COST VARIANCE INVESTIGATION WITH FUZZY SETS

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1 INTRODUCTION

Cost variance investigation is critical to process control. Cost variances provide a report that indicates the deviation between actual costs and standard costs for process systems. Since standard costs are a priori estimates, it is easy to understand that actual and standard costs are not likely to be identical. Thus, the problem that faces managers who receive variance reports on these systems, is to determine which reported cost variances indicate that the system they represent should be examined further.

If a system is to receive further examination then a team of investigators will take samples from the actual behavior of the system to estimate if the system is in-control or out-of-control. Thus, managers must determine the extent of the work that is done to determine the state of the system.

Ideally, in-control systems are not investigated and only out-of-control systems are investigated. However, in-control systems may be investigated and out-of-control systems may not be investigated because the variance report is only a probabilistic evaluation of the state of the system. If the system is in-control and the system is investigated then a cost is incurred when it is not necessary to incur a cost. However, if an out-of-control system is not investigated then the system will continue to be out-of-control and the organization will incur the costs of the system being out-of-control.

Since there are only limited time and monetary resources, and interrupting a process to investigate has a cost, management's team cannot investigate every system with a reported deviation or variance. Alternatively, if the manager does not decide to investigate the system based on the reported variances then the control function of the standard cost system is eliminated and the value of the system has been overestimated (ref. 1).

The optimal policy is somewhere between investigating all systems with variances or investigating no systems with variances. Thus, the problems facing the manager are to decide which systems to investigate, to determine how much investigation should be made and to estimate if the system is in-control or out-of-control.

However, these sets of systems that are to be judged "in-control" or "out-of-control" are in Zadeh's (ref. 6) words, "fuzzy." Rather than "in-control" the system may be "more or less in-control." Rather than "out-of-control" the system may be "almost out-of-control." Thus, these sets have poorly defined boundaries, i.e., the membership criteria are imprecise.

This is in contrast to the variance information sampled from system behavior which are oftentimes "clear-cut" or "objective." For example, cost expenditures, obtained during a system investigation are either outside their pre-established control boundaries or they are not outside the established boundaries. This information, in turn, is used to draw inferences about system behavior attributes, e.g., deviation of costs from standard rates, which also are unambiguous.

This paper uses Bayesian decision theory to consider the problem of fuzziness with variance reporting systems that indicate that process systems are either "in-control" or "out-of-control." The Bayesian approach provides a framework for explicitly working with system behavior attributes, the prior knowledge of the management team, the fuzzy judgement of the team's members of whether the system is in-control or out-of-control and formal modification of this knowledge as further information becomes available.

This paper proceeds as follows. Section 2 introduces the basic cost variance investigation model for which a process is either "in-control" or "out-of-control." Section 3 discusses the aspects of fuzzy set theory that are used in the fuzzy set model of cost variance investigation. Section 4 discusses a previous fuzzy set-based approach to cost variance investigation. Section 5 presents the primary results of the paper: a fuzzy set model of cost variance investigation. Section 6 illustrates the model with an example. Section 7 provides a summary and a discussion of some extensions of the paper.

2 BASIC COST VARIANCE INVESTIGATION MODEL

There are a number of decision models that can be used to estimate the importance of variance reports in the investigation of process systems. One approach is to use control charts to provide "in-control" and "out-of-control" information for cost control (ref. 1). An extension of the control chart model is to incorporate the expected costs and benefits from variance investigation. A single period model of this process is developed in Kaplan (ref. 1). That model assumes that the process is either "in-control" (IC) or "out-of-control" (OOC).

There are a number of assumptions in this model. It is assumed that if an investigation is undertaken when a process is judged OOC then the cause can always be found, that there is no ambiguity associated with the evidence, that there are no errors in the assessment of the evidence, there is no fuzziness in the management team's judgement of the states of nature (IC and OOC) and that an OOC process can be reset to the IC state.

The model assumes that there is no cost associated with accepting a process that is in-control. The model also assumes that there is a cost C to investigate the variance report that includes the time spent to investigate the process, the cost to interupt the process and any cost to correct an OOC process. If the process is OOC then there is a benefit, B, associated with returning the process to standard operation. As a result, there is a cost of B for not investigating a process that is OOC. Thus, there is a net cost of C-B for those OOC processes that are investigated.

Let PR be the probability of the cost process being out of control. Since we would expect that we would investigate the process when the expected benefit exceeds the cost then we would investigate if PR*B > C. These costs are summarized in Table 1.

TABLE 1
Cost Table Associated with Cost Investigation

	State of Nature	
Action	In-Control	Out-of-Control
Accept as is (A)	Zero Cost(Y_1)	B (Y_2)
Investigate (I)	C (Y_3)	C-B (Y_4)

The problem then becomes that of estimating $PR = P(OOC|x)$, where x is the evidence of variance, e.g., actual aggregated usage or expenditures in the most recent month. We assume a discrete probability distribution, although the results could be easily extended to a continuous distribution. The two probabilities $P(OOC|x)$ and $P(x|IC)$ are related through Bayes' Theorem. Let $PR'=1-PR$.

$$PR = P(OOC|x) \tag{1}$$

$$PR = P(OOC \cap x)/P(x) \tag{2}$$

$$PR = P(x|OOC)P(OOC)/[P(x|OOC)P(OOC)+P(x|IC)P(IC)] \tag{3}$$

$$PR = P(x|OOC)PR'/[P(x|OOC)PR'+P(x|IC)(1-PR')] \tag{4}$$

Thus, in order to compute PR we need to know the properties of the probability distribution of x, the probability of being in-control or out-of-control and the properties of the distribution to compute $P(x|OOC)$.

3 FUZZY SET THEORY

Fuzzy set theory was first proposed by Zadeh (ref. 6) as an alternative to ordinary set theory, wherein the membership of objects is precisely determined. The central concept of fuzzy set theory is the membership function which represents numerically the degree to which an element belongs to a set. The following four fundamental definitions are related to operations of fuzzy sets in cost variance investigation:

3.1 Definition 1

Let X be the universe, $X=\{X_1, \ldots, X_n\}$, where X_i is an element of the X. The fuzzy set A of X is a set of ordered pairs $[X_i, f_A(X_i)]$ for all X_i in X. $f_A(X_i)$ is the membership function that takes its values in the membership set $X \text{-->} [0,1]$. The larger $f_A(X_i)$, the stronger the degree of membership of X_i in A. For example, if $X = \{.05,.10\}$ represents a set of possible deviation rates and A represents either IC or OOC then, e.g., $f_{IC}(.05)$ might equal .90, whereas $f_{OOC}(.05)$ would equal .10.

3.2 Definition 2

Let B and C be two fuzzy sets over X. The intersection of B and C, denoted $B \cap C$, is a fuzzy set A over the universe X, where the membership function for A is:

$$f_{B \cap C}(X_i)=f_A(X_i)=\min\,[f_B(X_i),f_C(X_i)], \quad (5)$$

for all X_i in X. For example, $f_B(X_i)$ and $f_C(X_i)$ can be used to represent the manager's and the staff's degree of membership functions, so that f_A would represent the overal membership function.

3.3 Definition 3

Let A be a fuzzy set over X. The complement of A, denoted A', is a fuzzy set over X, where the membership function is:

$$f_{A'}(X_i)=1-f_A(X_i), \quad (6)$$

for all X_i in X.

For example, f_A may represent the degree of membership in an in-control situation, whereas $f_{A'}$ would represent the membership function in an out-of-control situation.

3.4 Definition 4

Let A be a fuzzy set over X and $P(X_i)$ be the probability of the element X_i. The probability of the fuzzy set A is:

$$P(A)=E[f_A(X_i)]=\sum_{i=1}^{n} f_A(X_i)P(X_i). \quad (7)$$

For example, if $f_{IC}(X_i)$ represents the degree of membership in an in-control situation and $P(X_i)$ is the probability of a specific deviation rate occurrence, then P(A) is the probability of an in-control situation.

3.5 Fuzzy Sets--Summary

The first three sets of definitions were taken from Zadeh (ref. 5) and Yager and Basson (ref. 4), while the last definition was taken from Zadeh (ref. 6).

In trying to use concepts for actual problems, a procedure must be employed for inferring or estimating the membership functions corresponding to individual opinion of the management team members. Saaty (ref. 3) has developed an approach to this problem based a pairwise comparison matrix which is not the subject of this paper.

4 PREVIOUS FUZZY SET APPROACHES TO COST VARIANCE INVESTIGATION

Zebda (ref. 7) developed an approach to cost variance investigation that expanded the number of states of nature from the basic model discussed above from two to n and the set of actions from two to m. He then developed an empirical study that focused on n=3 and m=2. That empirical study illustrates the importance and the applicability of a fuzzy set theory-based approach in variance report analysis.

However, that approach has two primary limitations. First, it ignores the assessment of the system by the management team. Since the state of the system is a fuzzy varible and there are multiple team members this can be an important variable. An alternative formulation would allow the incorporation of the management team. Second, that approach requires the *a priori* assessment of the number of states of nature and the number of actions. As a result, this does not exploit the basic fuzzy nature of the processes and states of nature. An alternative approach would allow the nature of the deviations in the variance analysis to drive the problem formulation.

Thus, in Zebda's empirical study that was conducted, only the manager was incorporated in the study, the number of states was limited to three ("in-control", "more or less out-of-control" and "out-of-control") and the number of decisions was limited to two ("investigate" and "do not investigate").

The choice of three states and two decisions may indicate that managers think in fuzzy terms for states of nature and yet think of variance analysis decisions in a binary manner.

5 A FUZZY SET MODEL OF COST VARIANCE INVESTIGATION

After the variance report is received, the manager and staff identify the set of costs in table 1. The manager and staff also identify a set of system attributes, e.g., deviation rates $X=\{X_1, \ldots, X_n\}$ associated with the investigation into evidence that indicates the state of nature. The manager and staff also develop the membership functions, using Definition 1 and Definition 2, and probability density functions associated with those deviation rates.

Associated with each deviation rate is a probability of that deviation rate occuring, $P(X_i)$, such that the sum of those probabilities is equal to one. In addition, associated with each of those deviation rates, there is a fuzzy measure, $f_j(X_i)$, j=IC or OOC, for each member of the investigation team. Each $f_j(X_i)$ provides the estimate of the system membership in either IC or OOC categories, given the deviation rate. Definition 4 provides the method to combine those fuzzy measures to determine the fuzzy measures of the states of nature.

Then the following cost investigation steps will be conducted.

First, cost investigation evidence on the possible deviation rates is obtained through sampling from a discrete probability distribution for a particular sample size, n. This liklihood of sample result will be denoted as $P(k|X_i)$, where k is the possible outcome of deviation frequency (k=0,1, ...,n).

Second, posterior probabilities of X can be derived through Bayesian revision using the following formulas.

$$P(X_i|k)=[P(k|X_i)P(X_i)]/[\sum_i P(k|X_i)P(X_i)]. \tag{8}$$

Third, using the fuzzy set theory definition 4, the probabilities of IC and OOC, given X_i and k, can be computed as follows.

$$P(IC|X,k) = \sum_i f_{IC}(X_i)P(X_i|k), \tag{9}$$

$$P(OOC|X,k)= \sum_i f_{OOC}(X_i)P(X_i|k). \tag{10}$$

Fourth, the expected cost of each alternative can then be computed. The optimal decision under each sample outcome k will be determined by chosing the minimal quantity. Finally, the expected cost of sampling with size n will be determined by chosing the optimal quantity over all of the possible choices of k.

6 EXAMPLE

This section presents an example to illustrate the above approach. The example assumes a set of costs of possible state-action combinations in table 1 as follows (ref. 1):

Y_1= 0 in-control and not investigated

Y_2= 160 cost of failure to investigate
Y_3= 30 cost of investigation when in-control
Y_4= 130 cost of uncovering the reasons for being out-of-control

The management team identifies the number of system deviation rates to be considered, n=2, and the system deviation rates $X=\{X_1,X_2\}=\{.05,.20\}$. Generally, in most applications n is not limited to two alternatives.

We will assume a multinomial probability distribution. Since n=2, this means that the distribution is a binomial distribution.

$$P(k|X_i)=(n!/k!(n-k)!)\ P(X_i)^k\ [1-P(X_i)]^{n-k}. \quad (11)$$

For the example, we will assume that $P(X_1)=.9$ and $P(X_2)=.1$.

The management team also develops the overall membership functions by using Definition 4 as displayed in the following table.

TABLE 2
"In-control" membership functions

Deviation Rate	Manager	Assistant 1	Assistant 2	Membership
X_1	.95	.90	.99	.90
X_2	.35	.50	.30	.30

Using Definition 3, "Out-of-control" membership functions become $f_{OOC}(X_i)$= .10 and $f_{IC}(X_i)$= .70.

Assume that k=2. This means that equation (11) becomes
$P(k=2|X_1=.05)=(8!/2!6!)(.9)^2(.1)^6$ = .00002 and
$P(k=2|X_2=.20)=(8!/2!6!)(.1)^2(.9)^6$ = .1488.

Thus, equation (8) becomes
$P(X_1|X,k=2)=(.00002*.9)/(.1488*.1 + .00002*.9)= .001$
$P(X_2|X,k=2)=(.1488*.1)/(.1488*.1 + .00002*.9)= .999$

Equations (9) and (10) becomes
$P(IC|k=2)= .9 * .001 + .3 * .999 = .3006$
$P(OOC|k=2)= .1 * .001 + .7 * .999 = .6994$

For k=2, that is, two deviations in eight observations, the minimum cost is determined by chosing between accepting as is or investigating:
Accept as is cost = \$0 * .3006 + \$160 * .6994 =\$111.90
Investigate cost = \$30 * .3006 + \$130 * .6994 =\$99.94

Thus, for k=2 the optimal approach is to investigate. Then the overall optimium is determined by chosing the minimum cost over all k=0,...,8.

7 CONCLUSIONS

This paper has incorporated fuzzy set concepts into the cost investigation process by allowing the management team to assess the membership of the system as IC or OOC, based on the deviation rate of the data sample resulting from the investigation. Bayesian analysis provides the manager with an opportunity to make the best decision under uncertainty based on probability specification and evaluation of sample evidence structured in terms of expected payoffs.

The representation of the cost variance investigation problem using fuzzy sets has a number of potential advantages:

1. The fuzzy set approach conforms with the way that people actually discuss control processes. Managers use terms such as little, small, large, slightly or tending to describe cost variance processes.
2. The fuzzy set approach allows us to "measure" the membership of a process in a group. Fuzzy measurement provides an expanded measurement scale (How much "in-control" is a cost variance process?) over binary measurement (Is the cost variance procedure "in-control" or "out-of-control"?).
3. The amount of cost variance investigation is inherently imprecise because it depends on human interpetation. However, this paper provides a means for determining an appropriate amount of effort investigation.

There are at least three limitations of this paper. First, the model is only a single period model. Future research could develop multiple period models. Second, the model is developed using only multinomial sampling. Future research could expand to other distributions, including continous distributions. Third, this paper ignores such management concerns as the implications of organizational heirarchies on the membership measures inherent in organizational settings.

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