Promotional Chat on the Internet

Dina Mayzlin
Yale School of Management, 135 Prospect Street, New Haven, Connecticut 06520, dina.mayzlin@yale.edu

Chat rooms, recommendation sites, and customer review sections allow consumers to overcome geographic boundaries and to communicate based on mutual interests. However, marketers also have incentives to supply promotional chat or reviews in order to influence the consumers’ evaluation of their products. Moreover, firms can disguise their promotion as consumer recommendations due to the anonymity afforded by online communities. We explore this new setting where advertising and word of mouth become perfect substitutes because they appear indistinguishable to the consumer. Specifically, we investigate here whether word of mouth remains credible and whether firms choose to devote more resources promoting their inferior or superior products.

We develop a game theoretic model in which two products are differentiated in their value to the consumer. Unlike the firms, the consumers are uncertain about the products’ quality. The consumers read messages online that help them decide on the identity of the superior product. We find a unique equilibrium where online word of mouth is persuasive despite the promotional chat activity by competing firms. In this equilibrium, firms spend more resources promoting inferior products, in striking contrast to existing advertising literature.

In addition, we discuss consumer welfare implications and how other marketing strategies might interact with promotional chat.

Key words: advertising; word-of-mouth; source credibility; Internet marketing

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1. Introduction
In August 1999, teenagers who frequented online bulletin boards of Britney Spears, a teen pop star, began to receive messages that recommended a new singer: Christina Aguilera. The authors communicated in a style shared by the other members of the communities, which made it difficult for Britney’s fans to distinguish whether the messages they received came from other fans or from marketers. In fact, many of the posts came from employees of Electric Artists, a promotional firm that specializes in community-based marketing campaigns. As part of their campaign to promote Christina Aguilera’s debut album, Electric Artists’ team of “posters” surfed various chat rooms and fan sites in order to generate online discussion and to provide information to potential fans. The campaign was considered a success since Ms. Aguilera’s album debuted at No. 1 on the charts and reached double platinum status (White 1999).

A remarkable feature of this campaign was the means of communication used by the marketers: The Internet enabled the promoters to infiltrate and influence consumers’ conversations. At first, this sounds like a very attractive strategy for marketers of many types of products. After all, online communities have become an important resource for consumers: 84% of Internet users or about 90 million Americans have participated in online groups (Horrigan and Rainie 2001). One reason for Electric Artists’ success was the fact that consumers often offer unsolicited product recommendations online, lending credibility to chat about Christina.

The marketers’ ability to disguise their promotion as consumer recommendations is made possible by the anonymity enjoyed by participants of online communities. Ultimately, our identities as well as our incentives are obscure in the virtual world. Thus, manufacturers can easily listen to the conversations that take place between ostensible consumers as well as actively participate in these discussions. Despite marketers’ ability to pose as consumers, we might question the viability of such marketing efforts in the face of consumer skepticism. Consumers’ awareness of the existence of such anonymous promotion (referred to as “promotional chat” from now on in this paper) could cause them to discount online recommendations. Moreover, as in traditional advertising,
we would expect the competitor to engage in a similar promotion. This paper investigates the long-term viability of such a strategy in the face of competition and consumer skepticism.

Promotional chat is most prevalent in entertainment industries, such as film and music, “to the point that I would say that [buzz marketing] is almost an industry standard for new CD releases.” Besides music and film, “online buzz marketing initiatives have also been used widely for television shows and books, and are being developed for products in a wide range of categories.” Some non-entertainment companies that employed buzz techniques for their products include Bayer, Levi’s, Starwood Hotels (campaigns conducted by Electric Artists www.electricartists.com), Mazda, Green Party (UK-based DMC (www.dmc.co.uk)), Outlast All-Day Lipcolor (Tremor, an in-house division of Proctor & Gamble; www.internetnews.com/IAR/article.php/1490041), Urban Outfitters, Gatorade (M80 Interactive Marketing), KFC, Colgate, and Kraft (Brand Buzz, a division of Young & Rubicam; “Marketers Take It to The Streets,” TheStandard.com, Feb. 26, 2001).

One of the main differences between promotional chat online and traditional advertising is in the interactivity that is inherent in the former. This provides for a potentially more persuasive message compared to traditional advertising but also rules out automated promotion. “Team members send out postings and emails to grab attention in their own words, avoiding marketing-speak” (Friedman 2000). In addition, consumers seem to be aware of the existence of promotional chat. Thus, a credible promotional chat campaign requires specific skills and may be quite labor-intensive, “You can’t just waltz into these communities and start promoting something; promotion only works if it comes from a source that is a member of the community already, or at least has a very clear understanding of how the community works” (Jonathan Carson, BuzzMetrics).

Finally, unlike traditional advertising, there are few regulations governing promotional chat. All the executives we interviewed denied using online communities as a way to attack a client’s competitors, even though one executive mentioned that his posters make comparisons to competitors’ products (such as a film released at the same time as the client’s, for example) if it is relevant to the conversation.5

This paper poses the following three research questions. First, we ask whether word of mouth online remains persuasive to the consumers—consumers listen to online advice—in the presence of promotional chat by rival firms. In previous work (see Banerjee 1992, 1993; Bikhchandani et al. 1992; Avery et al. 1999), word of mouth remains persuasive despite the fact that there might be some noise in the transmission process due to preference heterogeneity or, as in Banerjee (1993), uncertainty whether previous consumers acted on new information or “herded.” However, here the noise in the recommendations is due to strategic firm behavior. Second, we ask whether promotional chat is most valuable for a firm whose product is more appealing to the target segment than the competitor’s product or for a firm whose product is less appealing. In traditional advertising models, firms spend more resources promoting their winners, which results in advertising being a credible signal of quality (see, for example, Nelson 1974, Milgrom and Roberts 1986, Kihlstrom and Riordan 1984). In these models the credibility of the signal is due to the high-quality firm’s ability to recover the costs in repeat purchases or the high-quality firm’s lower costs of production. That is, because the satisfied consumers will promote the high-quality product to their friends, the firm with a better product benefits more from higher sales in the first period. On the other hand, we also observe online recommendations of inferior products and questionable remedies. Third, we explore the relative consumer welfare benefits and losses delivered by promotional chat.

We specify a game theoretic model in which two competing firms hold private information concerning the value of their products to the target segment. The firms send recommendations to the consumer in order to influence her purchase decision. Early adopters who have knowledge about the identity of the superior product also post online recommendations. Thus, online discussions are a mixture of unbiased recommendations as well as promotional activity by interested parties, where the consumer is not able to distinguish the advertising from the unbiased content. The consumer makes an inference about the relative payoffs of the new products based on the recommendation she receives. Her inference is affected by the knowledge that the firms engage in promotional chat.

We find conditions under which firms choose to promote and where online recommendations are persuasive. Thus, the firms’ promotional activity does not turn chat rooms into noise: Consumers are still more likely to hear the truth. Second, we find that it is the firm with the low-quality product that spends more resources on promotional chat, compared to the firm with the high-quality product: firms lie in

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5 From a phone interview with Troy Rutman (VP, Media and Entertainment MKTG Services).

3 From an email interview with Jonathan Carson, CEO of BuzzMetrics.

4 From an email interview with David Balter, CEO of BzzAgent.
equilibrium. The latter is the opposite of the signaling literature result where firms find it more profitable to promote their winners. In Horstmann and Moorthy (2003), as in the present paper, it is also shown that high-quality firms might advertise less, but in their model this is driven by a technological relationship between production capacity and quality; here, it is driven by the fact that word of mouth can substitute for advertising for high-quality firms. The first and second results taken together are surprising: Despite the firms’ incentives to invest more into promoting the less appealing products, the consumers find chat persuasive. We also find that consumer welfare approaches its upper bound with the increase in volume of consumer-generated word of mouth. This finding has implications for regulatory policy.

2. Basic Model

There are two competing firms, A and B, and there are two types of consumers: one risk-neutral uninformed consumer (C) and one informed consumer (D). The firms offer substitute products that differ in their payoff to C and D: They derive a payoff of \( V^H \) from one and \( V^L < V^H \) from the other. We refer to the former as the superior product for expository ease. The firms as well as D observe which product is the superior one, but C does not. We denote the state of the world \( j \) where A is superior by \( j = AB \), and state of the world where B is superior by \( j = BA \). The prior probability on product A delivering more value is \( \text{Pr}(AB) = \theta \).  

The firms and D send out anonymous messages recommending one of the products. We assume that the D posters are motivated by a desire to share their experiences with others, an assumption that is consistent with most word-of-mouth studies cited in the section above. Thus, D posts truthful messages about the firms’ products, while firms post biased messages. After receiving the message online, C makes an inference on the products’ relative value and makes a purchase decision, taking into account that some of the messages may be biased. We constrain C to buying at most one product, but the results do not change as long as at least a fraction of consumers must choose between the two products. Figure 1 presents the timeline of the model.

2.1. Messaging Online

There are two possible types of messages: “AB” — messages that claim that A is the superior product, and “BA” — messages that claim that B is the superior product. Alternatively, we can interpret these messages, respectively, as positive (negative) and negative (positive) word of mouth about A (B). We model anonymity by assuming that C observes only the content of the message and not its source.

Firm \( f \) in state \( j \) chooses the number of messages to send praising its own product:

\[
N_f^I(m) \quad \text{where} \quad j \in \{AB, BA\} \quad \text{and} \quad \begin{cases} m = "AB" & \text{if } f = A \\ m = "BA" & \text{if } f = B \end{cases}.
\]

Because we assume that A always sends "AB" and B always sends "BA," we abbreviate \( N_f^A("AB") \) as \( N_f^A \) and \( N_f^B("BA") \) as \( N_f^B \).

D sends messages praising the better product:

\[
N_D^D(m) \quad \text{where} \quad \begin{cases} m = "AB" & \text{if } j = AB \\ m = "BA" & \text{if } j = BA \end{cases}.
\]

In addition, we assume that \( N_D^D("AB") = N_D^D("BA") \equiv N_D^D > 0 \). Thus, D sends the same number of messages praising the better product, regardless of which it is. This simplifies the model but is not restrictive.

2.2. Firms’ Problem

C observes one message, which she picks at random from the existing pool of messages. The probability that C observes \( AB \) is the ratio of all messages praising A to the total number of messages. This simplifying assumption captures the intuition that increasing the number of messages sent by the firm increases the probability that its message is heard. The sampling of messages is a realistic assumption if we consider the fact that due to constraints on time, a consumer typically visits a small subset of all online communities.

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As will be shown later, we find that the high-quality product in equilibrium receives a higher total volume of praise because the word of mouth supporting it consists of the firms’ paid messages as well as the unbiased word of mouth. This ensures that in equilibrium, online word of mouth is persuasive.

In this section, we consider a simple model where the target segment (represented by one consumer) has homogenous preferences. The basic results do not change as some heterogeneity is added to the model.

For example, consider the competition between the latest Kodak and Sony digital cameras. The firms are aware of the merits of their own products as well as that of the competition. D in this case is the segment of early adopters who are more technology-savvy than the average user.

The only exception is Avery et al. (1999), who considers the problem of providing incentives to reviewers.
Following the reception of a message, C updates her prior on the identity of the better product. In a pure strategy equilibrium, her strategy space is a decision on a purchase of one of the products based on the type of message received. Let \( \Pr(AB|m) \) be the consumer’s posterior following the reception of the message. Applying Bayes’ rule, we have

\[
\Pr(AB|"AB") = \frac{\theta \Pr("AB"|AB)}{\theta \Pr("AB"|AB) + (1 - \theta) \Pr("BA"|BA)},
\]

(2)

\[
\Pr(AB|"BA") = \frac{\theta \Pr("BA"|AB)}{\theta \Pr("BA"|AB) + (1 - \theta) \Pr("BA"|BA)},
\]

(3)

where the probabilities of receiving each message in the two states of the world are summarized in Table 1. Note that C updates her beliefs, taking into account firms’ optimal promotional strategies in the two states of the world.

The recommendations are persuasive if and only if

\[
\Pr(AB|"AB") \geq \frac{1}{2} \quad \text{and} \quad \Pr(AB|"BA") < \frac{1}{2}.
\]

(4)

In this case C perceives that the recommended product is indeed likely to be superior and hence maximizes her expected utility by buying this product.\(^{11}\) It can be argued that it is rational for the consumer to “follow the message” if and only if that message is persuasive.\(^{12}\)

### 2.4. Perfect Bayesian Equilibrium

Putting together the firms’ and the consumer’s problem, we look for pure strategy equilibria where the consumer’s beliefs following the reception of a message are consistent with optimal messaging behavior by the firms, and firms’ behavior is consistent with equilibrium consumer beliefs. We can contrast this approach to existing models where advertising serves as a credible signal of quality. In most advertising signaling models, consumers can observe the level of advertising investment by the firms: \( N_{ij}^f \) is observed. Here, the consumer does not observe \( N_{ij}^f \), she infers it only in equilibrium. Off-the-equilibrium-path deviations in these quantities by the firms will elicit no response from C because those deviations are not observed. Such deviations change the probability distribution of messages, but upon receiving a message, the consumer’s updating rule is still governed by the equilibrium expectations regarding the \( N_{ij}^f \).

### 2.3. Consumer’s Problem

Following the reception of a message, C updates her prior on the identity of the better product. In a pure strategy equilibrium, her strategy space is a

\[^{10}\] More formally, our model only supports pooling in price (in equilibrium). This is because the firms do not differ in their marginal costs, and the category does not support repeat purchase.

\[^{11}\] We assume “market coverage”—the prices are low enough so that the consumer would buy either good under all beliefs: \( P < (1/2)V_m + (1/2)V_i \).

\[^{12}\] In fact, if \( \Pr(AB|"AB") = 1/2 \), then the consumer ought to be indifferent between choosing A or B. In that case, and only in that case, she may randomize. As tie-breaking rule (and without loss of generality), we assume that in that case the consumer buys the recommended product.
In equilibrium, neither firm should have an incentive to deviate while recognizing that the deviations won’t be observed. The imperfect observability on the part of C conveys the idea that there is room for firms’ manipulation of C’s perceptions.13

Proposition 1. If \( \theta^0 = 1/2 \) and the parameters satisfy one of the following inequalities, an unique perfect Bayesian equilibrium exists, and it is persuasive. In this equilibrium, the firm with the superior product has a higher expected profit. The equilibrium is described as follows.

1. (EQ1) If

\[
(N^{UI} + a)\sqrt{(2a - N^{UI})^2 + 4P - 2a^2 - (N^{UI})^2 + 3aN^{UI} < 2P},
\]

the firms promote in both states of the world but promote more heavily when their product is inferior. In equilibrium, \( N^A_{BA} + N^{UI} > N^A_{AB} > N^A_{NA} > 0 (N^B_{BA} + N^{UI} > N^B_{AB} > N^B_{NA} > 0). \]

2. (EQ2) If \( aN^{UI} < P \), neither firm invests in promotional chat.

Note that if the parameters do not satisfy any of the inequalities above, no equilibrium exists.14

Note that in EQ1 and EQ2, the superior firm receives more favorable messages (a mix of consumer recommendations and firm promotion) in equilibrium despite the fact that it invests less in promotion. Thus, rationally, the consumer interprets a favorable message as a positive signal of quality and is more likely to buy the product that is praised online. As can be seen from the statements above, an important driver of the results is the substitutability between promotion and consumer-generated word of mouth, the result of the anonymity assumption. (Of course, if the consumer could identify the source of the message, she would interpret firm-generated word of mouth as a negative signal.) Specifically, this substitutability endogenously differentiates the marginal costs of messaging that the firm faces across the two states, as we demonstrate below.

To provide some intuition for the results, we illustrate the maximization problem faced by Firm A in the two states of the world in Figure 2. We make a few simplifying assumptions. First, we assume that C’s beliefs are such that she buys A if and only if she receives “AB.” Second, we assume that the total number of ‘BA’ messages is the same across the two states of the world. This is done to ensure that the marginal benefit of sending messages is fixed across the two states of the world, which allows us to focus on the differences in marginal costs. Because we do not take into account B’s equilibrium behavior here, the illustration represents a partial equilibrium.

As shown in Figure 2, the marginal benefit (MB) of messaging—the change in probability of receiving “AB”—is decreasing in the total number of “AB” messages sent, \( T(AB) \): as bulletin boards become saturated with messages of one type, the benefit of sending these messages decreases. The marginal costs faced by the firm differ due to the anonymity assumption. In state AB, A faces a marginal cost of zero for the \( N^{UI} \) consumer-generated messages and a marginal cost that consists of a constant \( (a) \) and a linear component with slope 1 for each additional message. In state BA, A faces a marginal cost that consists of a constant \( (a) \) and a linear component with slope 1 for all messages generated. The intersection of the marginal cost and marginal benefit curves determine the firm’s behavior in equilibrium.

If \( a \) is small, we obtain an interior solution (see Figure 2, EQ1). Thus, we can see that there will be more positive messages about A in state AB: \( N^A_{AB} + N^{UI} > N^A_{BA} \), and hence the consumer rationally interprets the reception of “AB” as a signal of A’s superiority. On the other hand, we see that the firms spend more resources promoting their products in the state of the world where their products are inferior: \( N^A_{AB} < N^A_{BA} \). For intermediate levels of \( a \) (Figure 2, EQ2), we obtain a corner solution that is qualitatively equivalent to the interior solution. Thus, the amount of firm promotion is still greater for the inferior product: \( N^A_{BA} > N^A_{AB} = 0 \), but the “AB” message is still persuasive because in total there are more messages praising A in state AB: \( N^A_{AB} + N^{UI} > N^A_{BA} \). For very high levels of \( a \) (Figure 2, EQ3), we obtain another corner solution. There, the firm finds it optimal never to promote its product. In this scenario, only D sends out messages; there is no noise in the signal, and recommendations are persuasive.

Note that as demonstrated by the discussion above, the costs of messaging are essential for determining whether a persuasive equilibrium exists. Essentially, because the messaging is costly, the inferior firm finds it suboptimal to produce high volumes of promotion.
In addition, as can be seen from Figure 2, the declining marginal benefit (concave benefit) of messaging is crucial for the result that firms invest more heavily in their inferior products. The concavity of benefit is due to the micro model of consumer’s sampling of messaging. However, we expect that a number of alternative specifications would similarly result in the concavity of the benefit function.

The existence of unbiased reviews is also critical to the model because no persuasive equilibrium exists as \( N^{U} \) goes to zero. As we see in the next section, \( N^{U} \) is also an important parameter from the consumer welfare perspective. Next, we introduce some comparative statics within the regions where promotional chat exists (EQ1 and EQ2).

**Proposition 2.** The likelihood ratio of the signals, \( \frac{\Pr(\text{"AB"}|\text{BA})}{\Pr(\text{"AB"}|\text{BA})} \), is decreasing in \( P \) and increasing in \( N^{U} \).

The results are intuitive. Thus, an increase in \( P \) increases the incentive for the firms to engage in promotional chat, thereby introducing more noise into the system. On the other hand, an increase in \( N^{U} \) increases the ratio of signal to noise.

Finally, in the conclusion we discuss some interesting implications of the fact that the model can be easily extended to the case of asymmetric priors.

2.5. Consumer Welfare and Value of Information

In the basic model, we present two sides of public online discourse from the consumer perspective: Online advice is biased but still persuasive. In this section, we quantify the positive and the negative aspects of promotional chat. We do so by comparing consumer welfare under the persuasive equilibria with promotional chat, EQ1 and EQ2, to two alternative systems: (1) \( \hat{S} \), a system with word of mouth but no promotional chat; and (2) \( \bar{S} \), a system with neither word of mouth nor promotional chat.

We can think of \( \hat{S} \) as a “regulated” system where the consumer receives a perfect signal because anonymous promotion is outlawed: an upper bound on consumer welfare. \( \bar{S} \), on the other hand, is the system with no online recommendations—C decides solely based on prior beliefs. This is a lower bound on consumer welfare among all models where consumers optimally utilize information available in word-of-mouth recommendations. The difference in expected welfare between EQ1 or EQ2 and \( \hat{S} \) is the value of information that the existing system provides. The difference in expected welfare between \( \bar{S} \) and EQ1 or EQ2 is the welfare loss due to the noise introduced by the anonymous promotion.

**Proposition 3.** The value of information (welfare loss) due to anonymous promotion is increasing in \( V_{H} - V_{L} \), increasing (decreasing) in \( N^{U} \) and decreasing (increasing) in \( P \).

Hence, forums benefit the consumers when there is a large variance in benefit derived from different alternatives, which we expect to vary across categories. In addition, one implication of this analysis is that there is more of a need of regulation of online promotional chat if for a certain category \( N^{U} \) is low and \( P \) is high. This might be the case for forums that deal with cancer treatments, for example.
3. Discussion and Conclusion

The model presented concentrates solely on promotional chat. In reality, firms have a variety of marketing tools available to them. Here we apply the results of earlier analysis to explore the sources of firm strength in the model and suggest how the model may incorporate other marketing strategies. First, the firm that markets the superior product is likely to enjoy other sources of competitive advantage. For example, the consumer might receive information from other sources praising the superior product. Consider a scenario where C may receive a number of noisy signals on the state of the world prior to receiving a promotional chat message, such as third-party product reviews, advertising, or other (imperfect) signaling strategies by the firms. In this sense, the concept is a very general one. The set of all possible signals (excluding promotional chat) is $S = \{s_1, \ldots, s_N\}$. The set of consumer’s posterior probabilities on state $AB$ associated with each signal is $\Theta = \{\theta_1, \ldots, \theta_N\}$ where $\theta_1 < \cdots < \theta_N$. Using a generalization of Proposition 1 (described in a footnote to the proposition), we can show that promotional chat equilibrium takes place after the reception of a signal $s_i$ as long as $\Pr(AB^i | AB) \geq \max(1 - \theta_i, \theta_i)$. That is, as long as the third-party signals are sufficiently noisy, consumers listen to promotional chat. Thus, we can easily extend the model to more realistic settings where other sources of information are available to the consumer.

Another aspect of firm strength that is explicit in the model is the volume of $NU$ that supports the superior firm. We can show that the superior firm has an incentive to increase $NU$ because its expected profit is increasing in the amount of consumer support for its product. In this sense, we can extend the model to one where the amount of unbiased word of mouth is endogenously determined by the superior firm’s action prior to the promotional chat. This, in turn, would decrease the amount of messaging by both firms, thus decreasing the amount of noise in promotional chat equilibrium. Note that one marketing strategy that could achieve an increase in unbiased word of mouth is product give-aways, because it might generate excitement among the informed segment.

This study contributes to the advertising and marketing literature by studying the novel context of promotion under anonymity. Anonymity allows the firms to directly manipulate consumer-to-consumer conversations, but we show that in equilibrium received messages can still be informative because of the actions of legitimate chatters. An interesting feature of promotional chat, we argue, is that unlike traditional models of advertising, here firms with lower quality engage in more promotional chat. They promote more because they have more reason to promote: They don’t get the free publicity from legitimate chatters.

We believe that the applicability of this paper transcends the specific context studied here. The larger question is the relationship between advertising and word of mouth: Are word of mouth and advertising substitutes or complements? The present paper argues that they are substitutes when advertising messages are anonymous.

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Appendix

Proof of Proposition 1. First, we consider all possible equilibrium candidates.

1. $C$ is more likely to buy $A$ ($B$) following both “$AB$” and “$BA$.” This would imply that neither firm would have any incentive to send out messages → the “$BA$” message is perfectly informative because they come from an unbiased source → $C$ is more likely to buy $B$ ($A$) following “$BA$” (“$AB$”), a contradiction. This set of beliefs cannot be consistent.

2. $C$ is more likely to buy $A$ following “$BA$” and more likely to buy $B$ following “$AB$.” Neither $A$ nor $B$ has any incentive to send out messages → both “$AB$” and “$BA$” are perfectly informative → $C$’s decision rule is inconsistent with firms’ actions. This is not an equilibrium.

3. $C$ is more likely to buy $A$ following “$AB$” and more likely to buy $B$ following “$BA$.” This results in the following four maximization problems for the firms:

\[ A \text{ in state } AB: \max_\alpha \frac{N^A_{AB} + N^U_{AB} - aN^A_{AB} - \frac{(N^A_{AB})^2}{2}}{N^A_{AB} + N^U_{AB} + N^B_{AB}} \]
\[ \text{s.t. } N^A_{AB} \geq 0. \]
\[ (A1) \]

\[ B \text{ in state } AB: \max_\alpha \frac{N^B_{AB} + N^U_{AB} - aN^B_{AB} - \frac{(N^B_{AB})^2}{2}}{N^A_{AB} + N^U_{AB} + N^B_{AB}} \]
\[ \text{s.t. } N^B_{AB} \geq 0. \]
\[ (A2) \]

We can show that the firms’ profit functions are convex in their actions. We look for the solution in state $AB$. (The solution for state $BA$ is symmetric.) To simplify the notation,

\[ \frac{a}{2} \left( \frac{\alpha}{\beta} \right)^2 \]

The author thanks the referee for suggesting a discussion on the sources of firm strength in the model.

15 The author thanks the referee for suggesting a discussion on the sources of firm strength in the model.
There are four different candidates for a solution:

(1) Interior solution: $y > 0, x > 0$. The conditions for this are $Py/(N^U + y)^2 - a > 0, P/(N^U + x) - a > 0$

Corner solutions:

(2) $y > 0, x = 0$. (This holds iff $Py/(N^U + y)^2 - a < 0$ and $P/N^U - a < 0$)

The exact conditions on the parameters for (1) and (2) will be derived below.

(3) $y = 0, x > 0$. (This can be ruled out since according to $A$'s reaction function, $x(y = 0) = 0$)

(4) $y = 0, x = 0$. (This holds iff $P/N^U - a < 0$)

**EQ1:** $y > 0, x > 0$. FOCS are

$$Py/(x + N^U + y)^2 = a + x; \quad (A3)$$

$$P(x + N^U)/(x + N^U + y)^2 = a + y. \quad (A4)$$

We next show that there exists a unique positive solution to the Equations (A3) and (A4). If we add (A3) and (A4), we can solve for

$$x + y = \frac{-2a - N^U + \sqrt{(2a - N^U)^2 + 4P}}{2} \equiv w. \quad (A5)$$

(This is the only potentially positive solution.) If we divide (A3) by (A4) (we can do this because $a + y > 0$), we get

$$y(a + y) = (a + x)(x + N^U). \quad (A6)$$

Note that any solution that satisfies (A3) and (A4) must also satisfy (A5) and (A6).

There exists a unique positive solution if and only if $w = \frac{-2a - N^U + \sqrt{(2a - N^U)^2 + 4P}}{2} > 0 \Rightarrow -2a - N^U + \sqrt{(2a - N^U)^2 + 4P} > 0$. Graphically (Figure A1), we see that (A5) describes a line on the $(x, y)$ plane, whereas (A6) defines a hyperbola. The intersection gives us the equilibrium values. From the graph below, we see that there exists a unique positive solution if and only if $w = \frac{-2a - N^U + \sqrt{(2a - N^U)^2 + 4P}}{2} > 0$. The equilibrium values are

$$y \equiv N^B_{AB} = N^B_{BA} = \left[2a^2 - aN^U + 2P - a\sqrt{(2a - N^U)^2 + 4P}\right]/\left[2\sqrt{(2a - N^U)^2 + 4P}\right];$$

$$x \equiv N^A_{AB} = N^A_{BA} = \left[2a^2 + (N^U)^2 - 3aN^U + 2P - (a + N^U)\sqrt{(2a - N^U)^2 + 4P}\right]/\left[2\sqrt{(2a - N^U)^2 + 4P}\right].$$

We can show that $x + N^U - y > 0$ and $y - x > 0$: The firm makes more profit when its product is superior because then it spends less money on promotion and is more likely to sell. The condition $x > 0$ defines the region where the interior solution exists.

Next, we check that the consumer’s decision rule is optimal, given the firms’ actions. Due to symmetry, $Pr(AB^* | AB) = Pr(AB^* | BA)$. The condition for persuasive equilibrium reduces to $Pr(AB^* | AB) > 0.5$ or $(x + N^U)/(x + N^U + y) > 0.5$. This is satisfied because $x + N^U - y > 0$.

**EQ2:** $y > 0, x = 0$. From the firms’ reaction functions, the expressions that define this corner solution are

$$P/(N^U + y)^2 < a \quad \text{and} \quad PN^U/(N^U + y)^2 = a + y$$

(also implies that $P > aN^U$)

can be re-written as

$$P/(N^U + y)^2 = b$$

(where $b$ is to be determined, s.t. $0 < b < a$) and

$$PN^U/(N^U + y)^2 = a + y. \quad (A7)$$

After manipulating the system above, we arrive at the following expressions:

$$f \equiv y = (P - 2bN^U - aN^U)/(b + N^U) \quad \text{and} \quad g \equiv y = (-a + \sqrt{a^2 + 4bN^U})/2. \quad (A8)$$

To derive the condition for the existence and uniqueness of this solution, we look for values of $b$ that such that (A8) holds $\Rightarrow y = f(b) = g(b)$ for $0 < b < a$. We can show that $\partial f/\partial b < 0$ as long as $P > aN^U$ and $\partial g/\partial b > 0 \Rightarrow$ there is at most one solution s.t. $f(b) = g(b)$.

$$f(b = 0) \equiv y = (P - aN^U)/N^U > 0, \quad g(0) = 0,$$

$$f(a) = [P - 3aN^U]/[a + N^U],$$

$$g(a) = [-a + \sqrt{a^2 + 4aN^U}]/2 > 0.$$
EQ3: \( y = 0, x = 0 \). If \( y = 0 \), \( A \)'s marginal benefit of promotion is \( P(y/(x+N^A + y)^2) = 0 \); \( A \) does not promote. If \( x = 0 \), \( B \)'s marginal benefit of promotion is \( P(x + N^B)/(x + N^B + y)^2 = PN^B/(N^B + y)^2 \) and is decreasing in \( y \). The marginal cost is \( a + y \) and is increasing in \( y \rightarrow 0 \) if \( P/N^A - a < 0 \). (As a consequence of this, in regions where the other solutions exist, \( P/N^A - a > 0 \). Thus, EQ3 does not intersect with either EQ1 or EQ2.)

This equilibrium is persuasive because the only messages are sent out by the truthful unbiased source. From the conditions on the solutions, we can show that EQ1 and EQ2 are disjoint. The region where no equilibrium exists is the complement of the other three regions.

For the case of asymmetric priors, we can re-derive the proof above. The only difference is that the consumer’s problem is slightly more complicated because the prior belief must be taken into account. The conditions in (4) reduce to (due to symmetry)

\[
\frac{\Pr(AB^1|AB)}{1-\Pr(AB^2|AB)} = \frac{1-\theta^0}{\theta^0} \quad \text{and} \quad 1 - \frac{\Pr(AB^2|AB)}{\Pr(AB^1|AB)} < \frac{1-\theta^0}{\theta^0} \Rightarrow \Pr(AB^2 | AB) > \max \theta^0
\]

\[
\rightarrow N^U + N^A = \max \frac{\theta_0}{1-\theta_0} N^A.
\]

**Proof of Proposition 2.**

EQ1: We can demonstrate simple comparative statics on \( \Pr(AB^1|AB) = (x + N^U)/(x + y + N^B) \). This expression is increasing (decreasing) in a parameter if and only if \( (x + N^U)/y \) is increasing (decreasing) in that parameter. Here, \( (x + N^U)/y = 1 + 2(N^U/(-N^U + \sqrt{(N^U - 2a)^2 + 4P}) \). In this region, this expression is decreasing in \( P \) and increasing in \( N^U \). The same holds for \( \Pr(AB^1|BA)/\Pr(AB^2|BA) = \Pr(AB^1|BA)/(1 - \Pr(AB^2|BA)).
\]

EQ2: Here \( \Pr(AB^2|AB) = N^U/[y + N^U] \), \( \partial \Pr(AB^2|AB)/\partial N^U = (y - (\partial y)/(\partial N^U)N^U)/(y + N^U)^2 \). To obtain the expression for \( \partial \Pr(AB^2|AB)/\partial N^U \) we use the Implicit Function Theorem, where the first-order condition is \( PN^U/(N^U + y)^2 - a - y = 0 \) (we use the same method for other parameters). This allows us to derive that \( \Pr(AB^2|AB) \) is decreasing in \( P \) and increasing in \( N^U \).

**Proof of Proposition 3.** Assume without loss of generality that \( \theta^0 \geq 0.5 \). Below, we present the expected consumer welfare (EW) in each system.

Under promotional chat (PC),

\[
\begin{align*}
\text{EW}^{PC} &= \theta^0[\Pr(AB^1|AB)V_1 + \Pr(AB^2|AB)V_1] \\
&+ (1-\theta^0)[\Pr(AB^1|BA)V_1 + \Pr(AB^2|BA)V_1] - P \\
&= V_1 + (V_H - V_L)Pr(AB^2|AB) - P.
\end{align*}
\]

Under the upper bound (UB),

\[
\begin{align*}
\text{EW}^{UB} &= V_H - P \\
\text{EW}^{UB} - \text{EW}^{PC} &= (V_H - V_L)[1 - \Pr(AB^2|AB)] > 0.
\end{align*}
\]

Under the lower bound (LB),

\[
\begin{align*}
\text{EW}^{LB} &= V_L + \theta^0(V_H - V_L) - P \\
\text{EW}^{PC} - \text{EW}^{LB} &= (V_H - V_L)[\Pr(AB^2|AB) - \theta^0] > 0,
\end{align*}
\]

as shown in Proposition 1. Next, we can use the results on comparative statics on \( \Pr(AB^2|AB) \) in Proposition 2 to demonstrate that the rest of the results of Proposition 3 hold.

**References**


Friedman, E. 2000. Street marketing hits the Internet. *Advertising Age* 71(19) 32.


