1 Introduction

Blogging used to be a personal hobby in the private domain. However, skilled bloggers can make a living by publicizing their blogs and amassing hundreds of thousands of readers and followers. Many blogs offer information and news related to a specific area such as technology, health, and lifestyle. Among the most successful blogs, TechCrunch, which offers news and information on things related to technology, was sold to AOL for $25 million in 2010. Other well-known blogs include Smashing Magazine, Business Insider, and The Daily Beast, all of which have a wide reader base and massive daily traffic. One thing these blogs have in common is that they offer tips and advice dedicated to a specific area. For example, Lifehacker specializes in hints and tips to make people’s daily life easier; Smashing Magazine offers advice and resources to web designers and developers. There are many others that thrive in areas such as travel, food, and fashion.

Bloggers provide information to consumers by making product recommendations. More generally, any kinds of recommendation and reviews sites, guides, and ratings systems (e.g. Consumer Reports, Lonely Planet, Angies List, etc) are effectively making recommendations to consumers. Reviews are also a form of recommendation in that a good review is essentially a positive recommendation. However, all these review and recommendation systems are not retailers, nor are they usually affiliated with the sellers of products. The majority of these blogs’ revenue comes from
pay-per-view and pay-per-click ads. Therefore in order to be profitable, the blogs must generate traffic and attract visitors. And one of the best ways to attract eyeball is to provide useful information to consumers that help them with their purchase decision. Consumers are willing to follow the recommendation of a blogger known to give good advice, as this could reduce their search cost. However, not all bloggers give good advice, and consumers cannot always tell whether a blogger is good, especially if the blogger is new and has not had an established reputation. Although a blogger may recommend a product without interacting with the firm that sells it, it may be advantageous for the firm to secure the recommendation of a reputable blogger who can sway the consumers. But in blogosphere, we don’t always observe blogger-firm affiliations whereby the blogger would make product recommendations exclusively for a firm. On the contrary, we often find that bloggers don’t work for any particular firm and make recommendations for many different products. Since the recommendations are a form of online word of mouth, intuitively we may think that a firm would want a blogger with a very good reputation to recommend only its own product. But a blogger’s good reputation comes from making as many good recommendations as possible. How could the firms and the blogger reconcile their seemingly conflicting incentives? Under what conditions would the firm want to pay the blogger to make a recommendation? And how would the consumers react to a bloggers recommendations with and without firm sponsorships?

2 Literature Review

The effects of recommendations on consumers’ purchase decision have been studied in prior literature. Mayzlin (2006) investigated a setting in which consumers receive a mixture of unbiased recommendations from other consumers and biased promotional messages that are indistinguishable. However, in our model the consumers can actually observe the source of the recommendations – whether they come from independent or sponsored bloggers.
Bloggers also act as media through which firms inform the consumers about the product. Similar to an advertising medium such as a TV station, bloggers offer information to consumers and attract advertisers. However, making recommendations is not the primary way for the bloggers to generate income. Instead, recommendations are the “programming” that attract visitors to the blogs. Therefore, the blogger’s recommendations are different from the “advertisement” in Gal-Or and Dukes (2003). Also, unlike other advertising media and platforms, the bloggers do not receive a share of the profits on product sales; hence they do not care about the sales of the products but improving their reputation. This distinguishes the issue we address from the standard advertising problem.

The current topic is also related to product endorsement. Very well-known bloggers are effectively cyber-celebrities who enjoy a certain level of fame and recognition that make them comparable to conventional celebrities. When a blogger recommends a product, it is as if she “endorsed” the product, especially when she is paid to make the recommendation. But a blogger’s reputation is tied to her product recommendations to a greater extent relative to a celebrity. A movie star cares about how others perceive her skills as an actress, which would be not compromised by her act of endorsement. But a bloggers reputation is based on how others perceive her ability to recommend good products. Therefore, a blogger’s recommendation directly affects her reputation. It is not uncommon for individuals to engage in word of mouth as a means to improve others perception of them (Campbell, Mayzlin, and Shin, 2013).

Similar to conventional celebrities, bloggers could be sponsored by firms to recommend products. Product endorsement and promotion by conventional celebrities has been a broadly studied topic in the consumer behavior literature. The majority of these studies explain the often observed exclusivity in endorsement relationships from a credibility angle; they generally agree that an effective endorsement requires a credible endorser, and non-exclusivity hurts the credibility of the endorser. Tripp, Jensen, and Carlson (1994) finds that a higher number of products endorsed de-
creases the perceived credibility of the celebrity, as well as the likability of and attitude toward that celebrity. When a celebrity endorses a large volume of different products, consumers infer that the celebrity is motivated by monetary gains only and not a genuine belief in the quality of the products (Carroll, 2009). Furthermore, Spry, Pappu, and Cornwell (2009) suggests that endorser credibility indirectly affects brand equity. While conventional wisdom says that endorsing multiple products hurts the endorsers reputation as well as the consumers perception of the products, the current paper finds the opposite. We find that if a blogger makes an exclusive recommendation, it actually hurts her reputation.

3 Model

There is one firm that sells a product to a consumer at price $p$. The product may or may not be a good fit for the consumer, and neither the firm nor the consumer observe the product’s fit ex ante. That is, let $F$ denote the actual fit of the product, $F \in \{G, B\}$, where $G$ stands for “good fit” and $B$ stands for “bad fit.” The consumer receives utility $v$ from the product if it is a good fit, and 0 otherwise: $U(G) = v$ and $U(B) = 0$. The prior probability that the product is a good fit is $\rho_0$.

By examining the product, the blogger can costlessly receive a noisy binary signal on the product’s fit to the consumer: $s \in \{g, b\}$, where $g$ stands for “observed good fit” and $b$ stands for “observed bad fit.” We assume that some bloggers are more reliable judges on what consumers will like. To model this idea we assume that there are two types of bloggers: the high-skilled (type $H$) blogger receives a more precise signal on the product fit than the low-skilled (type $L$) blogger. That is, the blogger of type $H$ receives a signal $s_H$ where $\text{Prob}(g_H | G) = \text{Prob}(b_H | B) = \gamma_H$, and the blogger of type $L$ receives a signal $s_L$ where $\text{Prob}(g_L | G) = \text{Prob}(b_L | B) = \gamma_L$. To model the fact that the high-skilled type gets a more precise signal we assume that $\gamma_H > \gamma_L$. The blogger may choose to post her review of the product. We denote a positive recommendation by $m = g$, a
negative recommendation by $m = b$, and no post by $m = \emptyset$.

The firm may make a take-it-or-leave-it ($R > 0$) offer to the blogger in exchange for a positive recommendation. If the blogger accepts the offer, $(a = 1)$, she will have to write a positive review for the firm’s product regardless of the signal $(m = g)$, but she must also publicly disclose her affiliation with the firm. After observing the blogger’s recommendation and her relationship with the firm, $a \in \{0, 1\}$, the consumer makes a purchase decision. We assume that subsequently the actual fit of the product is always revealed.

The timing is the following. At $t = 1$, Nature chooses product fit and blogger type, which is not observable to any of the players. At $t = 2$, the firm may make a take-it-or-leave-it offer to the blogger in exchange for a positive recommendation, which the blogger can accept or reject. At $t = 3$, the blogger receives a signal on the product fit. If the blogger accepted the firm’s offer, at $t = 4$ she publicly posts a positive recommendation of the product and reveals her affiliation with the firm. If the blogger did not accept the firm’s offer, at $t = 4$ she can choose whether to post a review of the product, which may be positive or negative. At $t = 5$, the consumer observes whether the blogger posted a review, whether the review is positive or negative, and whether the blogger is independent or affiliated with the firm. He then makes a purchase decision based on his posterior belief on the product fit. At $t = 6$ the true fit of the product is revealed, which allows the consumer to make an inference on the blogger’s type.

3.1 The Consumer’s Inference on Product Fit

Let’s examine the consumer inference on product fit at $t = 5$ after he observes the blogger’s affiliation and product recommendation. At this stage the consumer’s information set is, $\psi = [a \in \{0, 1\}, m \in \{\emptyset, g, b\}]$. The consumer will purchase the product if and only if the expected utility
is (weakly) positive: \( EU(\psi) = \omega(\psi)v - p \geq 0. \)

### 3.1.1 The Blogger is Independent

Let’s first consider consumer inference in the scenario where he observes that the blogger is independent: \( a = 0. \) Since the blogger is independent of the firm, she can share either a positive \((m = g)\) or a negative \((m = b)\) recommendation. Let’s assume for now (and show later) that here the blogger posts a positive review if her signal was positive \((m = g \text{ if } s = g)\) and posts a negative review if her signal was negative \((m = g \text{ if } s = g)\).

The consumer’s posterior belief \((\omega_{a=0})\) on the product fit after seeing the blogger’s recommendation are the following

\[
Prob(G \mid g, a = 0) \equiv \omega_{a=0}^+ = \frac{\text{Prob}(g \mid G) \cdot \text{Prob}(G)}{\text{Prob}(g \mid G) \cdot \text{Prob}(G) + \text{Prob}(g \mid B) \cdot \text{Prob}(B)} \quad (1)
\]

\[
Prob(G \mid b, a = 0) \equiv \omega_{a=0}^- = \frac{\text{Prob}(b \mid G) \cdot \text{Prob}(G)}{\text{Prob}(b \mid G) \cdot \text{Prob}(G) + \text{Prob}(b \mid B) \cdot \text{Prob}(B)} \quad (2)
\]

The consumer updates her belief upwards after seeing \( m = g \) and lowers it after seeing \( m = b: \omega_{a=0}^- < \rho < \omega_{a=0}^+ \). Note that the posterior belief on product fit depends on the accuracy of the blogger’s signal - \( \text{Prob}(g \mid G) \) and \( \text{Prob}(b \mid G) \). Since we assume that some bloggers receive more accurate signals, and the blogger’s type is not known a priori, the probability that the blogger receives the correct signal \( (g \text{ if the product is a fit and } b \text{ if the product is not a fit}) - \text{Prob}(g \mid G) = \text{Prob}(b \mid B) = \gamma_H \theta + \gamma_L (1 - \theta). \) Conversely, the probability that she receives the wrong signal \( (g \text{ if the product is not a fit and } b \text{ if the product is not a fit}) - \text{Prob}(g \mid B) = \text{Prob}(b \mid G) = (1 - \gamma_H) \theta + (1 - \gamma_L)(1 - \theta). \)

Combining this with Equations 1 and 2 yields the following expressions for the posterior
beliefs following a positive and a negative recommendation, respectively:

\[
\omega^+_{a=0} = \frac{[\gamma_H \theta + \gamma_L (1 - \theta)]\rho}{[\gamma_H \theta + \gamma_L (1 - \theta)]\rho + [(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)](1 - \rho)} 
\]  

(3)

\[
\omega^-_{a=0} = \frac{[(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)]\rho}{[(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)]\rho + [\gamma_H \theta + \gamma_L (1 - \theta)](1 - \rho)} 
\]  

(4)

If the blogger makes no recommendation, the consumer’s belief will depend on the equilibrium behavior of the blogger. For now we simply denote it as \(\omega^0_{a=0}\)

The consumer’s expected utility equals the probability that the product is a good fit times the value to the consumer when the product is a good fit. The expected utility of the consumer is:

\[ EU_{\text{consumer}} = \rho v \]  

(5)

Let \(p\) be the price of the product. The consumer will buy as long as his expected utility of the product exceeds its price: \(EU_{\text{consumer}} - p \geq 0\). Equivalently, the consumer will buy as long as his belief on the product is high enough. Given the consumer’s prior belief of the product, the consumer is indifferent between buying and buying:

\[ \rho_0 = \frac{p}{v} \]  

(6)

The actual fit of the product will eventually be revealed as information gets diffused. The belief on the blogger type will be updated depending on the actual fit of the product. If the blogger independently made a positive recommendation, and the recommendation is correct, then
the consumer’s posterior belief on the blogger type is $\text{Prob}(i = H \mid g, G)$, which is equal to

$$
\frac{\text{Prob}(g, G \mid i = H) \cdot \text{Prob}(H)}{\text{Prob}(g, G \mid i = H) \cdot \text{Prob}(H) + \text{Prob}(g, G \mid i = L) \cdot \text{Prob}(L)}
$$

$$
= \frac{\gamma_H \theta}{\gamma_H \theta + \gamma_L (1 - \theta)} \equiv \mu^+ \quad (7)
$$

However, if the product turns out to be a bad fit, then the consumer’s posterior belief on the blogger type is $\text{Prob}(i = H \mid g, B)$, which is equal to

$$
\frac{\text{Prob}(b, G \mid i = H) \cdot \text{Prob}(H)}{\text{Prob}(b, G \mid i = H) \cdot \text{Prob}(H) + \text{Prob}(b, G \mid i = L) \cdot \text{Prob}(L)}
$$

$$
= \frac{(1 - \gamma_H) \theta}{(1 - \gamma_H) \theta + (1 - \gamma_L)(1 - \theta)} \equiv \mu^- \quad (8)
$$

where $\mu^-$ is a negative signal the blogger sends. If the consumer does not buy, then he won’t be able to observe the actual fit of the product and thus he won’t be able to update his belief about the blogger’s type.

However, if the blogger made a paid recommendation, the consumer’s inference will depend on the equilibrium behavior of the blogger. In other words, it depends on whether there is a separation on blogger type regarding whether the blogger would want to work for the firm. For example, if a high type blogger would work for the firm in equilibrium, then the consumer would infer that the blogger is a high type whenever he sees a paid recommendation.

### 3.2 The Blogger’s Incentive

$\theta$ could be interpreted as the blogger’s reputation. The higher the prior belief that the blogger is the high type, the more likely the consumer will follow the blogger’s recommendation. The blogger benefits from having a high reputation. Consumers will be more inclined to follow the blogger’s advice when the blogger has a reputation of offering good advice. In the real world, the profitability
of a blog is directly tied to site traffic and the number of followers. The higher the traffic and the larger the number of followers, the more revenue the blog generates. Therefore, the blogger’s goal is to improve her reputation.

Suppose there is a some cutoff value $\theta$ above which the blogger will experience a gain in signaling her type. This could be that once her reputation exceeds a certain level, she will attract more readers and her site traffic will increase. Let $\mu$ be the posterior belief on the blogger type, then the blogger’s payoff to signaling is:

\[
H(\mu) = \begin{cases} 
    h & \text{if } \mu \geq \theta^* \\
    0 & \text{if } \mu < \theta^* 
\end{cases}
\]  

(11)

After the true product fit gets revealed, if the blogger makes a correct recommendation, the posterior on the blogger is $\mu^+$; if the blogger makes an incorrect recommendation, the posterior on the blogger is $\mu^-$. If $\mu^- > \theta^*$, then the blogger will have nothing to lose even when she is wrong about the product and sends a negative signal $\mu^-$ on her type. If $\mu^+ < \theta^*$, the blogger will never benefit from signaling even when she sends a positive signal, so she would not want to signal. Therefore, the blogger would want to signal only if $\mu^- < \rho < \mu^+$. She is indifferent between signaling and not when $\theta = \theta^*$. This is because in this case the blogger does not have any extra gain from sending a positive signal, but she will be worse off if when sends a negative signal. Therefore, if the blogger’s prior is too high, she will not want to signal and risk ending up with a lower posterior. Hence $\theta^*$ is upper bound on the prior for which the blogger would want to signal. On the other hand, the blogger’s prior could be too low such that even if she sends a positive signal, her posterior will jump above $\theta^*$, in which she would not want to signal either. Let $\underline{\theta}$ be the lower bound on the prior for which the blogger would still want to signal. Then the posterior given $\underline{\theta}$ should be greater than $\mu^+$.

\[
\frac{\gamma_H \underline{\theta}}{\gamma_H \underline{\theta} + \gamma_L (1 - \underline{\theta})} = \theta^* \implies \underline{\theta} = \frac{\theta^* \gamma_L}{\gamma_H - \theta^* (\gamma_H - \gamma_L)} 
\]  

(12)
Therefore, the blogger would want to signal if her prior is between \( \theta \) and \( \theta^* \).

The blogger’s reputation can only improve when she is right about a product. Therefore, the blogger wants to recommend as many correct recommendations as possible. However, when the blogger chooses to signal, there is a risk that her reputation will be hurt when she makes a mistake, as the blogger is unsure about her type and the signal she receives from examining the product is noisy. The blogger’s value from signaling is her expected gain from signaling, which equals the probability that she is right times the reward when she is right:

\[
EH(\mu) = (\theta \gamma_H + (1 - \theta) \gamma_L) h
\]  

(13)

### 3.3 The Firm’s Profits

The firm has two actions, either pay the blogger to write a positive recommendation or not. Let the firm’s payment to the blogger be a function of the blogger’s prior: \( S(\theta) \). Then the firm’s profit is:

\[
\Pi_{\text{pay}} = \begin{cases} 
p - S(\theta) & \text{if consumer buys} \\
-S(\theta) & \text{if consumer does not buy} 
\end{cases}
\]  

(14)

### 3.4 Equilibrium

If the blogger accepts the firm’s offer, she is paid to make a positive recommendation. But then the consumer can no longer make an inference on blogger type based on the blogger’s recommendation. This is because the blogger does not know her type before taking the firm’s offer, so it could not be the case that a high type or low type blogger will always make a paid recommendation in equilibrium. The consumer’s belief on the blogger type will remain the same. Therefore, by accepting the firm’s offer, the blogger loses the opportunity to signal her type. Hence, the firm must pay the blogger.
enough such that she is indifferent between being able to signal and not. When the blogger’s initial
reputation is sufficiently low (θ < θ) or sufficiently high (θ > θ∗), she is indifferent between signaling
and not. However, when the blogger’s reputation is in the intermediate range (θ < θ < θ∗),
the blogger has a positive expected gain from signaling relative to no signaling and needs to be
compensated for the loss of this gain when she cannot signal. Therefore, the firm’s payment to the
blogger is:

\[
S(\theta) = \begin{cases} 
0 & \text{if } \theta < \theta \\
(\theta \gamma_H + (1 - \theta) \gamma_L) h & \text{if } \theta < \theta^* \\
0 & \text{if } \theta > \theta^* 
\end{cases} 
\]  

(15)

Proposition 1. A blogger benefits from signaling her type to the consumer only if θ < θ < θ∗.

If the blogger is independent, she would want to signal only when her prior lies in [θ, θ∗]. If
her prior is greater than θ∗, she would not want to signal and expose herself to the risk of a lower
posterior. If her prior is less than θ, she has no incentive to signal either because even with a positive
update her posterior is still not high enough for her to receive any benefits. If the firm wants to pay
the blogger, it needs to pay her the expected benefit from signaling, as shown in (16).

Proposition 2. Let \( ρ = \frac{\frac{\Delta \gamma}{\gamma_H + \gamma_L} - \frac{\gamma_L}{\frac{\gamma_L}{\frac{\gamma_L}{\gamma_H + \gamma_L}}}}{1 - \frac{\theta \Delta \gamma + \gamma_L}{\frac{\gamma_L}{\frac{\gamma_L}{\gamma_H + \gamma_L}}}} \), and \( \bar{\rho} = \frac{\frac{\gamma_L}{\frac{\gamma_L}{\gamma_H + \gamma_L}}}{1 - \frac{\theta \Delta \gamma + \gamma_L}{\frac{\gamma_L}{\frac{\gamma_L}{\gamma_H + \gamma_L}}}} \).

where Δγ = (γH − γL). Let \( Π_{pay} \) and \( Π_{don’t\ pay} \) denote the firm’s expected profits. Then, the firm’s
optimal strategy is as follows:

1. If \( ρ \in [0, \bar{\rho}] \), then \( Π_{pay} = -S(\theta) \) and \( Π_{don’t\ pay} = 0 \). The firm will not pay the blogger.

2. If \( ρ \in [\bar{\rho}, \frac{\Delta \gamma}{\gamma_H + \gamma_L}] \), then \( Π_{pay} = -S(\theta) \) and \( Π_{don’t\ pay} = Prob(+) \cdot p \), where \( Prob(+) \) is the probability
   that an independent blogger makes a positive recommendation. The firm will not pay the
   blogger.

3. If \( ρ \in \left[\frac{\Delta \gamma}{\gamma_H + \gamma_L}, \bar{\rho}\right] \), \( Π_{pay} = p - S(\theta) \) and \( Π_{don’t\ pay} = Prob(+) \cdot p \). The firm will pay the blogger if
(a) \[ |p(1 - 2\rho)| > h \text{ and } \theta > \frac{\gamma_L - [\gamma_L(1 - 2\rho) + \rho]p}{(\gamma_H - \gamma_L)(h - p(1 - 2\rho))}, \text{ or} \]

(b) \[ [\gamma_L(1 - 2\rho) + \rho]p > \gamma_L h \text{ and } p(1 - 2\rho) < h \text{ for all } \theta. \]

4. If \( \rho \in [\bar{\rho}, 1] \), then \( \Pi_{\text{pay}} = p - S(\theta) \) and \( \Pi_{\text{don't pay}} = p \). The firm will not pay the blogger.

Proof. See Appendix.

Proposition 2 says the following: 1) when the prior on the product is very low such that even if an independent blogger gives a positive recommendation that leads to a higher posterior on the product, it is still not high enough to induce a purchase. Since paying the blogger does not improve the firm’s profit in this case, the firm will not pay the blogger. 2) when the prior on the product is a little higher such that an independent blogger’s positive recommendation may lead to a posterior on the product fit that is high enough to induce a purchase, the firm will not pay the blogger. 3) the prior on the product is high enough such that the consumer will buy without the blogger’s recommendation. But in this case a negative recommendation may dissuade the consumer from buying. Then, if the cost of paying the blogger is not prohibitively high, the firm will pay the blogger to eliminate the such a risk. 4) when the prior on the product is very high such that even if the consumer updates his belief on the product downward, his posterior will still be high enough for him to buy the product. In this case, there is no risk to the firm, and thus the firm will not pay the blogger.

4 Conclusion

The success of a blogger relies on the quality of the information she provides, which in turn determines her credibility or reputation. Therefore, although sometimes they may voluntarily promote products without interacting with the firms, they have no intention to act in the interest of the sellers of the products. In this model, the main objective of the blogger is to signal that she is skillful and good
at discerning products. In the previous section we have shown that it is not always more profitable for a firm to have a blogger exclusively recommend the product. When the prior on the product is low, the consumer will not buy without learning. In this case, it is important for the firm to allow credible signaling on the part of the blogger.

However, when the prior belief on the product is high, there is a chance that the consumer’s belief on the product will update downward upon a negative recommendation. By paying the blogger, the firm can eliminate a potential downward update so that the consumer will make a purchase as if there was no learning.

One key feature of this paper is that the firm’s and the blogger’s incentives are clearly misaligned. The blogger is indifferent between what kind of recommendation she makes, as long as she is right about the product. In contrast, the kind of recommendation has an impact on the firm’s sales. A positive recommendation may lead to purchase while a negative recommendation may not. The blogger wants to improve the belief on her type and does not care about whether the consumer would buy the product, whereas the firm wants the consumer to buy the product and does not care about what others think about the blogger to the extent that it does not affect sales\textsuperscript{1}. However, the firm can achieve coordination by “buying off” the blogger — paying her enough so that she is willing to give up signaling.

\section{Future Research}

To enrich this model, the assumption that the actual fit of the product gets automatically revealed at some point in the future should be relaxed, and the process of information diffusion should be incorporated into a dynamic model. One way to model this is to allow the consumer to have private information about the product fit after they purchase the product. This may cause the

\footnote{\textsuperscript{1}although the firm would still indirectly care about the blogger’s reputation as the firm’s payment to the blogger is a function of $\theta$}
blogger’s recommendation to be biased, resulting in an asymmetry between positive and negative recommendations, since the consumer would not be able update his belief on the blogger without a purchase. In this case, a high reputation blogger may be biased toward negative recommendation in order to reduce the risk of being wrong about the product. Lastly, in the simple setting of this paper, the blogger is only intersecting with one firm. But in reality, the blogger does not just consider a single product but many so that before she can decide what kinda of recommendation to make, she first needs to decide which products to look at.

6 Appendix

Proof. Proposition 2

1. Case I: $\rho \in [0, \bar{\rho}]$. In his case the prior on the product is very low such that even with a positive update, the consumer will not buy. Let $\rho$ be the minimum value of $\rho$ such that $\omega^+ = \frac{p}{v}$.

$$\frac{[\gamma_H \theta + \gamma_L(1 - \theta)]\rho}{[\gamma_H \theta + \gamma_L(1 - \theta)]\rho + \{(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)(1 - \rho)\}} = \frac{p}{v} \quad (16)$$

$$\rho = \frac{\frac{p}{v} (\theta \Delta \gamma + (1 - \gamma_L))}{\theta \Delta \gamma + \gamma_L - \frac{p}{v} (2 \theta \Delta \gamma + 2 \gamma_L - 1)}, \text{ where } \Delta \gamma = (\gamma_H - \gamma_L) \quad (17)$$

2. Case II: $\rho \in [\bar{\rho}, \frac{p}{v}]$. The prior of the product is still low, but with a positive update on the product, the consumer’s posterior belief $\omega^+$ may just be high enough that he will buy. If the firm pays the blogger, the consumer will have no update on the product and thus not buy, and the firm’s expected profit is $\Pi_{pay} = -S(\theta) < 0$. If the firm does not pay the blogger, then the blogger will make a positive recommendation with probability $\rho[\gamma_H \theta + \gamma_L(1 - \theta)] + (1 - \rho)[(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)] = \text{Prob}(+)$. Since this probability is greater than zero, the firm’s profit if it does not pay the blogger is $\Pi_{don't pay} = \text{Prob}(+) \cdot p > 0$.

3. Case III: $\rho \in [\frac{p}{v}, \bar{\rho}]$. The prior on the product is high enough for the consumer to buy. But if
the blogger makes a negative recommendation, the consumer’s posterior on the product will fall below the cutoff such that the consumer will not buy. Let the $\bar{\rho}$ be the maximum level of $\rho$ below which a negative recommendation will lead to non-purchase. If the firm pays the blogger, the consumer will buy for sure as there will be no change to the belief on the product, and $\Pi_{pay} = p - S(\theta)$. If the firm does not pay the blogger, $\Pi_{don’t pay} = \text{Prob}(+)p$, where $\text{Prob}(+)$ is the probability that the blogger makes a positive recommendation.

$$\Pi_{pay} > \Pi_{don’t pay} \implies p - S(\theta) > \text{Prob}(+)p \quad (18)$$

$$\quad (1 - \text{Prob}(+))p > S(\theta) \quad (19)$$

$$\quad \text{Prob}(-)p > S(\theta) \quad (20)$$

$$\rho[(1 - \gamma_H)\theta + (1 - \gamma_L)(1 - \theta)] + (1 - \rho)[\gamma_H\theta + \gamma_L(1 - \theta)]p > \theta(\gamma - \gamma_L)h \quad (21)$$

$$\rho < \frac{(\theta(\gamma_H - \gamma_L) + \gamma_L)\theta}{\theta(2\gamma_H - \gamma - \gamma_L) + 2\gamma_L - 1} \equiv \bar{\rho} \quad (22)$$

Since $\gamma_H > 1/2$, $\gamma_L < 1/2$, $\theta(2\gamma_H - \gamma - \gamma_L) + 2\gamma_L - 1 > 0$. If $\rho_0 < \bar{\rho}$, consumers will not buy, and the firm will have an incentive to pay the blogger.

4. Case IV: $\rho \in [\bar{\rho}, 1]$. When the prior on the product is very high such that even with a negative update on the product, the consumer will still. In this case, if the firm pays the blogger, $\Pi_{pay} = p - \$s(\theta)$, and $\Pi_{don’t pay} = p$. The firm will not pay the blogger.

$\square$

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