

On the Monetization of Mobile Apps

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Abstract

Though the mobile app market is substantial and growing fast, most firms struggle to monetize apps profitably. Monetizing apps is done in two ways: a) selling advertising space within a free version of the app, and b) selling a paid version, termed *freemium* or *in-app purchase* strategy. In this paper, we present a framework for monetization of mobile apps, using two central empirical regularities concerning the relationship between users and their mobile apps: a) Uncertainty: While consumers have some prior knowledge of their fit with the app, they remain uncertain regarding their exact utility until they are using it; and b) Satiation: The utility of using the app may decrease with time. While work on the monetization of digital goods has largely overlooked the role of satiation and the consequent retention issues, we show that in combination with uncertainty, it elucidates the role of the segments of consumers that download the free vs. paid version of the app, and how to balance these two segments so as to monetize mobile apps. We encounter two distinct scenarios: In the first, advertising drives most of the revenues; while in the second, revenues are driven by the paid version of the app. We explain how uncertainty and satiation affect the prevalence of the respective scenarios and impact the share of revenues from the paid vs free version of the app. We also demonstrate that a firm can profit from offering a free version with ads even if advertisers are not paying for these ads. In other words, the firm benefits from offering a “damaged good” version of the app that includes ads, even if this version is free to consumers, and the advertisers are not paying for the ads.

Keywords: advertising; freemium; mobile apps; online strategy; pricing; retention; satiation

1. Introduction

The number of mobile apps offered to consumers is rising rapidly, and by 2018, consumers could choose among nearly six million apps in Google or Apple stores (Statista 2018). Industry reports indicate that over 90% of mobile apps start as free, over 90% of the profits from mobile apps come from apps that began as free, and that the share of free apps is expected to grow (AppBrain 2018). Although these apps are “free”, market size is notable, at more than \$80 billion in 2017 and expected to almost double by 2022 (Cheney and Thompson 2018). The largest app category is mobile games, whose revenues reached \$70 billion in 2018, exceeding both video console games and PC games (Wijman 2018).

Monetizing apps is mainly done in two ways: a) selling advertising space within a free version of the app, and b) selling a paid version, or an in-app purchase strategy that offers additional features for pay (Needleman 2016). As app creators struggle to remain profitable, the question of choosing the correct business model and marketing mix becomes more pressing (Natanson 2016). While advertising and pricing decisions have always been interwoven (Assmus, Farley, and Lehmann 1984), in mobile apps, the interrelationship is magnified significantly due to the specific segments that download each version of the app. Potential users consider adopting the app in its two versions – free, and paid – and make these decisions over time (Gu, Kannan, and Ma 2018). Indeed, many “sample” the free version while considering the paid version (Deng, Lambrecht, and Liu 2018).

In this paper we present a framework for optimal app management in this complex environment. We do so by looking at two aspects of app monetization that should be jointly taken into account when considering marketing of mobile apps: *satiation*, and *uncertainty*.

Satiation. Customer retention is a key issue confronting the mobile apps industry (Jacobson

2018). Mobile app retention rates are typically much lower than observed retention rates for classic products: Industry reports suggest that across all categories, over 70% of all app users may churn within 90 days (Perro 2018). This introduces additional complexity into the decision making, as the user who is targeted by advertising might have churned already in early periods of the app life cycle. Thus, pricing and advertising decisions have a direct effect on the number of users in each segment in both the current and future periods.

Low retention rates in the industry at large can be related to low switching costs and the large and growing selection in the apps stores. At the individual level, marketers realize that churn is closely related to declining engagement over time, which lowers users' utility from apps (AppsFlyer 2017). From a behavioral standpoint, such decline is consistent with abundant evidence on satiation's role in consumption patterns of products, hedonic products in particular (Galak, Kruger, and Loewenstein 2013; Sevilla, Zhang, and Kahn 2016; Galak and Redden 2018). It is well recognized that repeated consumption of the same item over time is likely to produce a decline in liking even for a positive experience, and in turn reduction and cessation of usage. Therefore, under satiation, customer satisfaction will not necessarily lead to higher revenue, as consumer lifetime value gradually declines (Voss, Godfrey, and Seiders 2010). Satiation is thus a threat to the firm's ability to generate future profits from its customers.

It is thus not surprising that satiation has been found to affect the consumption behavior of mobile apps such as games (Han, Park, and Oh 2016; Hui 2017). However, the extent of the phenomenon may differ considerably among app categories that likewise vary considerably in their retention rates (App Annie 2016). The app research firm Flurry, for example, found that the high-retention categories are health and fitness, weather, magazines, and business and finance; while categories such as action games, sports, family, entertainment, and puzzles have much

lower retention probabilities (Klotzbach 2016). This difference may be thus related to the extent to which apps are perceived more as hedonic vs utilitarian (Schulze, Schöler, and Skiera 2014).

Uncertainty. While apps are clearly not the only product where uncertainty affects decision, few are the markets where consumers face such an extensive choice with relatively little product knowledge. This is due to the sheer number of apps available and the general absence of advertising budgets for such low-ticket items. Thus, while consumers have some prior knowledge of their fit with a given app, they remain uncertain about their exact match-utility until they are using it. Consequently, the firm has to design two versions – paid, and free – knowing that some consumers will sample the product and decide about purchasing either version in future periods once uncertainty is resolved as a result of the sampling procedure.

We show how monetization policy creates the market reality of paid and free apps, taking into account the joint effect of customer uncertainty and satiation. The equilibria in our setting result from consumers maximizing utility by choosing free or paid versions, and the app producer maximizing profits by optimally choosing price and advertising. We see that this setting in fact leads to two distinct scenarios: In the first, advertising drives most of the revenues; while in the second, revenues are driven by the paid version of the app. We will demonstrate how uncertainty and satiation affect the prevalence of the respective scenarios.

In particular, our analysis elucidates the importance of satiation in the market. Satiation's role is notable given that academic discussion of app monetization to date has largely overlooked the relevance of user duration with the app. We show that the optimal prevalence of free versus paid apps is related to the level of satiation: When satiation increases, initially, the paid version's share of the app decreases; then, when satiation is high enough, the firm offers only the ad-supported version of the app. The following analysis helps us to understand how and why this

happens, and how persistence also creates value through its effect on advertising and price.

We also show that the firm benefits from offering a free version of the app that contains ads even if advertisers are not paying for these ads. The logic behind the production of this “damaged good” is that the free version acts as a sample that enables consumers to learn their fit with the app. The firm would rather add advertising to this version in order to keep the consumers who derive high value from the app away from the sample, and push them toward the paid version. Overall, our analysis demonstrates how the respective shares of paid and free apps are a function of a complex process that combines app features and the marketing mix to yield user utility. Managers need to understand this process in order to set realistic expectations, understand the impact of various parameters, and design strategies to increase profitability.

2. Related literature

Our work is relevant to research efforts aiming at understanding firm behavior in free digital markets. Research in this area has largely focused on the choice between content and advertising in the context of media markets, where the basic tradeoff is that moving from an advertising-only revenue model to charging for content, may reduce usage and thus hurt advertising revenues (Lambrecht et al. 2014). Specifically, our work is related to recent calls for better understanding of freemium product design to maximize customer lifetime value (Kannan and Li 2017).

Empirical results regarding the consequences of such a choice vary. Profitability in such contexts may depend on factors such as the type of promotions used (Pauwels and Weiss 2008), temporal changes in demand (Lambrecht and Misra 2016), and users’ ability to bring in new business through referrals (Lee, Kumar, and Gupta 2017). It had been also suggested that the existence of free apps may affect the speed of paid alternatives’ growth (Arora, Hofstede, and Mahajan 2017; Deng, Lambrecht, and Liu 2018). Analytically, noticeable effort has centered on

the questions of profit from content vs. advertising in two-sided media markets such as newspapers (Halbheer et al. 2014). The tradeoff between paid content and advertising may depend on competitive intensity (Godes, Ofek, and Sarvary 2009), consumer heterogeneity in willingness to pay (Prasad, Mahajan, and Bronnenberg 2003), or the extent to which consumers dislike advertising (Tåg 2009).

Viewing the use of a free product as a sampling mechanism, recent work has suggested that advertising's effectiveness, coupled with consumers' expectations regarding quality, can determine which revenue source firms should focus on in attempting to enhance profitability (Halbheer et al. 2014). Moreover, it has been shown that the appeal of the free option is influenced by the effectiveness of word of mouth (Niculescu and Wu 2014), and by quality and other design parameters (Li, Jain, and Kannan 2019). These research efforts, however, do not consider customer satiation's role and its effect on retention in the subsequent market state.

Our work is also relevant to the literature on product line design (Moorthy 1984; Moorthy and Png 1992; Biyalogorsky and Koenigsberg 2014; Gu, Kannan, and Ma 2018). The main driving force behind product line design and pricing is the fact that consumers derive differing valuations from the product's attributes. The firm knows the distribution of preferences, yet not individual preferences, and thus cannot engage in standard monopoly (first-degree) price discrimination. Instead, it offers a line of products and relies on self-selection of consumer segments thereto, thereby permitting partial discrimination among consumers. This self-selection process relies on incentive-compatibility constraints that deter one segment from purchasing the "wrong" product that was designed for another segment.

As Gu, Kannan, and Ma (2018) demonstrate, questions of app market planning, for example extending the premium product line, can be analyzed from a product line standpoint. Indeed, our

model features this type of price discrimination, where the firm offers two product versions (paid, and free), and consumers self-select, where the low-valuation consumers download the free version (with the nuisance of ads), and the high-valuation consumers choose the paid version. What complicates the analysis considerably is the introduction of uncertainty: As Biyalogorsky and Koenigsberg (2014) indicated, uncertainty complicates the design problem of developing products that satisfy the incentive-compatibility constraints. In our case, the firm has to design both versions, knowing full well that some consumers will sample the product and decide about purchasing either version in the subsequent period, once uncertainty is resolved.

Given our emphasis, our work is relevant to the avenue of research that addresses customer retention's consequences (Libai, Muller, and Peres 2009; Ascarza et al. 2018), and how optimal retention efforts may differ based on market characteristics (Musalem and Joshi 2009; Shin and Sudhir 2012; Subramanian, Raju, and Zhang 2013). In the context of online behavior, researchers have considered the antecedents of users' tendencies to stick with an app (Hsu and Lin 2016). Yet the dynamics of how satiation, and consequently retention, affect monetization in free digital markets, have not been examined.

Finally, as the consumers in our setting learn about the app, our work addresses the demand-side literature on consumers' learning (Erdem et al. 1999; Iyengar, Ansari, and Gupta 2007). Our learning structure is not on specific attributes of the app, but rather on the idiosyncratic utility that the consumer can expect from experiencing the app, similar in spirit to Akerberg (2003). Free samples in technological products have been shown to be an effective way to convey information, lowering consumers' risk associated with adoption, and consequently accelerating adoption (Foubert and Gijbrecchts 2016; Li, Jain, and Kannan 2019). Here we show how such learning combines with post-adoption satiation to affect market outcomes.

3. The model

To clarify the exposition, we chose the following example of a mobile game of a popular category: Role Playing Games (RPG). RPG is an app category intended for mobile devices, distributed on either Google's Play Store or Apple's iOS App Store as either free, or paid¹. The game, *Doom and Destiny*, is designed in the 8-bit graphics style of Japanese RPG (JRPG) by Italian developer Heartbit Interactive. The game tells the story of four nerds who get drawn into a fantasy world. Mistaken for heroes, they must battle their way through an adventure in order to defeat a villain. It contains nerdy humor and strong language, so that advocacy group and text monitoring app SaferKid recommended 12+ as a safe age to play the game. The game was launched on Android in 2013, and currently has over 500k installs and 70k reviews for the free version, and over 50k installs and 18k reviews for the paid version. It is also quite popular, as for example, the paid version is #9 in the US in the Role Playing category (as of November 2018). In terms of the costs to consumers, in user time or out-of-pocket expenses, in the free version, an ad appears every 3 minutes, covering the entire screen for 5 seconds; while the paid version costs \$4.99. The only difference between the two is lack of advertising in the paid version.

In the next subsection, we use *Doom and Destiny* as a running example in order to better elucidate the main factors affecting consumers' utility such as uncertainty, advertising, and price.

3.1 Uncertainty, advertising, and price

We begin by describing the factors affecting consumers' utility such as uncertainty, advertising, and price; then proceed with the formal utility maximization problem; and conclude with the resultant demand and firm maximization.

¹ The term "free" app is somewhat of a misnomer, as even the "free" version has a cost, i.e., the consumer pays with her time by viewing advertising. However, for convenience' sake, we refer to the ad-supported version as "free".

Uncertainty. Although consumers might have some prior knowledge about the app, they remain uncertain about the idiosyncratic utility they can expect from using the app (Morvinski, Amir, and Muller 2017). In the above-described example of *Doom and Destiny*, judging from the reviews, there's uncertainty about the user's match with the app until s/he begins playing the game. The reason for this might be that the description is rather terse and generic: The user might consider herself a nerd, yet cannot be sure that the humor fits or about the crudity of the language, or some idiosyncrasies of the game's technical aspects. Thus we find reviews that state, "I didn't expect this to be a gem..." and "...love the dialog, cursing included...", and "...new way to play with the control setup being something I've never used before..."

Product fit uncertainty is a major impediment to online markets in general (Hong and Pavlou 2014). Consumers' uncertainty about apps is amplified, as the majority of apps have been developed by small businesses with unknown reputations (Arora, Hofstede, and Mahajan 2017). Market reports suggest that given the absence of monetary cost to download apps, consumer learning often occurs post-adoption, rather than pre-adoption as in classic consumer markets (Klotzbach 2016). Yet consumers are not entirely in the dark with respect to their match with any given app. For example, some consumers know a-priori that they prefer "social games" over "brain games". However, as each app is unique, even after reading an app's description, consumers are uncertain about the fit between their preferences and the app.

The costs and benefits of advertising. The free version of the app is bundled with advertising. Exposure to advertising is irksome to the consumer, arousing annoyance and irritation, and thus the intensity of advertising (denoted by γ) depreciates the consumer's utility (Wilbur 2008). Advertising creates inconvenience for the consumer in that it effectively reduces the level of content that s/he can enjoy from the app, in addition to inflicting other costs, such as distraction

and opportunity cost of time, resulting in a negative effect on utility and retention (Tåg 2009; Goldstein et al. 2014). Indeed, in *Doom and Destiny*, users have complained about the ads in reviews such as: “...*I’m thinking of buying the full game to get rid of those pesky ads...*” and “*This game is awesome... though there were scientology ads which made confused deeply. [sic]*”

As the aforementioned scientology advertising demonstrates, firms tend to have limited control over ad content, which is typically outsourced to ad exchanges that match ads to apps (e.g., Google’s Marketing Platform; Yahoo’s Flurry; OpenX). However, the firm can select the level of advertising intensity, and thus Heartbit Interactive, for example, can decide to have an ad every 2 minutes rather than every 3 in *Doom and Destiny*.

These additional ads come with a cost, though: Firstly, ad exchanges pay developers on a cost-per-click basis. Posting more ads leads to lower click-through rates, leading to lower ad conversion rate, and thus to lower revenues from advertising (Babu 2018). Moreover, there can be a more general effect on the brand in general. Given the strong resentment of many individuals toward ads in games (Lagace 2018), heavy advertising can harm the brand name such that it interferes with firm’s ability to acquire customers and retain them. There is a limit to how often a firm can advertise without causing such damage.

Paid version price. Consumers may choose to eliminate the annoyance of ads, for a price. The firm offers a paid version of the app for a price $p \geq 0$. If consumers choose to pay this price, the ads are removed and the consumers enjoy the app’s full utility. Hence Heartbit Interactive can decide to price *Doom and Destiny* at \$3.99 rather than the current \$4.99. As we show later, choice of price and advertising levels has a profound effect on the size and revenues of the consumer segments that download the paid vs the free version of the app.

Satiation and persistence. As discussed previously, satiation is expected to affect consumption over time (Galak and Redden 2018). Our model considers satiation to be a feature of the app. Various app-based factors may influence an app’s satiation level, including the app’s quality and its ability to create consumer engagement, commitment, and trust (Hsu and Lin 2016). Market studies suggest that observed retention rates (which follow the extent of satiation) depend on the specific type of app, and on variety seeking and boredom on the part of the target market (Klotzbach 2016).

Consistent with the customer duration literature that often highlighted retention rather than churn, we define the fraction of the utility that is still experienced in the second period, and label this parameter (denoted by δ) the *persistence* parameter. Persistence is the opposite of satiation, of course, as churn is to retention: the higher the persistence, the lower the satiation and the more utility remains by the second period. Note that satiation (or persistence) reflects changes in utility over time. While it is naturally a key antecedent of user retention, it is not the only one: Other market factors, as we discuss here, may play a role in the final consumer decision. The actual churn decision depends upon the overall utility, if it is negative.

In our example, while we do not have access to *Doom and Destiny* usage data, we can find out the mobile games category’s average usage. In order to assess the level of persistence, we make two assumptions²: Firstly, a “period” in our model is a week; and secondly, usage is a reasonable proxy for utility. Under these assumptions, we utilize an Adjust Mobile Benchmark Q3 2016 study (the latest available) to compute average usage, in minutes per day, of the first and second week for Android users. This if we use the average usage time as a proxy for utility,

² As this is just an example for computation of satiation, the assumptions are not inherent to our analysis, and are used solely for this example. Nonetheless, both assumptions are reasonable: As industry reports indicate the time frame used for these fast-moving games are either daily or weekly; and usage, while not perfect, might be a good proxy for the engagement and therefore utility enjoyed by the player.

then the average time a user spends on a game is 15.3 minutes per day in the first week, and 8 minutes per day in the second, leading to a persistence parameter of $\delta = \frac{8}{15.3} = 0.53$.

3.2 Consumer's expected utility

In each period ($t = 1, 2$) the user faces three alternatives with three corresponding utility levels:

$$(1) \begin{cases} \text{(a) not use the app with resultant utility of } u_{0,t} = 0 \\ \text{(b) use the free app with utility } u_{a,t} = v\delta^{t-1}(1 - \gamma) \\ \text{(c) use the paid version of the app with utility } u_{p,t} = v\delta^{t-1} - pI\{t\} \end{cases}$$

where $I\{t\} = 1$ if the consumer bought the app in period t and 0 otherwise; v is the experience utility, γ is the intensity of advertising, and δ is the app persistence parameter. The utility gained from the app is v in the first period and $v\delta$ in the second period. In the example given in the previous subsection, if $v = 15.3$ in the first period, then $v\delta = 8$ in the second (to simplify notations, we normalize utility as we show later). Utility is multiplied by $1 - \gamma$ if the free version is used. The indicator function I captures the fact that the price is paid only once. Note that γ and δ are bounded between 0 and 1.

Formalizing uncertainty. To capture uncertainty, we let it be additive, in that each consumer is assigned a base utility of α , and then nature either adds or subtracts ε to or from α (with probability of 1/2 to each option). Thus let $v = \alpha + \mu$ where v denotes the experience utility of the app, α the base utility – an attribute that is known ex-ante – and μ the match utility, a horizontal attribute that is learned after use in the first period. This process is common knowledge, and each consumer knows her own α , yet does not know whether nature added or subtracted ε in her case. The distributional assumptions are that μ has a Bernoulli distribution with parameter $p = \frac{1}{2}$ on $\{-\varepsilon, \varepsilon\}$, where $\varepsilon \geq 0$, and that α and ε are distributed uniformly in the

interval $(0,1)$. Thus, the parameter ε represents the degree of uncertainty that the consumer faces with respect to her true base utility. This uncertainty is resolved when the consumer uses the app, irrespective of whether she uses its free or paid version.

Next, we formalize the consumer's utility when using the app's paid and free versions.

Consumer chooses paid version. Note that when forming expectations, the consumer takes into account the consequences of her choices in the first period on her state in the second in two ways: Firstly, using the app resolves uncertainty, and the consumer knows whether his utility is $\alpha - \varepsilon$, or $\alpha + \varepsilon$. Secondly, paying for the app in the first period renders it free for use (without ads) in the second period. Thus, to calculate the user's lifetime expected utility, we need to describe her second-period utilities and choices.

We begin by considering the case in which the user chooses the paid version in the first period. In such a case, she has two options in the second period: (a) continue to use the paid version or (b) not use the app at all. The option of using the free version in the second period is not appealing to her, as it contains ads, which decrease her utility, and she has already paid for the ad-free version. Her utility would be either $(\alpha - \varepsilon)\delta$, or $(\alpha + \varepsilon)\delta$ if she continues using the paid version, depending on the realization of μ , which will be known to her at that point, or else 0 if she does not continue using the app. Therefore, the choice between using the app or not, which maximizes her utility, can be characterized as follows:

1. The consumer, regardless of her intrinsic valuation, uses the paid version of the app in the first period (first term of Equation 2).
2. If $\alpha < \varepsilon$, she will use the paid version in the second period only when her realization of μ is positive, with probability $1/2$ (second term of Equation 2).
3. If $\alpha > \varepsilon$, the user will use the app in the second period irrespective of her realization of μ (third term of Equation 2).

Accordingly, her expected utility is given by:

$$(2) \quad U_p(\alpha) = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon)I\{\alpha < \varepsilon\} + \alpha\delta I\{\alpha > \varepsilon\}$$

Consumer chooses free version. Next, we consider the case in which the user chooses the free version in the first period. In such a case, he has three options in the second period: a) stop using the app; b) keep on using the free version; or c) pay for the premium version. Therefore, the choice between using the app or not can be characterized as follows:

1. The consumer, regardless of his intrinsic valuation (α), uses the free version in the first period (first term of Equation 3).
2. If $\alpha < \varepsilon$, he will use the app in the second period only when his realization of μ is positive, with probability $\frac{1}{2}$ (second term of Equation 3).
3. If $\varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon$, he will use the free version in the second period irrespective of the realization of μ (third term of Equation 3).
4. If $\alpha > \frac{p}{\gamma\delta} - \varepsilon$, he will buy the app in the second period in case his realization is positive, and use the free version if the realization is negative (last term of Equation 3).

Accordingly, his expected utility is given by³:

$$(3) \quad U_a(\alpha) = \alpha(1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon)(1 - \gamma)I\{\alpha < \varepsilon\} + \alpha\delta(1 - \gamma)I\left\{\varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon\right\} \\ + \frac{1}{2}[((\alpha + \varepsilon)\delta - p) + \delta(\alpha - \varepsilon)(1 - \gamma)]I\left\{\alpha > \frac{p}{\gamma\delta} - \varepsilon\right\}$$

3.3 Demand and profits

The consumer's decision is simple: Given her base utility, she selects the option that yields the highest expected utility. This decision yields four scenarios that depend on the model's parameters, to be discussed presently. As we wish to discuss these scenarios in detail, including the firm's choices, we must define the firm's objective function. The firm chooses the price and

³ For this equation and the rest of the analysis, we need both γ and δ to be strictly positive (positive δ for some of the cases).

level of advertising intensity so as to maximize its profits from the two periods. We normalize the overall market potential to 1. Let P_1 and P_2 be the share of consumers who use the *paid* version of the app, and A_1 and A_2 the share of consumers who use the *free* version of the app, in Periods 1 and 2 respectively.

In addition, we need to define a conversion parameter for the firm's profit function: As the price is expressed in monetary terms such as dollars, euros or yuans, we need a parameter, denoted by k , to convert advertising intensity γ to the same monetary term. The advertising parameter k thus represents the monetary payment received by the firm from advertisers per unit of ad intensity. It thus measures the advertising's effectiveness, which might be a function of the quality of the app developer's audience. As we discussed earlier, there are costs associated with advertising, which we assume to be convex. These convex costs are equivalent to assuming diminishing marginal returns to investment in advertising (Pindyck 1982; Vakratsas and Ambler 1999)⁴. The objective function of the firm is thus:⁵

$$(4) \quad \Pi(p, \gamma) = p(P_1 + P_2) + k\gamma(A_1 + A_2) - \gamma^2$$

The profit is a function of the two decision variables of the firm – price, and advertising intensity – which of course depend on the model's parameters: k , ε , and δ . Following common practices and contractual settings in actual app markets, we assume that price and advertising intensity do not change in the second period. Specifically, in the real app market, the price and ad intensity are embedded in the software and are rarely changed. For example, the business press

⁴ While we could have added a parameter ω to convert advertising costs into monetary terms, it is straightforward to show that ω is just a scale parameter that changes both decision variables at the same proportions, and does not have any effect on the scenarios presented next. We thus, without loss of generality, assumed that $\omega = 1$.

⁵ While consumers who buy the paid version may or may not stay for the second period, they do not affect the profit function, as the paying consumers do not generate any revenues in a later period. Thus, consumers pay for the app in the second period only if they switched from the free version to the paid one in that period.

reports that changing the in-app purchase price of an app is a complicated, resource-intensive process that can intimidate developers (Ogg 2013). The complexity of price and advertising changes is also driven by apps' relatively short life cycles, and by the fact that advertising contracts are entered into in advance with third parties.

4. Equilibrium scenarios

When considering the equilibria that result from consumers maximizing their utility by choosing the free vs paid version in the two periods, and the producer maximizing profits by optimally choosing price and advertising intensity, we encounter two distinct scenarios that are summarized in Table 1.

Table 1: Ad- versus pay-supported scenarios

scenario	% revenues from paid users	% paid users	Prevalence (% of all cases)	average k	average ε	average δ
Ad supported	24%	17%	77%	0.56	0.40	0.48
Pay supported	64%	45%	23%	0.30	0.83	0.56

The analysis is based on analytical solutions of optimal price, advertising, and profits, in addition to supplementary numerical simulation as follows. The resulting profit function (after plugging in the optimal levels of price and advertising) is too complex for additional analysis (for example, in order to compare profits for various scenarios). Thus, we calculated the value of the profit function over a million times, each time plugging in a different set of parameters and conducting some of the analysis numerically based on these calculations. Specifically, we used a full factorial design to generate sets of parameters for 101 levels (from zero to one) for each parameter (k , ε , and δ), resulting in $101^3 = 1,030,301$ cases. These calculations enabled us to determine which parameters lead to a scenario in which most of the profit comes from ads, and

those in which most of the profit comes from pay, and then characterize these differing scenarios.⁶

The scenarios in Table 1 are divided into two distinct groups: In the first, advertising drives most of the revenues, while in the second, revenues are driven by the paid version of the app (second column of Table 1). Note that regardless of whether revenues are driven by advertising or the paid version, the majority of users of the app download the free version (second column of Table 1). The two scenarios also differ substantially in their prevalence, as depicted in the third column of the table, in that the ad-supported scenario constitutes more than three quarters of all cases. In addition, the scenarios include one additional special case each, to be discussed next.

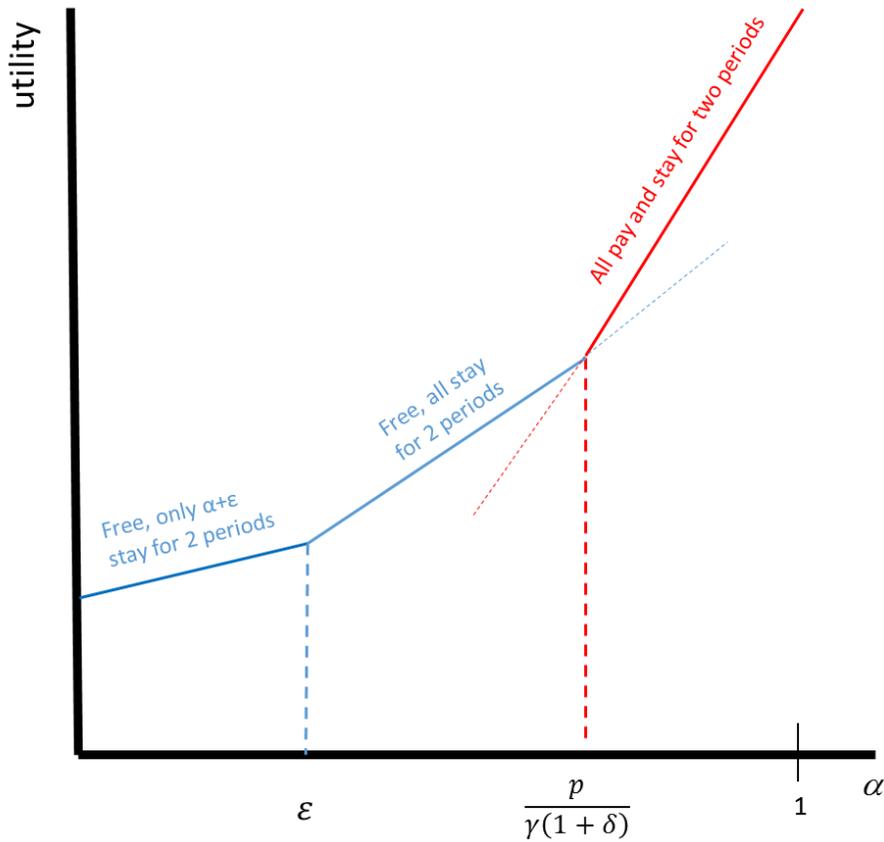
Observing the average parameter values for each scenario, the ad-supported scenario is characterized by high advertising effectiveness (k), and low uncertainty (ε). The reason for high k is obvious: higher advertising effectiveness implies better and more efficient use of resources devoted to advertising, and, thus *ceteris paribus*, more revenues will be yielded by this more efficient tool. With respect to uncertainty, consumers who use the *paid* version are betting on a positive outcome of the realization of μ ($+\varepsilon$ with probability $1/2$). If this happens, they will enjoy the full benefit of this occurrence, as they do not pay for the app in the second period. Thus under higher uncertainty, consumers will tend to use the paid version of the app. Lastly, though the difference is not large, the same reasoning applies to satiation, which implies higher second-period utility, ergo higher likelihood of choosing the paid version.

⁶ Note that a) the first three columns contain figures that are outputs of the equilibrium, while the last three contain the inputs that lead to the equilibrium, and b) the partition into price- vs. ad-supported scenarios is based on the results, i.e., after calculating the profit, we grouped together the cases that lead to the two rows of the table; and c) the partitions into cases were continuous, in that they gradually switched from one case to the next. Thus for example, all cases in which $\varepsilon \leq 0.44$ resulted in the ad-supported scenario. Cases in which $\varepsilon \geq 0.45$ resulted in gradual increase in the number of pay-supported cases as the parameter k decreased from one to zero.

4.1 Ad-supported scenario

This scenario, as well as the pay-supported scenario, is best explained via figures that enable us to derive the demand for the app from its two segments: paid, and free. Figure 1 presents the relationship between the intrinsic utility α and the resulting utility.

Figure 1: Demand derivation, ad-supported scenario



The red line in Figure 1 represents the users whose intrinsic utility α is large enough that it is optimal for them to download the paid version and stay for two periods. From Equation 2 it is clear that the user's utility is $\alpha - p + \alpha\delta$ where the first term ($\alpha - p$) is her utility in the first period, while the second term ($\alpha\delta$) summarizes her utility in the second period. If her intrinsic utility α is slightly lower, then s/he will download the free version and stay for two periods with resultant utility of $\alpha(1 - \gamma) + \alpha\delta(1 - \gamma)$, as per Equation 3. It is straightforward to check that

these two lines intersect when $\alpha = p/\gamma(1 + \delta)$. Accordingly, it is easy to check that the two blue lines intersect when $\alpha = \varepsilon$.

Thus, we have three segments of consumers of differing sizes:

1. For high α , users download the paid version and stay for two periods. The size of this segment is $1 - p/\gamma(1 + \delta)$.
2. For intermediate α , users download the free version and stay for two periods. The size of this segment is $p/\gamma(1 + \delta) - \varepsilon$.
3. For low α , users download the free version and only those whose realization of μ is positive ($+\varepsilon$ with probability $1/2$) will stay for two periods. Their size is thus $\varepsilon/2$.

When we add these segments, each multiplied by price or advertising as per Equation 4, we obtain the profits for the app in the two periods as follows:

$$(5) \Pi(\text{ad supported scenario}) = p \left(1 - \frac{p}{\gamma(1 + \delta)} \right) + k\gamma \left(\frac{p}{\gamma(1 + \delta)} - \frac{\varepsilon}{2} \right) - \gamma^2$$

To obtain optimal price and advertising, we differentiate Equation 5 with respect to p and γ and equate to zero – see Web Appendices A and B for details⁷.

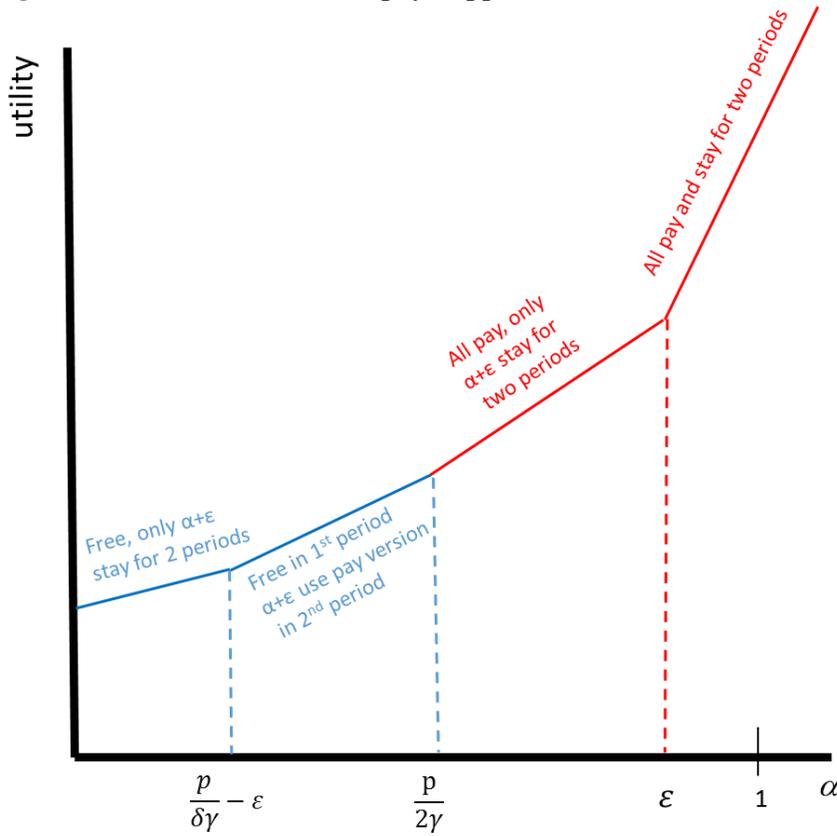
There is a special case of this scenario in which the intersection point of the paid and free versions (red and blue lines) is greater than or equal to one ($p/\gamma(1 + \delta) \geq 1$). Thus in this case we're left with only two segments (2 and 3 of the list above). The rest of the analysis follows through in the same manner, *mutatis mutandis*, see Web Appendices A and B.

4.2 Pay-supported scenario

In the same manner as in the ad-supported scenario above, we can construct Figure 2 with its four segments:

⁷ It is straightforward to check that for both Equations 5 and 6, the Hessian matrices are negative semidefinite, and thus second-order conditions for maximization of the profit functions are indeed satisfied.

Figure 2: Demand derivation, pay-supported scenario



First note that as ϵ is high, all users whose intrinsic utility (α) is lower than ϵ and whose realization is negative ($-\epsilon$ with probability $\frac{1}{2}$) will not use the app in the second period. This includes all segments except for the first.

1. For high α , users download the paid version and stay for two periods. The size of this segment is $1 - \epsilon$.
2. For intermediate-high α , users download the paid version and only those whose realization of μ is positive ($+\epsilon$ with probability $\frac{1}{2}$) will stay for two periods. The size of this segment is thus $(\epsilon - \frac{p}{2\gamma})$.
3. For intermediate α , users download the free version and those whose realization of μ is positive switch to the paid version in the second period. The size of this segment is $\frac{p}{\delta\gamma} - \frac{p}{\delta\gamma} + \epsilon$ for the free version, and half this number for the paid version.
4. For low α , users download the free version and only those whose realization of μ is positive will stay for two periods. Their size is thus: $(\frac{p}{\delta\gamma} - \epsilon)/2$.

When we add these segments, each multiplied by price or advertising as per Equation 4, we obtain the profits for the app in the two periods as follows:

$$(6) \Pi(\text{pay supported scenario}) = p \left(1 - \frac{p}{2\gamma}\right) + \frac{p}{2} \left(\frac{p}{2\gamma} - \frac{p}{\delta\gamma} + \varepsilon\right) + k\gamma \frac{p}{2\gamma} + \frac{k\gamma}{2} \left(\frac{p}{\delta\gamma} - \varepsilon\right) - \gamma^2$$

In order to obtain optimal price and advertising, we differentiate Equation 5 with respect to p and γ and equate to zero – see Web Appendices A and B for details.

From Figure 2 it is clear that for Segment 3 to exist (users who download the free version and whose realization of μ is positive, and switch to the paid version in the second period), requires that $\frac{p}{\delta\gamma} - \varepsilon \leq \frac{p}{2\gamma}$. There is a special case of this scenario in which the reverse inequality holds, and thus we're left with only three segments (1, 2, and 4 of the list above). The rest of the analysis follows through in the same manner, *mutatis mutandis*, see Web Appendices A and B.

5. Main results

In this section, we present the main results of our analysis beginning with the effects of the model's parameters on the firm's optimal choices in terms of price and advertising.

5.1 Price and advertising sensitivities

One of the advantages of analytical solutions is that we can derive optimal decision variables' sensitivity as a function of the model's parameters in each of the cases: Ad supported, and pay supported. The results are given in Web Appendices A and B and presented in Table 2 and Result 1:

Table 2a: Ad-supported scenario: Effects of advertising effectiveness, uncertainty, & persistence*

	Sensitivity to ad effectiveness (k)	Sensitivity to uncertainty (ε)	Sensitivity to persistence (δ)
Advertising (γ)	Strictly Positive	Negative	Positive
Price (p)	Positive	Negative	Positive

Table 2b: Pay-supported scenario: Effects of advertising effectiveness, uncertainty, & persistence

	Sensitivity to ad effectiveness (k)	Sensitivity to uncertainty (ε)	Sensitivity to persistence (δ)
Advertising (γ)	Strictly Positive	Strictly Positive	Positive
Price (p)	Strictly Positive	Strictly Positive	Strictly Positive

* The sensitivity is strictly positive if for all values of the parameters, the derivative is strictly positive. It is positive if the derivative is positive, yet a combination of the parameters might make it zero, or if in one of the special cases it is zero. Similarly for negative.

Result 1: *Optimal price and advertising increase ad effectiveness and persistence in both scenarios. They increase uncertainty in the pay-supported scenario, and decrease it in the ad-supported scenario.*

The reason that advertising and price increase with an increase in persistence (δ) is because such increase makes the user's utility in the second period higher, ergo her willingness to pay increases. This willingness to pay can materialize in one of two ways: She is willing to pay more for the app, or she is willing to be exposed to more advertising while using the app. The reason that advertising and price increase with an increase in advertising effectiveness is that such increase induces the firm to increase the level of advertising. Given the increase in ad intensity, the paid version becomes more appealing, thus enabling the firm to charge a higher price.

The rationale behind uncertainty's increasing effect on price and advertising is a bit different: At first glance, it appears as if ε 's role is symmetric, as it is represented by $\alpha - \varepsilon$ or $\alpha + \varepsilon$ in the consumer's utility. Yet this is misleading, as if the utility realization is $\alpha - \varepsilon$, then the consumer might not use the app at all in the second period; while if the utility realization is

$\alpha + \varepsilon$, the consumer might either continue to use the free app, or switch to the paid version.

As explained in Figure 1, in the ad-supported scenario, an increase in uncertainty causes Segment 3 to increase at the expense of the Segment 2, that is, the number of those who stay increases only if their utility realization is positive at the expense of those who stay regardless of their utility realization. The net effect is thus negative, causing the firm to lower the costs to consumers in terms of price and advertising.

As shown in Figure 2, in the pay-supported scenario, uncertainty is high, and all users whose utility is lower than ε and whose realization is negative ($-\varepsilon$ with probability $\frac{1}{2}$) will not use the app in the second period. This includes three of the four segments. Thus when uncertainty increases, there is positive asymmetry, as most users would not have used the app in the second period already. Now that uncertainty has increased, these users' utility increases in the second period, causing the firm to increase costs to consumers in terms of advertising and price.

5.2 Share of Revenue shares from free vs paid versions of the app

As we discussed in Section 4.1, there is a special case of an ad-supported scenario in which there are only two segments, both downloading the free version. Thus, the paid version of the app is not made available to the users. This is a relatively large case, or about 25% of all cases. To better understand the reasons behind the firm's (optimal) decision to offer a free version only, we note that this sub-scenario is characterized by a very low persistence parameter level (average $\delta = 0.33$) implying high customer satiation, and relatively high advertising effectiveness (average $k = 0.84$). When persistence is low, second-period utility is low, and many consumers cannot see a justification in paying for the app when it is highly unlikely that they will stay for a second period. Thus, the company focuses on the free version, wherein more consumers are more likely to remain, even with low levels of persistence. In this scenario, revenues are

generated via ads only, and as advertising effectiveness is high, ads have a good conversion rate, rendering them more profitable for the firm. This, together with Figure 3b, is summarized by the following result:

Result 2: *When persistence decreases (satiation increases), first, the share of the paid version of the app decreases, and then with low enough persistence levels (coupled with effective levels of advertising), the firm offers an ad-supported version of the app only.*

Note that persistence in our model is correlated with retention, as it affects second-period utility, which in turn directly correlates with the decision to stay for the second period. Some anecdotal evidence supports this relationship: Firstly, in a three-part series of notes about retention in mobile apps, Balfour (2017) listed four reasons why retention spurs apps' revenue growth, the third being that retention improves monetization. The examples given involve four monetization modes: Advertising, Subscription, Transactional, and Freemium, where in the latter, he reasons that revenues grow as a result of increased retention, as it increases upgrades from free to paid versions of apps, which is exactly the model described here.

Secondly, we obtained a dataset from an established publisher of multiple casual mobile game apps for children, several of which reached the Top 100 in the major app stores in recent years. Our dataset consists of data on the adoption and retention of six apps. Indeed, we observe a correlation between the percentage of users of the paid versions of the games and retention as summarized in Table 3⁸.

⁸ This cannot be construed as an empirical analysis, as we do not have data on control variables that should be controlled for in an empirical analysis. Thus this is model-free evidence for the correlation.

Table 3: Correlation between retention and % users of paid version of mobile games*

Game	% paid users	2 nd week retention rate
1	2.2%	51.1%
2	0.5%	45.0%
3	2.2%	49.8%
4	1.9%	45.2%
5	2.4%	54.3%
6	1.1%	44.3%
Correlation	80.7%	

* to preserve confidentiality, all numbers are multiplied by a constant $1 \leq \theta \leq 1.3$

5.2.1 Advertising, uncertainty, and app versions

To obtain the systematic change in the revenue share of paid vs free apps, observe Figure 3 and Result 3:

Result 3a. *With an increase in ad effectiveness, total revenues, as well as share of revenues from advertising, increase. Moreover, for low levels of ad effectiveness, most revenues are derived from the paid version of the app, while ad revenues completely dominate revenues when ad effectiveness increases.*

Result 3b. *With an increase in uncertainty, total revenues, as well as share of revenues from advertising, decrease.*

An increase in *advertising effectiveness* clearly increases advertising levels, and in turn the appeal of the ad version of the app. Thus, the share of revenues from the ad version increases. At very high levels of ad effectiveness, the scenario becomes pure advertising without the paid version (as per Result 2), which explains the sharp increase in the share of advertising for these high levels of k . As an increase in advertising effectiveness also increases price (as per Result 1), the app's overall profitability increases.

Figure 3a: Effect of ad effectiveness on revenues from free vs paid app versions

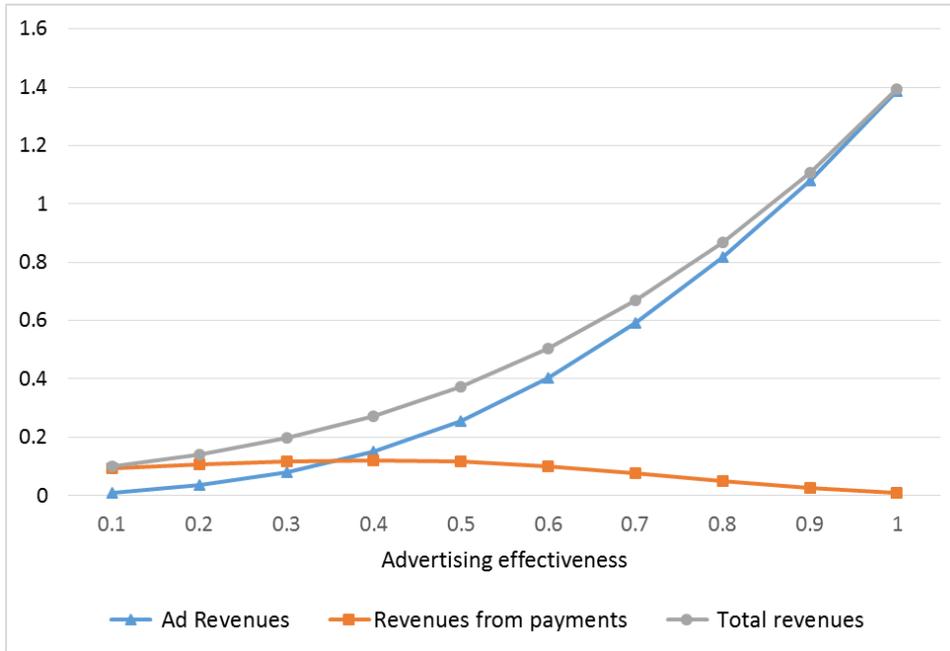
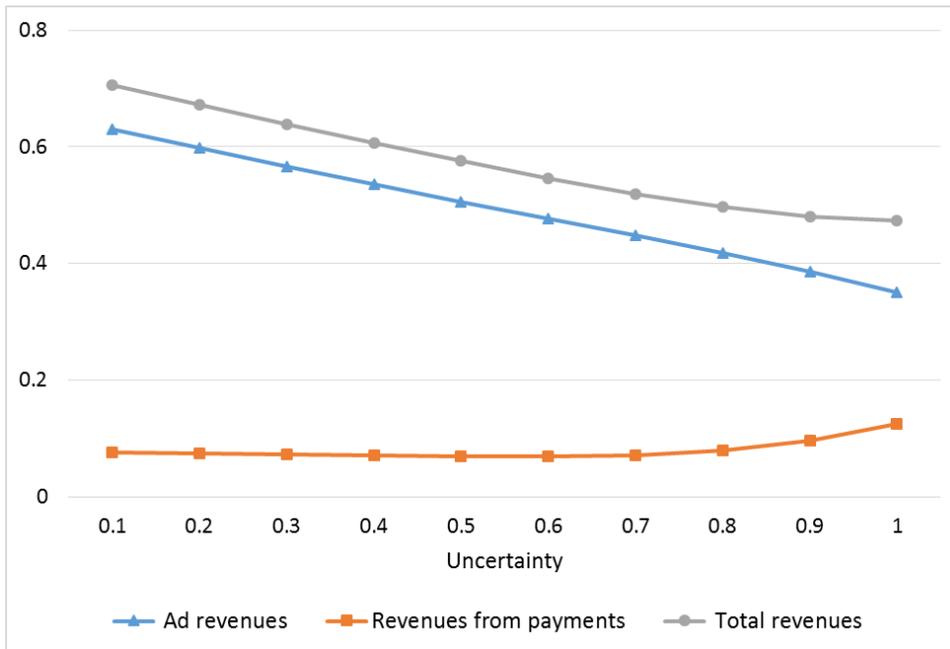


Figure 3b: Effect of uncertainty on revenues from free vs paid app version



With an increase in *uncertainty*, we obtain two opposing effects: (1) in the pay-supported scenario, price and advertising increase, and in this scenario, where uncertainty is high to begin with, most users will not use the app in the second period. Thus when uncertainty increases,

there is a positive effect, as most users would not have used the app in the second period already.

(2) In the ad-supported scenario, advertising and price decline, and moreover in this scenario, an increase in uncertainty increases the number of those who stay only if their utility realization is positive at the expense of those who stay regardless of their utility realization. The effect is thus negative. As the proportion of ad-supported scenarios is far larger than that of price-supported ones, the total effect on profitability is negative. However, this is the overall effect, and the two scenarios differ in this respect: In the ad-supported scenario, uncertainty's effect on profits is negative; whereas in the pay-supported scenario, Web Appendix C demonstrates that if k is smaller than δ , profits increase under uncertainty. As Table 1 shows, in the pay-supported scenario, indeed k is smaller than δ . We thus checked numerically, and indeed in the two cases of the pay-supported scenario, the conditions set forth in Web Appendix C hold, and thus we have the following:

Result 4. *When uncertainty increases, profits in the pay-supported scenario increase, while profits in the ad-supported scenario decrease.*

5.3 The role of advertising in supporting price: Damaged Good production

In a two-period model, the free version contributes to the firm's profits not only via advertising revenues, but also by acting as sort of a "sample": Specifically, some consumers who would otherwise not use the paid app (as the paid version is too costly for them) download and use the free version to learn their fit with the app. Some of these are likely to switch in the second period to the paid version. Thus, even if advertisers pay little for ads, the firm can benefit from offering a free version with ads. However, offering this version also has some costs: Some consumers, who would otherwise buy the paid version, might find the free version appealing. Thus, the optimal level of advertising balances between two incentives, so that the firm's

objective function depends on ad intensity even when $k = 0$. Furthermore, *the free version with ads can yield profits even if advertisers are not paying for these ads.*

To examine this idea – that the free version with ads can yield profits even if advertisers are not paying for these ads – we focused our analysis on all cases in which $k = 0$. Indeed, in all cases wherein a paid version exists (all except for the subcase of an ad-supported version with no paid version) we find that the optimal advertising level when $k = 0$ is strictly positive. The firm has two sources of income: the consumers who downloaded the paid version in the first period; and the consumers who chose to download the free version in the first period, then switch to the paid version in the second period. The firm does not realize any revenues from the latter segment in the first period (as $k = 0$), but does realize some profits from these consumers in the second period. This is summarized in the next statement:

Result 5. *When advertising effectiveness is zero, the firm still advertises, despite the fact that it sees no revenues from these ads.*

This situation bears similarity to the production of damaged goods (Deneckere and McAfee 1996; Halbheer et al. 2018), where firms deliberately “damage” an entire line of products of their offering in order to better price discriminate among their customers. An iconic example is IBM introducing the low-cost Model E laser printer that was identical to the high-cost IBM laser printer, except for the insertion of a chip that slowed printing from 10 pages per minute to 5 (Deneckere and McAfee 1996). In our case, the app-with-ads is a damaged good, yet the motivation is not price discrimination: The firm constructs this app knowing that it will not obtain any direct benefit from it, as it hopes to get some consumers to download it for the sake of learning its fit to their preferences. The firm makes money only from the paid version of the app, despite setting an optimal level of advertising for the free version, so that the users will switch to the paid version where their utility is higher, as this version is free of advertising.

6. Discussion

When looking at app markets, managers and consumers may see two broad types of apps. Some apps are by nature short term, i.e., consumers' utility may be high early on, yet soon they can expect to move to the next one. Many games and other hedonic apps fit this description. Other apps are longer-term in nature: If they fit consumers, they may stick with them for a while. Business, functional, and other utilitarian apps may better fit this category. What we showed here is that this distinction may lead to a fundamental difference in optimal management of the monetization process due to differences in propensity to stay.

We focused on the tension between the main monetization sources: paying consumers versus paying advertisers. While the tension between the monetization options is recognized as fundamental to app success (Pozin 2014), in-depth answers as to the drivers of the respective use of each are lacking. Furthermore, to the best of our knowledge, no previous studies have analyzed this issue in a customer duration dynamic setting. We show that such a setting, i.e., the combination of satiation and uncertainty, leads to some important insights on price levels, advertising intensity, profitability, and profitability sources. We summarize our findings as follows:

- We show that as satiation decreases, prices and advertising intensity increase, and the share of revenues accruing from the paid version increases. The reason is that an increase in persistence increases consumers' willingness to pay, which manifests in one of two ways: The user is willing to pay more for the app, or he is willing to be exposed to more advertising while using the app. This pushes some of those who are using the free version toward the paid version in both periods, or in Period 2. As a result, the share of consumers who use the paid version increases.
- As uncertainty increases, we see two opposing effects: (1) in the pay-supported scenario where uncertainty is high to begin with, all users whose intrinsic utility is low and whose realization of uncertainty is negative, will not use the app in the second period. Thus when

uncertainty increases, there is a positive effect, as most users would not have used the app in the second period in any case (with the lower uncertainty). Thus in the pay-supported scenario, profits increase under uncertainty. (2) In the ad-supported scenario, an increase in uncertainty increases the number of those who stay only if their utility realization is positive, at the expense of those who stay regardless of their utility realization. The effect is thus negative. As the proportion of the ad-supported scenarios is far larger than the pay-supported ones, the total effect on profitability is negative.

- We demonstrate that a firm can profit from offering a free app version with ads even if advertisers are not paying for those ads. In other words, the firm benefits from offering a version of the app that includes ads even if the consumers are not paying for the app and the advertisers are not paying for the ads. The logic behind offering this “damaged good” is that in a dynamic setting, the free version acts as a sort of “sample” that enables consumers to learn their fit with the app. The firm would rather add ads to this version (even if they’re not paid for) in order to retain the consumers who have high valuation of the app away from the “sample” and buying the paid version.

6.1 Managerial implications

The question of how to treat apps via the two main mechanisms of free (advertising) and paid (premium version, or in-app purchase) versions, is key to app management, in particular given many apps’ low profitability. Thus, managers are encouraged to study the profitability outcome of the various options and understand their sources (Nathanson 2016). Our work carries a number of implications that contribute to this discussion.

Setting realistic expectations

How many paid vs free downloads should app makers expect? To what extent does the percentage of each signal an app’s success? A straightforward way to look at composition of paid versus unpaid apps uses a simple segmentation scheme under which there are two kinds of users: Those who see limited value in the app, and thus will not be willing to pay; and those who see greater value and are willing to pay. Because the app maker may find it hard to assess the

nature of newcomers, they will offer a free version up front, and charge some price for a minority that desires higher utility (Nathanson 2016).

While this approach helps to portray a simple-to-understand picture of the market, our analysis demonstrates that the respective shares of paid and free apps are in fact a function of a complex process. Uncertainty, satiation, and the marketing mix elements of price and advertising effectiveness combine to determine the market outcome. The advantage of a formal analytical approach lies in the ability to portray the dynamics that shape the utility function, and understand the impact of the various factors that create the emerging dynamics. This view has a direct effect when forming expectations of the profitability created by apps. For example, an ongoing debate among investors and venture capitalists surrounds the viability of freemium business models (Maltz 2012). Understanding the complexity of the situation will enable entrepreneurs and investors alike to form more educated expectations based on the app-related models; identify strengths and weaknesses; and formulate business strategies that take into account the endogenous ways in which the shares of paid versus free versions could be forecasted and realized.

Realistic expectations should also be formed regarding the effectiveness of marketing mix elements in the app monetization environment. As Result 5 suggests, advertising's effectiveness should be considered in a wider perspective that takes into account its overall effect on profitability in the free versus paid app monetization environment.

Carefully balancing investments in paid vs. free app versions

Result 2 demonstrates the delicate balance that must be struck between the free vs paid app versions in terms of investment in satiation reduction, price promotion, and advertising levels. Any change in one of these variables in turn alters the composition of the segments that

download the free vs paid app version. Furthermore, this composition has a direct effect on the app's profitability, as Result 3 demonstrates. Moreover, the composition has an effect on retention, as paid users have higher retention rates (see Result 2 and Table 3).

An interesting issue is the effect of uncertainty: Note that the majority of users download and use free app versions, where revenues and retention, and thus lifetime value, are lower. Thus firms are trying to contrive ways to move users from free to paid app versions. Having said this, the free version plays a major role in the face of uncertainty: It is used by consumers as a sampling device, so as to better learn the app's idiosyncratic value to them. Later on they can decide to continue using the free version, or switch to the paid one, or else drop the app from further consideration. It is this role of sampling that renders uncertainty profitable under the pay-supported scenario – higher uncertainty induces users to try the free version and then switch to the paid version of the app, as in this scenario, the cost of advertising to the consumer is high (relative to price).

Investing in satiation reduction

Our results highlight in particular satiation's role, and how persistence with the app affects profitability. The contribution of customer duration to app profitability and survival is increasingly acknowledged by industry observers (Jacobson 2018). App makers are encouraged to consider user retention at the expense of acquisition, and formulate ways to deliver more value to users and be in touch with them so they will stay (Perro 2018). These efforts are particularly notable for mobile apps given the difficulty in retaining adopters of free products (Datta, Foubert, and Van Heerde 2015).

Given the expected effect of satiation on app use duration, as is also cited here, app creators can draw from the emerging behavioral literature on satiation. For example, to decrease satiation,

managers may want to encourage users to change their consumption rates (Galak, Kruger, and Loewenstein 2013), or to think of future variety (Sevilla, Zhang, and Kahn 2016). Apps may want to manage the similarity of new experiences the users get (Lasaleta and Redden 2018), or limit the times for which the products are available (Sevilla and Redden 2014).

Yet the road from persistence to retention is not always linear and straightforward. As Result 2 shows, satiation is correlated with the share of users who download the free version of the app, which in turn affects both profitability from current users as well as retention, as free users have lower retention rates. What we see is the indirect way in which satiation affects the marketing mix. The classic effect of retention on profit is considered to be direct, through customer lifetime value (Ascarza et al. 2018). Persistence will certainly have an effect in this direction. Yet as we see in Result 1, persistence also has a positive impact on advertising levels and on the price to be charged for the paid version. App creators should thus deeply examine ways to decrease satiation, and consider carefully the positive consequences of these efforts.

6.2 Limitations

Our attempt to capture a complex market situation with a relatively parsimonious model clearly has a number of limitations. One issue is the lack of explicit modeling of competitive activity, which takes into account alternative space, time, and competitive apps. The added complexity thereof is beyond our scope here. It should also be mentioned that it is not trivial to model explicit competition, due to the complex meaning of what constitutes a “competitor” in this market. Many free products compete in a general sense for users’ attention and time, and not necessarily with products to which they are very similar. However, there may be specific categories in which clear, direct competitors emerge.

We examined a specific freemium model in which the product’s premium version had no

advertising. There are more complex free business models in which, for example, users purchase in-app add-ons that change the product's utility in various ways. Modeling each of these scenarios would add complexity that we feel is unneeded at this stage, yet can of course be done in future explorations. In the same vein, we assumed a fixed price for the paid version. This assumption is supported by market observations and interviews with managers, which suggest that given the short time periods, the paid version price is largely fixed. Changing this assumption adds significant complexity, yet can be an interesting avenue for future research.

Finally, further analysis might wish to examine peer effects, which are absent from our analysis. When a customer leaves, it may affect not only the firm's ability to acquire other customers via word of mouth, but may also lead to the churn of other current customers (Nitzan and Libai 2011). This may further emphasize the importance of efforts to decrease satiation, and increase customer utility in general. Furthermore, existing paying customers may affect the other customers' utility by creating within-customer contagion processes (Bapna and Umyarov 2015). In this sense, there is still a way to go toward better understanding of the complex process in which profitability is created when monetizing free versus paid apps.

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Web Appendices

Web Appendix A: Consumers' choices of free vs paid app versions

In this web appendix, we consider the two options the user faces when she adopts the app. She can either choose the paid version with U_p utility, as described in Equation (2), or she can choose the free option, with utility U_a , as described in Equation (3). While U_p is straightforward, U_a describes two distinct cases. Both cases are equivalent for low and high values of α . For low values of α , only users with positive realization of μ will use the free app in the second period, and for high values of α , users with negative realization of μ will use the free app in the second period, while users with positive realization of μ will switch to the paid app in the second period. The difference between the cases lies in intermediate levels of α depending on two thresholds for α . The first is ε , the threshold for users with negative realization of μ for adopting the free app. The second is $p/\delta\gamma - \varepsilon$, the threshold for free users with positive realization for μ to switching from the free to the paid app. Thus we have two cases depending upon the relative values of these two thresholds. Note that the condition $\varepsilon < p/\gamma\delta - \varepsilon$ is satisfied if (and only if) $\varepsilon < p/2\gamma\delta$.

Case 1: $\varepsilon < p/2\gamma\delta$

In this case, the slopes of the utility segments of U_a , are given by the following equations:

$$(A1) \frac{\partial U_a}{\partial \alpha} = \begin{cases} (1 - \gamma)(1 + \frac{\delta}{2}) & \text{for } \alpha < \varepsilon \\ (1 - \gamma)(1 + \delta) & \text{for } \varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon \\ (1 - \gamma)(1 + \frac{\delta}{2}) + \frac{\delta}{2} & \text{for } \frac{p}{\gamma\delta} - \varepsilon < \alpha \end{cases}$$

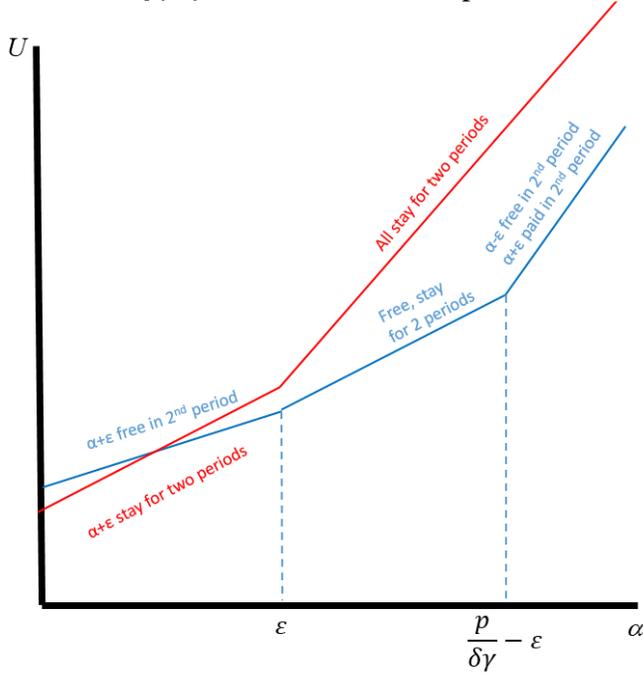
While the slope of U_p is:

$$(A2) \frac{\partial U_p}{\partial \alpha} = \begin{cases} 1 + \frac{\delta}{2} & \text{for } \alpha < \varepsilon \\ 1 + \delta & \text{for } \varepsilon < \alpha \end{cases}$$

Note that at the intersection point of U_p and U_a (i.e., for the same level of α), the slope of U_p is larger than that of U_a . It is also straightforward to see that when $\alpha = 0$, $U_a(0) > U_p(0)$, for all cases of Web Appendix B (except for the last case that we prove cannot exist). This will be evident in all subsequent figures in both web appendices. As the slope of U_p makes a discrete change at $\alpha = \varepsilon$, and this corresponds to the exact same point in the change of slope of U_a , the utilities can have three possible intersection locations as follows:

Case 1.1: $\varepsilon < p/2\gamma\delta$; Low intersection point: The intersection occurs at $\alpha < \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will always be greater than U_a , as its slope is greater – see Equations A1 and A2, and Figure A1.

Figure A1: $\varepsilon < p/2\gamma\delta$; Low intersection point



At the point of intersection,

$$U_p = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon) = \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon) \right] \cdot (1 - \gamma) = U_a$$

$$p = \gamma \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon) \right]; \text{ or } \alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$$

Since in this case, by definition, $\varepsilon < \frac{p}{\delta\gamma} - \varepsilon$ the lines intersect when

$\gamma \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon) \right] > 2\varepsilon\delta\gamma$, or when $\alpha > \frac{3\varepsilon\delta}{2 + \delta}$. Furthermore, here $\alpha < \varepsilon$, the lines intersect when:

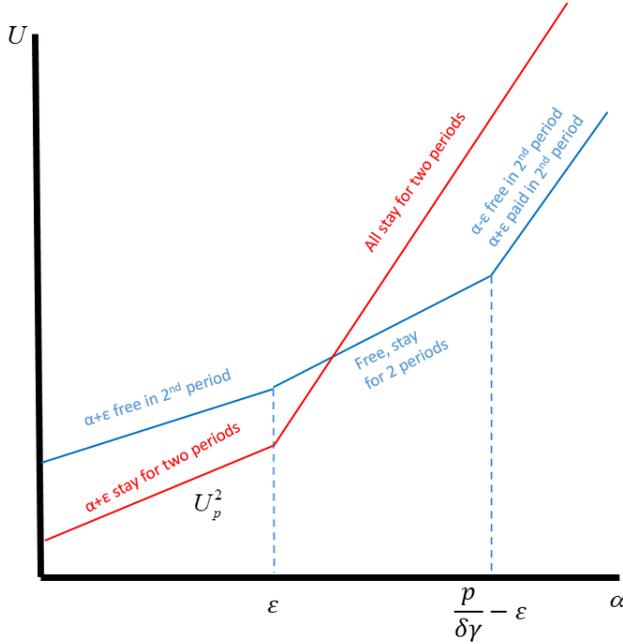
$\frac{3\varepsilon\delta}{2 + \delta} < \alpha < \varepsilon$. Replacing α with $\frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$ defines the range of ε for this case, where $\frac{3\varepsilon\delta}{2 + \delta} <$

$$\frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)} < \varepsilon; \text{ or } \frac{p}{\gamma(1 + \delta)} < \varepsilon < \frac{p}{2\delta\gamma}.$$

Case 1.2: $\varepsilon < p/2\gamma\delta$; Medium intersection point: The intersection occurs at $\varepsilon < \alpha < \frac{p}{\delta\gamma} - \varepsilon$.

Once this intersection occurs, the lines cannot intersect again, as U_p will remain above U_a , as its slope is greater – see Equations A1 and A2, and Figure A2.

Figure A2: $\varepsilon < p/2\gamma\delta$; Medium intersection point
At the point of intersection,



$$U_a = \alpha(1 - \gamma)(1 + \delta) = \alpha - p + \alpha\delta = U_p$$

$$p = \alpha\gamma(1 + \delta); \text{ or } \alpha = \frac{p}{\gamma(1+\delta)}$$

However, as the intersection is greater than ε , we also require $\alpha = \frac{p}{\gamma(1+\delta)} > \varepsilon$, defining the range of ε for this case as well.

Case 1.3: $\varepsilon < p/2\gamma\delta$; High intersection point: The intersection occurs at $\alpha > \frac{p}{\gamma\delta} - \varepsilon$. As we show here, this case does not exist, i.e., the conditions under which it exists violate the requirement of Case 1. Assume, *a contrario*, that the intersection does occur, and thus at the point of intersection,

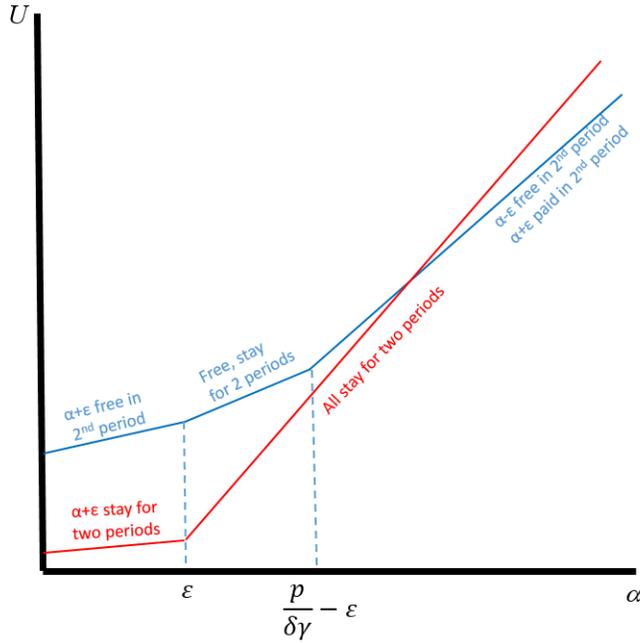
$$U_a = \alpha(1 - \gamma) + \frac{\delta}{2}(\alpha - \varepsilon) \cdot (1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon) - \frac{p}{2} = \alpha - p + \alpha\delta = U_p$$

$$\alpha(1 - \gamma) + \frac{\delta}{2}(\alpha - \varepsilon) \cdot (1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon) + \frac{p}{2} - \alpha(1 + \delta) =$$

$$= \alpha - \alpha\gamma + \left(\frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2}\right)(1 - \gamma) + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= \alpha - \alpha\gamma + \frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2} - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} = -\alpha\gamma - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{p}{2} = 0$$

Figure A3: $\varepsilon < p/2\gamma\delta$; High intersection point



$$p = \gamma[2\alpha + \delta(\alpha - \varepsilon)]; \text{ or } \alpha = \frac{p + \gamma\varepsilon\delta}{\gamma(2 + \delta)}$$

Here $\alpha > \frac{p}{\gamma\delta} - \varepsilon$, so that the lines intersect when $\frac{p + \gamma\varepsilon\delta}{\gamma(2 + \delta)} > \frac{p}{\gamma\delta} - \varepsilon$ or $p < \gamma\varepsilon\delta(1 + \delta)$;

However, in this case, $\varepsilon < \frac{p}{\gamma\delta} - \varepsilon$, that is, $p > 2\varepsilon\delta\gamma > \gamma\varepsilon\delta(1 + \delta)$, in contrast to the above condition on p . Thus the utility lines do not intersect, ergo Case 1.3 does not exist.

Case 2: $\varepsilon > p/2\gamma\delta$

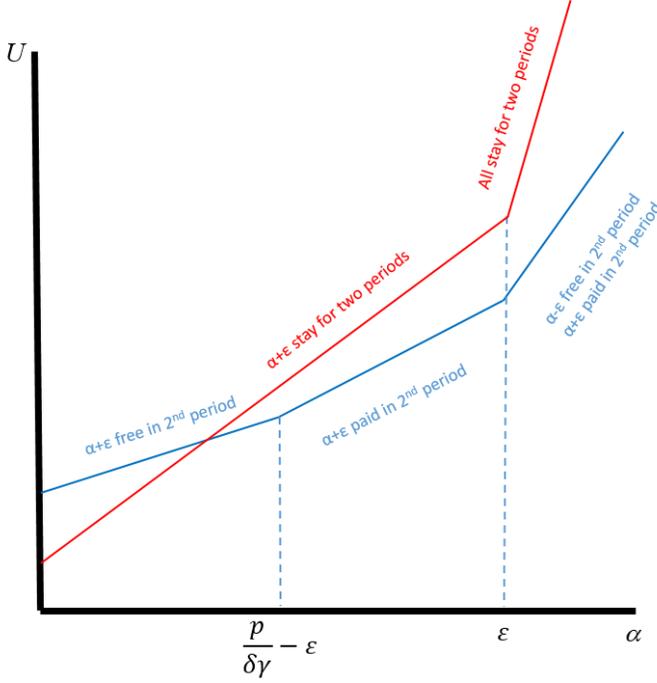
In this case, the slopes of the utility segments of U_a are given by the following equations:

$$(A3) \quad \frac{\partial U_a}{\partial \alpha} = \begin{cases} (1 - \gamma)(1 + \frac{\delta}{2}) & \text{for } \alpha < \frac{p}{\gamma\delta} - \varepsilon \\ 1 - \gamma + \frac{\delta}{2} & \text{for } \frac{p}{\gamma\delta} - \varepsilon < \alpha < \varepsilon \\ (1 - \gamma)(1 + \frac{\delta}{2}) + \frac{\delta}{2} & \text{for } \varepsilon < \alpha \end{cases}$$

While U_p 's slope is the same as in Equation (A2). As in Case 1, U_a starts at a higher value than U_p , and at the intersection point of U_p and U_a , U_p 's slope is greater than that of U_a , and as the slope of U_p changes at $\alpha = \varepsilon$, and this corresponds to the exact same point in the change of U_a 's slope, the utilities can have three possible intersection locations as follows:

Case 2.1: $\varepsilon > p/2\gamma\delta$; Low intersection point: The intersection occurs at $\alpha < \frac{p}{\gamma\delta} - \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will remain above U_a , as its slope is greater – see Equations A2 and A3, and Figure A4.

Figure A4: $\varepsilon > p/2\gamma\delta$; Low intersection point



At the point of intersection,

$$U_p = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon) = \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon) \right] \cdot (1 - \gamma) = U_a$$

$$p = \gamma \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon) \right]; \text{ or } \alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$$

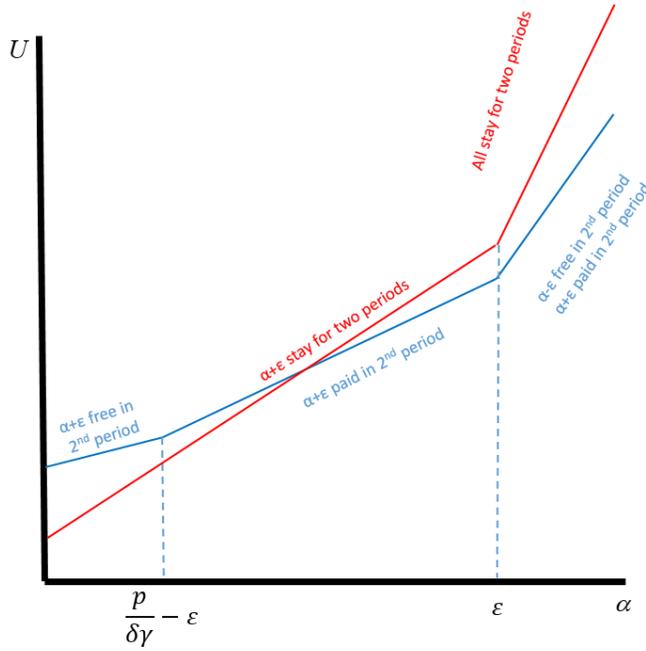
We still need to ascertain that $\alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)} < \frac{p}{\gamma\delta} - \varepsilon$, or $\varepsilon < \frac{p(2 - \delta)}{2\gamma\delta}$, including the case limit on ε

that defines ε 's range in this case: $\frac{p}{2\gamma\delta} < \varepsilon < \frac{p(2 - \delta)}{2\gamma\delta}$

Case 2.2: $\varepsilon > p/2\gamma\delta$; Medium intersection point: The intersection occurs at $\frac{p}{\gamma\delta} - \varepsilon < \alpha < \varepsilon$.

Once this intersection occurs, the lines cannot intersect again, as U_p will always be greater than U_a , as its slope is greater – see Equations A2 and A3, and Figure A5.

Figure A5: $\varepsilon > p/2\gamma\delta$; Medium intersection point



$$U_a = \alpha(1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon) - \frac{p}{2} = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon) = U_p$$

$$p = 2\alpha\gamma; \text{ or } \alpha = \frac{p}{2\gamma}$$

In this case, $\varepsilon > \frac{p}{2\gamma\delta}$, that is, $p < 2\varepsilon\delta\gamma$, so that the lines intersect when $2\alpha\gamma < 2\varepsilon\delta\gamma$ or $\alpha < \varepsilon\delta$.

Replacing α with $\frac{p}{2\gamma}$ defines ε 's range for this case, $\varepsilon > \frac{p}{2\delta\gamma}$.

Case 2.3: $\varepsilon > p/2\gamma\delta$; Large intersection point: The intersection occurs at $\alpha > \varepsilon$. As we show here, this case does not exist, that is, the conditions under which it exists violate the requirement of Case 2. Assume, *a contrario*, that the intersection does occur (see Figure A6), and thus at the point of intersection,

$$U_a = \alpha(1 - \gamma) + \frac{\delta}{2}(\alpha - \varepsilon) \cdot (1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon) - \frac{p}{2} = \alpha - p + \alpha\delta = U_p$$

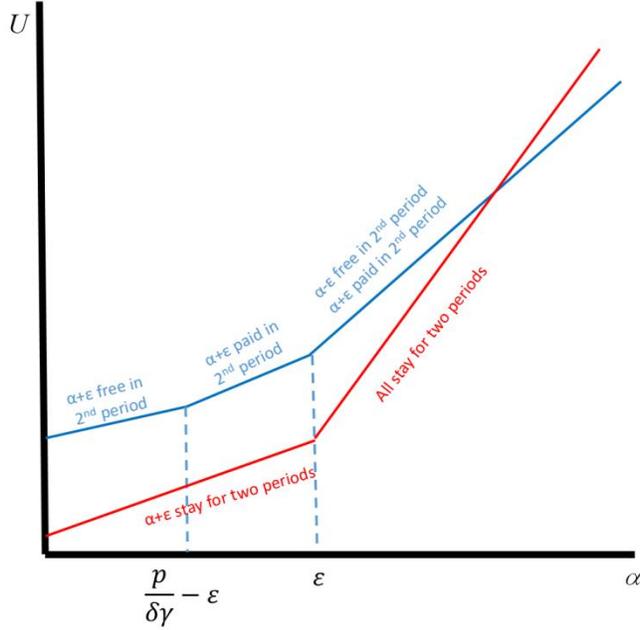
$$\alpha(1 - \gamma) + \frac{\delta}{2}(\alpha - \varepsilon) \cdot (1 - \gamma) + \frac{\delta}{2}(\alpha + \varepsilon) + \frac{p}{2} - \alpha(1 + \delta) =$$

$$= \alpha - \alpha\gamma + \left(\frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2}\right)(1 - \gamma) + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= \alpha - \alpha\gamma + \frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2} - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= -\alpha\gamma - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{p}{2} = 0$$

Figure A6: $\varepsilon > p/2\gamma\delta$; Large intersection point



$$p = \gamma[2\alpha + \delta(\alpha - \varepsilon)]; \text{ or } \alpha = \frac{p + \gamma\varepsilon\delta}{\gamma(2 + \delta)}.$$

In this case, $\varepsilon > \frac{p}{2\gamma\delta}$, or $p < 2\varepsilon\delta\gamma$, so that the lines intersect when $p = \gamma[2\alpha + \delta(\alpha - \varepsilon)] < 2\varepsilon\delta\gamma$ or $\alpha < \frac{3\varepsilon\delta}{2 + \delta}$. However, $\frac{3\varepsilon\delta}{2 + \delta} < \varepsilon$, contradicting the fact that the intersection occurs at a point that is greater than ε . Thus Case 2.3 cannot happen.

Summary of Case 1: When $\varepsilon < \frac{p}{2\gamma\delta}$, U_p 's and U_a 's utilities can only intersect in Cases 1.1 and

1.2. Case 1.1's range is when $\frac{p}{\gamma(1 + \delta)} < \varepsilon < \frac{p}{2\delta\gamma}$, and then $\alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$, and Case 1.2's range is when $\varepsilon < \frac{p}{\gamma(1 + \delta)}$, and then $\alpha = \frac{p}{\gamma(1 + \delta)}$.

Summary of Case 2: When $\varepsilon > \frac{p}{2\gamma\delta}$, U_p 's and U_a 's utilities can only intersect in Cases 2.1 and

2.2. Case 2.1's range is when $\frac{p}{2\gamma\delta} < \varepsilon < \frac{p(2 - \delta)}{2\gamma\delta}$, and then $\alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$, and Case 2.2's range is when $\varepsilon > \frac{p}{2\delta\gamma}$, and then $\alpha = \frac{p}{2\gamma}$.

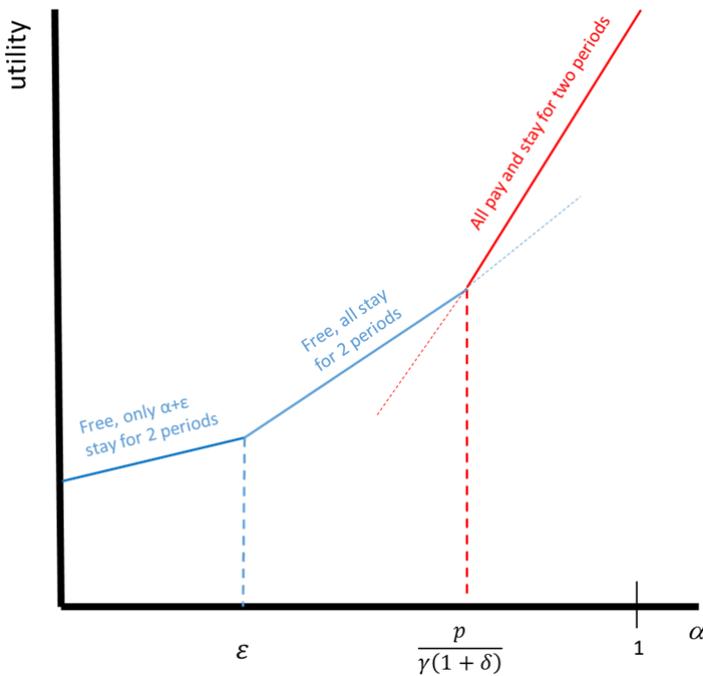
Web Appendix B: The firm's profit function

While in Web Appendix A we considered the two options that the user faces when he adopts the app, and found the four cases specified at the end of the web appendix; in this web appendix, we group the four cases into the two scenarios presented in the paper: Ad Supported, and Pay Supported, each with its own special case as described shortly. We begin with the Ad-Supported Scenario, that is, Case 1.2 in Web Appendix A, and its special case. We then proceed to the Pay-Supported Scenario - Case 2.2 of Web Appendix A, and its special case (Cases 1.1 and 2.1). We conclude by showing that a second special case of the Pay-Supported Scenario cannot exist, as it violates one of the conditions of the model's parameters.

Ad-Supported Scenario: Case 1.2 of Web Appendix A

Observe the following figure (Figure 1 in the text):

Figure B1: Ad-Supported Scenario



In this case, when $\alpha < \epsilon$, everyone uses the free version in the first period, and those with positive realization of μ stay for a second period. When $\epsilon < \alpha < \frac{p}{\gamma(1+\delta)}$, everyone uses the free version in both periods. Finally, when $\alpha > \frac{p}{\gamma(1+\delta)}$, everyone uses the paid version for both periods. We use the firm's objective function as defined in Equation (4). With one addition, we define P_{12}

as the share of those who paid in Period 1 and stayed in Period 2: While they do not generate any profit to the firm, and thus do not appear in Equation (4), we report their value here for completeness' sake.

$$\text{Here, } P_1 = P_{12} = 1 - \frac{p}{\gamma(1+\delta)}; P_2 = 0; A_1 = \frac{p}{\gamma(1+\delta)}; \text{ and } A_2 = \frac{2p - \gamma\varepsilon(1+\delta)}{2\gamma(1+\delta)}$$

$$\Pi = p \left(1 - \frac{p}{\gamma(1+\delta)} \right) + k\gamma \frac{4p - \gamma\varepsilon(1+\delta)}{2\gamma(1+\delta)} - \gamma^2$$

Differentiating with respect to p , equating to zero and collecting terms yield:

$$p_{opt} = \frac{\gamma}{2}(1 + 2k + \delta)$$

to be in this scenario requires $\gamma(1 + \delta) > \frac{\gamma}{2}(1 + 2k + \delta) > \varepsilon\gamma(1 + \delta)$,

$$\text{or: } \left(\varepsilon - \frac{1}{2} \right) (1 + \delta) < k < \frac{1+\delta}{2}$$

Differentiating with respect to γ , equating to zero and collecting terms yield:

$$\frac{p^2}{\gamma^2(1+\delta)} - \frac{k\varepsilon}{2} - 2\gamma = 0$$

Substitute the optimal p from the above equation, and solve to get the optimal γ :

$$\gamma_{opt} = \frac{(1 + 2k + \delta)^2 - 2k\varepsilon(1 + \delta)}{8(1 + \delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = (1 + 2k + \delta) \frac{(1 + 2k + \delta)^2 - 2k\varepsilon(1 + \delta)}{16(1 + \delta)}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{[(1 + 2k + \delta)^2 - 2k(1 + \delta)\varepsilon]^2}{64(1 + \delta)^2}$$

The partial derivatives of the optimal advertising, price, and profit levels are:

$$\frac{\partial \gamma}{\partial \varepsilon} = -\frac{k}{4} < 0$$

$$\frac{\partial \gamma}{\partial \delta} = \frac{(1+\delta)^2 - 4k^2}{8(1+\delta)^2} \geq 0, \text{ since } \frac{1+\delta}{2} > k$$

$$\frac{\partial \gamma}{\partial k} = \frac{2 - \varepsilon}{4} + \frac{k}{(1 + \delta)} > 0$$

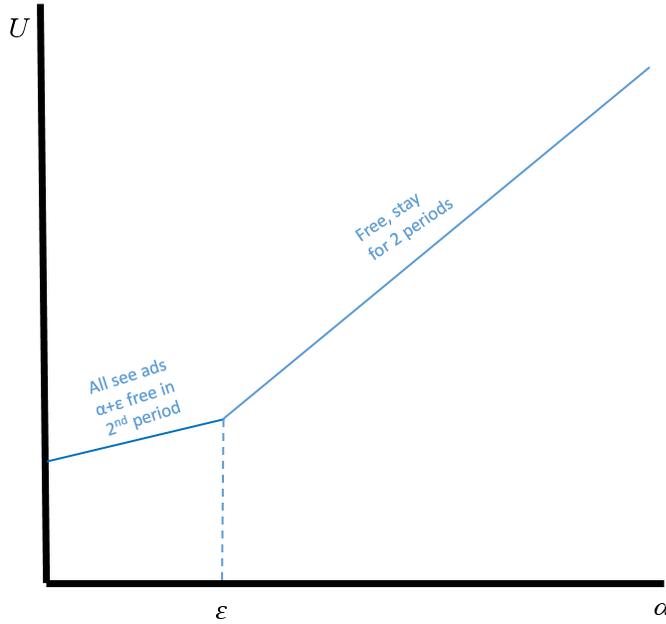
$$\frac{\partial p}{\partial \varepsilon} = -\frac{k(1 + 2k + \delta)}{8} < 0$$

$$\frac{\partial p}{\partial \delta} = \frac{(1+\delta)^3 - 4k^3 + k(1+\delta)^2(3-\varepsilon)}{8(1+\delta)^2} \geq 0, \text{ since } \frac{1+\delta}{2} > k, \varepsilon < 1$$

$$\frac{\partial p}{\partial k} = \frac{3(1+2k+\delta)^2 - \varepsilon(1+\delta)(1+4k+\delta)}{8(1+\delta)} > 0$$

Ad-Supported Scenario, special case: For Figure B1 to hold, we need $\frac{p}{\gamma(1+\delta)} < 1$ to be true. In this special case we address the situation wherein the reverse holds, that is, $\frac{p}{\gamma(1+\delta)} \geq 1$. Thus the two lines (paid and free) do not intersect, as the price is relatively high compared to the level of advertising, so that no one pays for the app – see Figure B2.

Figure B2: Ad-Supported Scenario, special case



In this case, when $\alpha < \varepsilon$, everyone uses the free version in Period 1, and those with positive realization of μ stay for a second period. When $\alpha > \varepsilon$, they stay for both periods.

Here, $P_1 = P_{12} = 0$; $P_2 = 0$; $A_1 = 1$; and $A_2 = 1 - \varepsilon + \frac{\varepsilon}{2}$

$$\Pi = k\gamma \left(2 - \frac{\varepsilon}{2}\right) - \gamma^2$$

Differentiating with respect to γ , equating to zero and collecting terms yield:

$$\gamma_{opt} = \frac{k(4 - \varepsilon)}{4}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{k^2(-4 + \varepsilon)^2}{16}$$

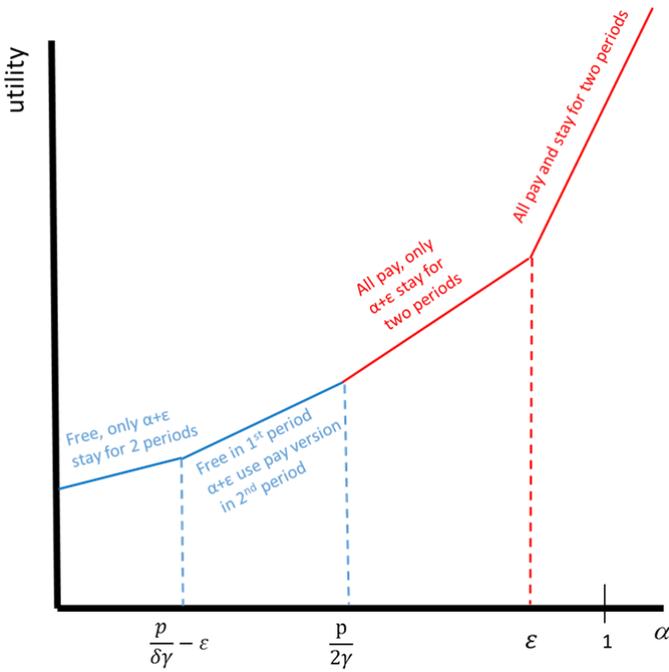
The partial derivatives of the optimal advertising and profit levels are:

$$\frac{\partial \gamma}{\partial \varepsilon} = -\frac{k}{4} < 0; \frac{\partial \gamma}{\partial \delta} = 0; \frac{\partial \gamma}{\partial k} = \frac{4-\varepsilon}{4} > 0$$

Pay-Supported Scenario: Case 2.2 of Web Appendix A

Observe the following figure (Figure 2 in the text):

Figure B3: Pay-Supported Scenario



In this case, when $\alpha < \frac{p-\delta\gamma\varepsilon}{\delta\gamma}$, everyone uses the free version in Period 1, and those with positive realization of μ stay for a second period. When $\frac{p-\delta\gamma\varepsilon}{\delta\gamma} < \alpha < \frac{p}{2\gamma}$, everyone uses the free version in Period 1, those with positive realization of μ switch to the paid version in the second period, and those with negative realization of μ churn. When $\frac{p}{2\gamma} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive realization of μ stay for a second period. Finally, when $\alpha > \varepsilon$, everyone uses the paid version for both periods.

Here, $P_1 = 1 - \frac{p}{2\gamma}$; $P_2 = \frac{1}{2} \left(\frac{p}{2\gamma} - \frac{p-\delta\gamma\varepsilon}{\delta\gamma} \right)$; $A_1 = \frac{p}{2\gamma}$; and $A_2 = \frac{1}{2} \cdot \frac{p-\delta\gamma\varepsilon}{\delta\gamma}$

$$P_{12} = 1 - \varepsilon + \frac{1}{2} \left(\varepsilon - \frac{p}{2\gamma} \right) = 1 - \frac{p}{4\gamma} - \frac{\varepsilon}{2}$$

$$\Pi = p \left(1 - \frac{p}{2\gamma} \right) + \frac{p}{2} \left(\frac{p}{2\gamma} - \frac{p - \delta\gamma\varepsilon}{\delta\gamma} \right) + k\gamma \frac{p}{2\gamma} + \frac{k\gamma(p - \delta\gamma\varepsilon)}{2\delta\gamma} - \gamma^2$$

Differentiating with respect to p , equating to zero and collecting terms yield:

$$p_{opt} = \frac{\gamma[k(1+\delta) + \delta(2+\varepsilon)]}{2+\delta};$$

Differentiating with respect to γ , equating to zero and collecting terms yield:

$$\frac{1}{4} \left[\frac{p^2(2+\delta)}{\gamma^2\delta} - 2k\varepsilon - 8\gamma \right] = 0$$

Substitute the optimal p from the above equation, and solve to get the optimal γ :

$$\gamma_{opt} = \frac{k^2(1+\delta)^2 + 2k\delta(2+2\delta-\varepsilon) + \delta^2(2+\varepsilon)^2}{8\delta(2+\delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = \frac{[k(1+\delta) + \delta(2+\varepsilon)][k^2(1+\delta)^2 + 2k\delta(2+2\delta-\varepsilon) + \delta^2(2+\varepsilon)^2]}{8\delta(2+\delta)^2}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{([k + (2+k)\delta]^2 - 2(k-2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^2}{64\delta^2(2+\delta)^2}$$

The partial derivatives of the optimal advertising and price levels are:

$$\frac{\partial \gamma}{\partial \varepsilon} = \frac{\delta(2+\varepsilon) - k}{4(2+\delta)} > 0$$

$$\frac{\partial \gamma}{\partial \delta} = \frac{[\delta(2+\varepsilon) - k][k(1+\delta) + \delta(2+\varepsilon)]}{4\delta^2(2+\delta)^2} > 0$$

$$\frac{\partial \gamma}{\partial k} = \frac{(1+\delta)[k + \delta(2+k)] - \delta\varepsilon}{4\delta(2+\delta)} > 0$$

For the following three derivatives, we show that they are positive using numerical simulations, wherein the derivatives are always positive for δ values that match the numerical simulations' solutions as described in section 4 of the paper.

$$\frac{\partial p}{\partial \varepsilon} = \frac{k^2(\delta^2 - 1) + 3\delta^2(2+\varepsilon)^2 + 2k\delta[2 - \varepsilon + \delta(4+\varepsilon)]}{8(2+\delta)^2} > 0$$

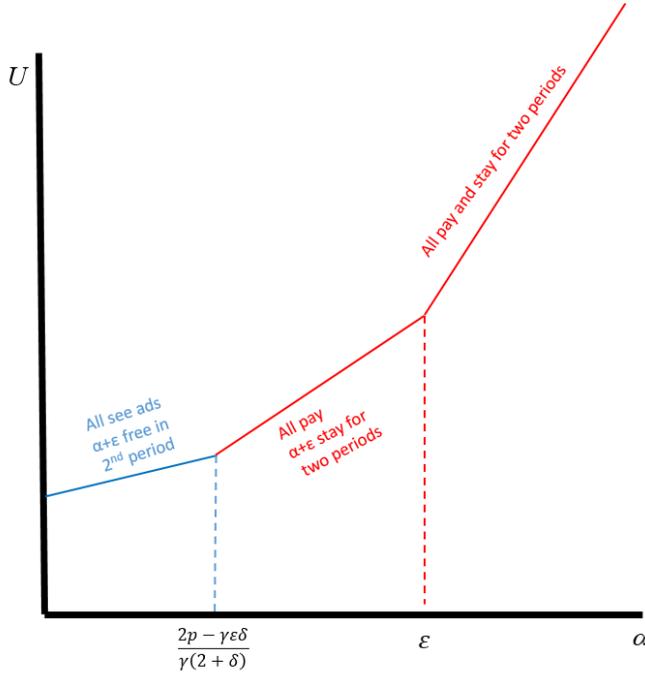
$$\frac{\partial p}{\partial \delta} = \frac{k^3(\delta-2)(1+\delta)^2 + 4\delta^3(2+\varepsilon)^3 + k\delta^2(2+\varepsilon)(12+18\delta-2\varepsilon+5\delta\varepsilon) + 2k^2\delta^2[6+\varepsilon+2\delta(3+\varepsilon)]}{8\delta^2(2+\delta)^3} > 0$$

$$\frac{\partial p}{\partial k} = \frac{3(1 + \delta)[k + \delta(2 + k)]^2 + 2\delta\varepsilon[2\delta - k + \delta^2(4 + k)] + \delta^2\varepsilon^2(\delta - 1)}{8\delta(2 + \delta)^2} > 0$$

Pay-Supported Scenario, special case 1: (Case 1.1 and 2.1 of Web Appendix A)

For Figure B3 to hold, we need $\frac{p}{\delta\gamma} - \varepsilon \leq \frac{p}{2\gamma}$ to be true. In this special case, we address the situation in which the reverse holds, as shown in Figure B4.

Figure B4: Pay-Supported Scenario, special case 1



In this case, we can combine Cases 1.1 and 2.1 of Web Appendix A, as they share the same intersection point. In this special case, when $\alpha < \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}$, everyone uses the free version in period 1, and those with positive realization of μ stay for a second period. When $\frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive realization of μ stay for a second period. Finally, when $\alpha > \varepsilon$, everyone uses the paid version for both periods.

The profit function components are:

$$\text{Here, } P_1 = 1 - \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}; P_2 = 0; A_1 = \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)}; \text{ and } A_2 = \frac{2p - \gamma\varepsilon\delta}{2\gamma(2 + \delta)}$$

$$P_{12} = 1 - \varepsilon + \frac{1}{2} \left(\varepsilon - \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)} \right) = \frac{\gamma(2 + \delta - \varepsilon) - p}{\gamma(2 + \delta)}$$

$$\Pi = p \left(1 - \frac{2p - \gamma\varepsilon\delta}{\gamma(2 + \delta)} \right) + k\gamma \frac{3(2p - \gamma\varepsilon\delta)}{2\gamma(2 + \delta)} - \gamma^2$$

Differentiating with respect to p , equating to zero and collecting terms yield:

$$p_{opt} = \frac{\gamma}{4}(2 + 3k + \delta + \varepsilon\delta);$$

Differentiating with respect to γ , equating to zero and collecting terms yield:

$$\frac{2p^2}{\gamma^2(2 + \delta)} - \frac{3k\delta\varepsilon}{2(2 + \delta)} - 2\gamma = 0$$

Substitute the optimal p from the above equation and solve to get the optimal γ :

$$\gamma_{opt} = \frac{(2 + 3k + \delta + \delta\varepsilon)^2 - 12k\delta\varepsilon}{16(2 + \delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = (2 + 3k + \delta + \varepsilon\delta) \frac{(2 + 3k + \delta + \delta\varepsilon)^2 - 12k\delta\varepsilon}{64(2 + \delta)}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{(9k^2 + 6k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)^2}{256(2 + \delta)^2}$$

The partial derivatives of the optimal advertising, price, and profit levels are:

$$\frac{\partial \gamma}{\partial \varepsilon} = \frac{\delta(2 + 3k + \delta + \delta\varepsilon) - 6k\delta}{8(2 + \delta)} > 0$$

$$\frac{\partial \gamma}{\partial \delta} = \frac{(2 - 3k + \delta + \delta\varepsilon)(2 + 3k + \delta + (4 + \delta)\varepsilon)}{16(2 + \delta)^2} > 0$$

$$\frac{\partial \gamma}{\partial k} = \frac{3(2 + 3k + \delta - \delta\varepsilon)}{8(2 + \delta)} > 0$$

$$\frac{\partial p}{\partial \varepsilon} = \frac{3\delta(-3k^2 + 2k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)}{64(2 + \delta)} > 0$$

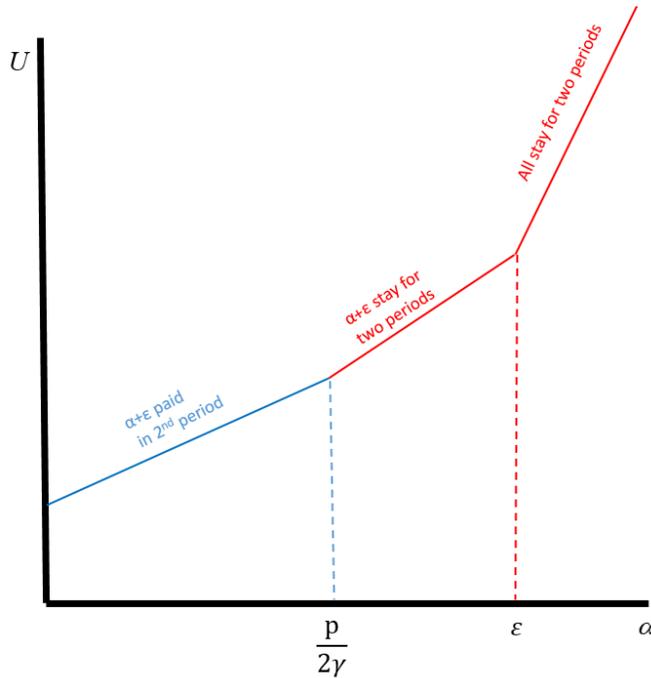
$$\frac{\partial p}{\partial \delta} = \frac{9k(2 + \delta)^2 - 18k^2\varepsilon + 6k(2 + \delta)^2\varepsilon - 3k\delta(4 + \delta)\varepsilon^2 + 2(2 + \delta + \delta\varepsilon)^2(2 + \delta + (3 + \delta)\varepsilon) - 27k^3}{64(2 + \delta)^2} > 0$$

$$\frac{\partial p}{\partial k} = \frac{9(2 + 3k + \delta)^2 + 6\delta(2 - 3k + \delta)\varepsilon - 3\delta^2\varepsilon^2}{64(2 + \delta)} > 0$$

Pay-Supported Scenario, special case 2

For Figure B3 to hold, we need $\frac{p}{\delta\gamma} - \varepsilon > 0$ to be true. In this special case we address the situation in which the reverse holds, as shown in Figure B5. As we show below, this case contradicts the requirement that $k \geq 0$, and therefore does not exist. Assume, *a contrario*, that the intersection does occur at $\alpha = \frac{p}{2\gamma}$ (see Figure B5), and thus when $< \frac{p}{2\gamma}$, everyone uses the free version in Period 1, those with $\alpha + \varepsilon$ switch to the paid version in the second period, and those with negative realization of μ churn. When $\frac{p}{2\gamma} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive realization of μ stay for a second period. Finally, when $> \varepsilon$, everyone uses the paid version for both periods.

Figure B5: Pay-Supported Scenario, special case 2



Here, $P_1 = 1 - \frac{p}{2\gamma}$; $P_2 = \frac{1}{2}\left(\frac{p}{2\gamma}\right)$; $A_1 = \frac{p}{2\gamma}$; and $A_2 = 0$

$$\Pi = p \left(1 - \frac{p}{2\gamma} + \frac{p}{4\gamma}\right) + k\gamma \frac{p}{2\gamma} - \gamma^2$$

Differentiating with respect to p and γ , equating to zero and collecting terms yield:

$$p_{opt} = (2 + k)\gamma; \gamma_{opt} = \sqrt[3]{\frac{p^2}{8}}$$

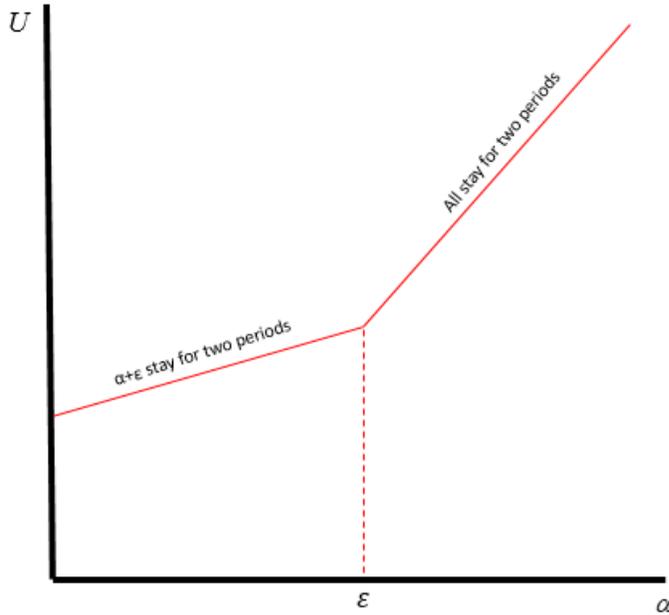
Validating that there are k values that satisfies the constraint on p for this case:

$$p_{opt} = (2 + k)\gamma < \gamma\varepsilon\delta, \text{ implying that } k < \varepsilon\delta - 2 < 0, \text{ a contradiction.}$$

Pay-Supported Scenario, special case 3 In this special case, $p < \varepsilon\gamma\delta/2$ and we address the situation in which $U_p(0) > U_a(0)$. As we show here, this case does not exist, that is, the optimal price does not conform to the requirement of this case.

Assume, *a contrario*, that $p < \varepsilon\gamma\delta/2$. As U_p 's slope is always greater than that of U_a , the two lines (paid and free) do not intersect, and everyone pays for the app in Period 1 – see Figure B6.

Figure B6: Ad-Supported Scenario, special case



In this case, when $\alpha < \varepsilon$, everyone pays in Period 1, and those with positive realization of μ stay for a second period. When $\alpha > \varepsilon$, everyone stays for both periods.

$$\text{Here, } P_1 = 1; P_{12} = P_2 = A_1 = A_2 = 0$$

$$\Pi = p - \gamma^2$$

This profit function is linear in price, and so the optimal solution is:

$$\gamma_{opt} = 0; p_{opt} = 1$$

However, $p < \frac{\varepsilon\gamma\delta}{2} < 1$ contradicts the optimal p .

Web Appendix C: Profits and uncertainty

In this appendix we show the conditions under which profits are increasing under uncertainty (ε).

Pay-Supported Scenario (main case)

The optimal profits are given by (see Web Appendix B):

$$\Pi_{opt} = \frac{([k + (2 + k)\delta]^2 - 2(k - 2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^2}{64\delta^2(2 + \delta)^2}$$
$$\partial\Pi_{opt}/\partial\varepsilon = \frac{2([k + (2 + k)\delta]^2 - 2(k - 2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^1}{64\delta^2(2 + \delta)^2} A$$

Where

$$A = -2(k - 2\delta)\delta + 2\delta^2\varepsilon = 2\delta(-k + 2\delta + \delta\varepsilon)$$

$\partial\Pi_{opt}/\partial\varepsilon$ is positive if

$$k < \delta(2 + \varepsilon)$$

Pay-Supported Scenario (special case 1)

The optimal profits are given by (see Web Appendix B):

$$\Pi_{opt} = \frac{(9k^2 + 6k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)^2}{256(2 + \delta)^2}$$
$$\partial\Pi_{opt}/\partial\varepsilon = \frac{2(9k^2 + 6k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)^1}{256(2 + \delta)^2} B$$

where

$$B = -6k\delta + 2(2 + \delta + \delta\varepsilon)^1\delta = 2\delta(-3k + 2 + \delta + \varepsilon\delta)$$

$\partial\Pi_{opt}/\partial\varepsilon$ is positive if

$$k < \frac{2 + \delta(1 + \varepsilon)}{3}$$

Numerical simulation of these two cases reveals that in all value parameters of the pay-supported scenario (main case), indeed $k < \delta(2 + \varepsilon)$, while in the special case, $k \leq (2 + \delta(1 + \varepsilon))/3$.