract

We derive the solution in the case of delegation, the solution in the case of employment is symmetric. The second-best decision is given by

$$d_A^{FB} = \frac{\gamma E(\theta_P|s_P,\theta_A)+\theta_A}{1+\gamma}.$$

Given normality of the random variables, their joint distribution is given by

$$\begin{pmatrix} \theta_P \\ s_P \\ \theta_A \end{pmatrix} \sim N\begin{pmatrix} \mu_P & \sigma_P^2 & \sigma_P^2 & \sigma_{AP} \\ \mu_P \;, & \sigma_P^2 & \sigma_P^2+\sigma^2_{\varepsilon_P} & \sigma_{AP} \\ \mu_A & \sigma_{AP} & \sigma_{AP} & \sigma_A^2 \end{pmatrix}.$$

Therefore, we can write

$$E\left(\theta_P|s_P,\theta_A\right)=\mu_P+\begin{pmatrix}\sigma_P^2 & \sigma_{AP}\end{pmatrix}\begin{pmatrix}\sigma_P^2+\sigma_{\varepsilon_P}^2 & \sigma_{AP}\\ \sigma_{AP} & \sigma_A^2\end{pmatrix}^{-1}\begin{pmatrix}s_P-\mu_P\\ \theta_A-\mu_A\end{pmatrix},$$

which after a little algebra simplifies to:

$$\frac{\mu_P\left(\sigma_{\varepsilon_P}^2\sigma_A^2\right)-\sigma_{AP}\sigma_{\varepsilon_P}^2\mu_A+\sigma_P^2\sigma_A^2\left(1-\rho^2\right)s_P+\sigma_{AP}\sigma_{\varepsilon_P}^2\left(\theta_A\right)}{\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2}.$$

Thus, the optimal decision can be written as:

$$d_A^{FB}=\frac{\gamma E(\theta_P|s_P,\theta_A)+\theta_A}{1+\gamma}=\frac{\gamma\mu_P\sigma_{\varepsilon_P}^2\sigma_A^2-\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\mu_A+\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)s_P+\theta_A\left(\sigma_{\varepsilon_P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}.$$

Given a contract $\alpha_P X_P+\alpha_D X_D+B$, the agent chooses a decision satisfying

$$d^A=\frac{\alpha_P s_P+\alpha_D\overline{d}+\theta_A}{\alpha_P+\alpha_D+1}.$$

The rest is simply matching the coefficients so that

$$\frac{\alpha_P s_P+\alpha_D\overline{d}+\theta_A}{\alpha_P+\alpha_D+1}=\frac{\gamma\mu_P\sigma_{\varepsilon_P}^2\sigma_A^2-\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\mu_A+\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)s_P+\theta_A\left(\sigma_{\varepsilon_P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}.$$

To simplify, let $\alpha=\frac{\alpha_P}{\alpha_P+\alpha_D+1},\beta=\frac{\alpha_D}{\alpha_P+\alpha_D+1},\delta=\frac{1}{\alpha_P+\alpha_D+1}=1-\alpha-\beta$, then

$$\alpha s_P+\beta\overline{d}+\delta\theta_A=\frac{\left(\gamma\mu_P\sigma_{\varepsilon_P}^2\sigma_A^2-\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\mu_A\right)+\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)s_P+\theta_A\left(\sigma_{\varepsilon_P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}.$$

Then, we have that

$$\alpha=\frac{\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)},\beta\overline{d}=\frac{\left(\gamma\mu_P\sigma_{\varepsilon_P}^2\sigma_A^2-\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\mu_A\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)},\ \delta=\frac{\left(\sigma_{\varepsilon_P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon_P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon_P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}.$$

Now,

$$\frac{\alpha}{\delta}=\frac{\frac{\alpha_P}{\alpha_P+\alpha_D+1}}{\frac{1}{\alpha_P+\alpha_D+1}}=\alpha_P=\frac{\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon_P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)}.$$

Similarly,

$$\delta=\frac{1}{\alpha_P+\alpha_D+1}\to\delta\alpha_D=(1-\delta)-\delta\alpha_P=1-\delta\left(1+\alpha_P\right),$$

so that

$$\frac{\left(\sigma_{\varepsilon P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}\alpha_D = 1-\frac{\left(\sigma_{\varepsilon P}^2\sigma_A^2+\left(1-\rho^2\right)\sigma_P^2\sigma_A^2+\gamma\sigma_{AP}\sigma_{\varepsilon P}^2\right)}{(1+\gamma)\left(\sigma_{\varepsilon P}^2\sigma_A^2+(1-\rho^2)\sigma_P^2\sigma_A^2\right)}\left(1+\frac{\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)}\right),$$

giving $\alpha_D = \frac{\gamma\sigma_{\varepsilon P}^2\left(\sigma_A^2-\rho\sigma_A\sigma_P\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)}$.

Finally, we have that

$$\frac{\beta\overline{d}}{\delta} = \alpha_D\overline{d} = \frac{\gamma\sigma_{\varepsilon P}^2\left(\sigma_A^2-\rho\sigma_A\sigma_P\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)}\overline{d} = \frac{\left(\gamma_P\sigma_{\varepsilon P}^2\mu\sigma_A^2-\gamma_P\sigma_{\varepsilon P}^2\sigma_{AP}\mu_A\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)},$$

which gives us

$$\overline{d} = \frac{\mu_P\sigma_A^2-\rho\sigma_A\sigma_P\mu_A}{\sigma_A^2-\gamma\rho\sigma_A\sigma_P} = \frac{\mu_P\sigma_A-\rho\sigma_P\mu_A}{\sigma_A-\gamma\rho\sigma_P}.$$

Thus, the optimal contract is given by:

$\alpha_P X_P + \alpha_D X_D + B$, where

$$\alpha_P = \frac{\gamma\sigma_P^2\sigma_A^2\left(1-\rho^2\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)},\ \alpha_D = \frac{\gamma\sigma_{\varepsilon P}^2\left(\sigma_A^2-\rho\sigma_A\sigma_P\right)}{(1-\rho^2)\sigma_P^2\sigma_A^2+\sigma_{\varepsilon P}^2\left(\sigma_A^2+\gamma\rho\sigma_A\sigma_P\right)},\ \overline{d} = \frac{\mu_P\sigma_A-\rho\sigma_P\mu_A}{\sigma_A-\gamma\rho\sigma_P}$$

The case of employment is symmetric, with an adjustment for γ. The expected loss follows by noting that we can write

$$\begin{aligned}
&E\left(\gamma\left(\theta_P-d^i\right)^2+\left(\theta_A-d^i\right)^2\right)\\
&= E\left(\gamma\left(\theta_P-d^{FB}+d^{FB}-d^i\right)^2+\left(\theta_A-d^{FB}+d^{FB}-d^i\right)^2\right)\\
&= \left((1+\gamma)\,E\left(\theta_A-d^{FB}\right)^2+(1+\gamma)\,E\left(d^{FB}-d^i\right)^2\right)
\end{aligned}$$

In the case of delegation,

$$E\left(d^{FB}-d^A\right)^2 = \left(\frac{\gamma\theta_P+\theta_A}{1+\gamma}-\frac{\alpha_P s_P+\alpha_d\overline{d}+\theta_A}{1+\alpha_P+\alpha_d}\right)^2.$$

To simplify the derivation, note from above that

$$\frac{\alpha_P s_P+\alpha_d\overline{d}+\theta_A}{1+\alpha_P+\alpha_d} = \frac{\gamma E(\theta_P|s_P,\theta_A)+\theta_A}{1+\gamma},$$

so we have that

$$E\left(d^{FB}-d^{A}\right)^{2}=\frac{\gamma^{2}}{(1+\gamma)^{2}}E\left(\theta_{P}-E\left(\theta_{P}|s_{P},\theta_{A}\right)\right)^{2}.$$

Recall that the conditional variance for a normally distributed variable is given by

$$\widetilde{\sigma}_{P}^{2}=\sigma_{P}^{2}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21},$$

and from above we have that

$$\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}=\begin{pmatrix}\sigma_{P}^{2} & \sigma_{AP}\end{pmatrix}\begin{pmatrix}\sigma_{P}^{2}+\sigma_{\varepsilon_P}^{2} & \sigma_{AP}\\ \sigma_{AP} & \sigma_{A}^{2}\end{pmatrix}^{-1}\begin{pmatrix}\sigma_{P}^{2}\\ \sigma_{AP}\end{pmatrix}=\frac{\sigma_{P}^{2}\left(\sigma_{P}^{2}\left(1-\rho^{2}\right)+\rho^{2}\left(\sigma_{\varepsilon_P}^{2}\right)\right)}{\sigma_{P}^{2}(1-\rho^{2})+\sigma_{\varepsilon_P}^{2}},$$

so

$$\sigma_{P}^{2}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}=\frac{\sigma_{P}^{2}\sigma_{\varepsilon_P}^{2}\left(1-\rho^{2}\right)}{\sigma_{P}^{2}(1-\rho^{2})+\sigma_{\varepsilon_P}^{2}}.$$

B.2 Proposition 5: Imperfect Information

The decisions (principal and second-best) are given by

$$d^{P}=\frac{\alpha_{A}s_{A}+\alpha_{D}\overline{d}+E_{P}(\theta_{P}|v_{P})}{1+\alpha_{A}+\alpha_{D}}\text{ and }d^{SB}=\frac{E_{P}(\theta_{A}|s_{A})+E_{P}(\theta_{P}|v_{P})}{2},$$

and

$$E_{P}\left(\theta_{A}|s_{A}\right)=\frac{\sigma_{\varepsilon_A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}\mu_{A}+\frac{\sigma_{A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}s_{A}.$$

Thus, the coefficients need to satisfy

$$d^{P}=\frac{\alpha_{A}s_{A}+\alpha_{D}\overline{d}+E_{P}(\theta_{P}|v_{P})}{1+\alpha_{A}+\alpha_{D}}=\frac{\frac{\sigma_{\varepsilon_A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}\mu_{A}+\frac{\sigma_{A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}s_{A}+E_{P}(\theta_{P}|v_{P})}{2},$$

which gives directly

$$\alpha_{A}=\frac{\sigma_{A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}\qquad\alpha_{D}=\frac{\sigma_{\varepsilon_A}^{2}}{\sigma_{\varepsilon_A}^{2}+\sigma_{A}^{2}}\qquad\overline{d}=\mu_{A}.$$

The expected loss follows as above. We can write the expected loss as

$$E\left(\left(\theta_P - d^{FB}\right)^2 + \left(\theta_A - d^{FB}\right)^2 + 2\left(d^{FB} - d^P\right)^2\right),$$

where $d^P = \frac{E(\theta_P|v_P)+E(\theta_A|s_A)}{2}$,

so

$$\frac{1}{4}E\left(\left(\theta_A - E\left(\theta_A|s_A\right)\right) + \left(\theta_P - E\left(\theta_P|v_P\right)\right)\right)^2 = \frac{1}{4}\left[\left(\frac{\sigma_A^2\sigma_{\varepsilon A}^2}{\sigma_A^2+\sigma_{\varepsilon A}^2}\right) + \left(\frac{\sigma_P^2\sigma_{\eta_P}^2}{\sigma_P^2+\sigma_{\eta_P}^2}\right)\right].$$

B.3 Proposition 6: Risk-Aversion and Incentive Strength

The principal's problem continues to be one of maximizing expected total surplus, which is now given by

$$\min_{\alpha_P} E\left[\left(\theta_P - d^A\right)^2 + \left(\theta_A - d^A\right)^2\right] + rVar\left[\alpha_P\left(\theta_P - d^A\right)^2 + \alpha_P\varepsilon_P + \left(\theta_A - d^A\right)^2\right]$$

$$s.t. \qquad d^A = \frac{\alpha_P\theta_P+\theta_A}{\alpha_P+1}.$$

Substituting in the decision, we have that:

$$\left(\theta_P - d^A\right)^2 + \left(\theta_A - d^A\right)^2 = \frac{1+\alpha_P^2}{(\alpha_P+1)^2}\left(\theta_P - \theta_A\right)^2$$

The problem becomes:

$$\min_{\alpha_P}\frac{1+\alpha_P^2}{(\alpha_P+1)^2}E\left(\theta_P-\theta_A\right)^2 + r\left(\frac{\alpha_P+\alpha_P^2}{(\alpha_P+1)^2}\right)^2 Var\left(\theta_P-\theta_A\right)^2 + r\alpha_P^2\sigma_{\varepsilon_P}^2$$

$$\min_{\alpha_P}\frac{1+\alpha_P^2}{(\alpha_P+1)^2}E\left(\theta_P-\theta_A\right)^2 + \frac{r\alpha_P^2}{(\alpha_P+1)^2}Var\left(\theta_P-\theta_A\right)^2 + r\alpha_P^2\sigma_{\varepsilon_P}^2.$$

Thus, the first-best incentive strength is implicitly defined by

$$\frac{1}{(\alpha_P+1)^3}\left[\left(\alpha_P-1\right)E\left(\theta_P-\theta_A\right)^2 + r\alpha_P Var\left(\theta_P-\theta_A\right)^2\right] + r\alpha_P\sigma_{\varepsilon_P}^2 = 0$$

Now, denote the distribution of $\theta_P - \theta_A$ by $N\left(\Delta\mu, \sigma_{\Delta\theta}^2\right)$. Then,

$$E\left(\theta_P-\theta_A\right)^2 = \Delta\mu^2 + \sigma_{\Delta\theta}^2 \text{ and } Var\left(\theta_P-\theta_A\right)^2 = 2\sigma_{\Delta\theta}^2\left(\sigma_{\Delta\theta}^2 + 2\Delta\mu^2\right),$$

so we can write the first-order condition as

$$\frac{1}{(\alpha_P+1)^3}\left[(\alpha_P-1)\left(\Delta\mu^2+\sigma^2_{\Delta\theta}\right)+r\alpha_P\left(2\sigma^2_{\Delta\theta}\left(\sigma^2_{\Delta\theta}+2\Delta\mu^2\right)\right)\right]+r\alpha_P\sigma^2_{\varepsilon_P}=0.$$

Thus, using the implicit function theorem we have that

$$\frac{\partial\alpha_P}{\partial\sigma^2_{\varepsilon_P}}<0 \qquad and \qquad sign\left(\frac{\partial\alpha_P}{\partial\sigma^2_{\Delta\theta}}\right)=sign\left(1-\alpha_P\left(1+4r\left(\Delta\mu^2+\sigma^2_{\Delta\theta}\right)\right)\right).$$