Retention and the Monetization of Apps

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Abstract

Though free apps dominate mobile markets, firms struggle to monetize such products and make profits, relying on revenues from two sources: paying consumers, and paying advertisers. Which source is more attractive and when? In this paper, we present a framework that shed light on these issues and on additional questions relating to the monetization of mobile apps. When formulating such a model one needs to pay attention to three central empirical regularities about the retention of mobile apps. a) The process is inherently dynamic; b) consumers have some prior knowledge of their fit with the app, yet they remain uncertain about their exact match-utility until they are using it; and c) the match-utility of using the app in the lower in the second period than in the first.

The resulting equilibrium in which user stickiness with the app plays a central role has a few insights on the monetization of apps. We find that in some cases it might be optimal for the firm to offer only one version of the app (i.e., free or paid). Specifically, when the drop in the match utility after the first period is low, it might be optimal to offer only the paid version. We discuss how this implication relates to hedonic products such as games, versus utilitarian products such as finance and business. Furthermore, we also demonstrate that in a dynamic setting, a firm can profit from offering a free version with ads even if advertisers are not paying for these ads.

Keywords: advertising; freemium; mobile apps; online strategy; pricing; retention; stickiness

1. Introduction

The number of mobile apps offered to consumers is rising rapidly, and by 2017, consumers could choose among more than two million apps in Google or Apple stores (Statista 2017a). Industry reports indicate that more than 90% of the mobile apps start as free, more than 90% of the profits of mobile apps come from products that began as free, and that the share of free products is only expected to grow in the coming years (AppBrain 2017; Taube 2013). Although these products are "free", market size is notable, with close to $90 billion in 2016 and it is expected to be doubled by 2020 (Takahashi 2017). In 2016, the worldwide revenue from mobile games, the largest app category, surpassed income from either video console games or PC games (Needleman 2016, Statista 2017b).

Monetizing apps comes in two versions: a) selling advertising space within a free version of the product, and b) selling a paid version, labeled *freemium* or *in-app purchase* strategy that can come as an ad-free version or offering as additional features to a subset of paying users (Needleman 2016)[1]. Offering an ad-free version as a premium alternative represents a salient, free product business model that is widely used by suppliers such as Spotify and other music services, as well as virtually all mobile games. Which approach is more effective? Is it better to offer only one version (free or paid) or both? And how do the answers to these questions depend on the circumstances? These questions are not only of academic interest. Practitioners are also being challenged by them as free product creators struggle to remain profitable (Marks 2012; Zhang 2015). Indeed, the business literature describes the question of monetization model choice and the level of co-existence of the main tools as "plaguing the industry" (Natanson 2016).

[1] There are other ways to monetize digital products, such as selling user information to a third party, or the creation of a user community that can be later sold to a third party. These are beyond the scope of our paper.

In this paper, we present a framework that shed light on these issues and on additional questions relating to the monetization of mobile apps. When formulating such a model one needs to pay attention to three central empirical regularities about the retention of mobile apps: a) The process is inherently dynamic; b) consumers have some prior knowledge of their fit with the app, yet they remain uncertain about their exact match-utility until they are using it; and c) the match-utility of using the app in the lower in the second period than in the first.

a) The process is inherently dynamic not only because retention is a temporal phenomenon, but mainly because consumers are considering adopting the app in its two versions – free and paid – over time. Indeed, many are "sampling" the free version while considering the paid version (Terwitte 2015; Cleary 2016). In other words, consumers can download a free version that includes ads, or a paid version that excludes ads, or sample the free version first, before deciding whether to buy the paid or free version in the next period.

Retention in these markets is unique. Specifically, app retention rates are much lower than the observed retention rates in classical products and services, with reports suggesting for that across all categories, more than 80% of all app users may churn within 90 days (Perro 2016; Marketing Charts 2017). Our model account for the low retention rate in two ways:

b) We account for consumers' heterogeneity in their match with the app and assume that they are initially uncertain about this match. Specifically, while consumers have some prior knowledge of their fit with the app, they remain uncertain about their exact match-utility until they are using it.

c) We allow the match-utility to drop after the first period, following user experience with the app. In addition to the match-utility factor, it is noted that many apps, in particular games, are built in a way that most utility is experienced early on. Thus and both factors can lead to a drop in utility. Consistent with industry practiced in such online environments that is looking at retention rather than churn, we label the tendency to retain a utility from the product over time as “stickiness” (Yaloz 2015; Shaul 2016). Both the uncertainty and the drop in the utility over time can explain the low retention rate.

While app retention may be low, app categories vary considerably in their retention rates. The app research firm Flurry, for example, found that the high retention categories are health and fitness, weather, news & magazine, and business and finance, while categories such as action games, sport games, family, entertainment, and puzzles have a much lower retention probability (Klotzbach 2016, see also Haslam 2016; App Annie 2016). The picture that emerges seems to be consistent with previous research that highlighted the need to distinguish between two types of products (i) those who are more hedonic, enjoyable, fun, and leisure and (ii) those that are more utilitarian, work, and functional (Babin, Darden, and Griffin 1994; Van der Heijden 2004; Schulze, Schöler, and Skiera 2014). The value from products in the first hedonic category stems from sensations derived from the experience of using, while the products in the second utilitarian category provide value manly through the functions they perform. What is notable about mobile apps is that an app typically supports only one class of activities: For instance, banking and productivity apps support utilitarian/goal directed outcome whereas games support hedonic/experiential outcomes (Furner, Racherla and Babb 2016).

Thus, the main unique characteristics of our model are a) temporal aspect in a form of a two-period dynamic model, b) uncertainty about the match with the app, c) the match-utility in the second period is lower than that of the first. The firm offers two versions of the app: ad-supported free version and a paid one, and the consumer decides at each period which version to adopt, if at all, while the firm sets the levels of price and advertising, given the consumers' heterogeneity and decision rules.

One of the main insights from the results relates to the dilemma faced by some firms over whether to offer two versions of the app (free and paid), or only one. Of course, when operating costs are ignored, offering two versions is more profitable. However, our analysis can identify cases in which the additional profit is minimal. In such cases, taking costs into account, offering just one version might be more appealing. Indeed, we find that when the retention level is high, the firm might benefit by focusing on the paid version and avoiding the free version.

This result is quite central with respect to the monetization of apps. It implies that when dynamics are taken into account, the attraction of the two versions of apps is quite different across categories. When it comes to apps with low stickiness (as discussed above, e.g., hedonic products such as games) the free version is more attractive and marketers might want to avoid offering the paid version, but when it comes to apps with higher stickiness (e.g., utilitarian products such as finance and business) the opposite holds – the paid version is more attractive and it might make sense to avoid offering the free version.

Our analysis also provides the mechanism behind the monetization decisions. Specifically, we show that as stickiness increases, so do prices and ad intensity, and firms earn more of their revenues from paying customers than they do from advertising. Specifically, we find that as stickiness increases, the firm's earnings from the paid version increase monotonically. However, its earnings from advertising first increase, then drop. Accordingly, the share of profit coming from the paid version increases with the stickiness level throughout.

Furthermore, we demonstrate that in a dynamic setting, a firm can profit from offering a free version with ads even if advertisers are not paying for these ads. In other words, the firm benefits from offering a version of the app that includes ads even if this version is free for consumers, and the advertisers are not paying for the ads. The logic behind this result is the following: In a dynamic setting, the free version is a kind of "sample" that enables consumers to learn their fit with the app. The firm would rather add ads to this version (even if they are not paid for) in order to keep the consumers who have high valuation of the app away from the "sample" and push them toward the pay version.

2. Related literature

Our work is particularly relevant to increasing research efforts aiming at understanding optimal firm behavior in free digital markets. Research in this avenue has largely focused on the choice between content and advertising in the context of media markets, where the basic tradeoff is that moving from an advertising-only revenue model to charging for content may reduce viewership and thus hurt advertising revenues (Lambrecht et al. 2014). Empirical results regarding the consequences of such a choice vary (Chiou and Tucker 2013; Pauwels and Weiss 2008). Profitability in such contexts may depend on factors such as the type of promotions used (Pauwels and Weiss 2008), temporal changes in demand (Lambrecht and Misra 2016), and users' ability to bring new business through referrals (Lee et al. 2015). The existence of free apps may slow the speed of a paid alternatives growth, yet this may change through the product life cycle (Arora, Hofstede and Mahajan 2017).

Analytically, noticeable effort has centered on the questions of profit from content vs. advertising in two-sided media markets such as newspapers (Halbheer et al. 2014). The tradeoff between paid content and advertising may depend on competitive intensity (Godes et al. 2009), consumer heterogeneity in willingness to pay (Prasad et al. 2003), or the extent to which consumers dislike advertisements (Tåg 2009). Looking at the use of a free product as a sampling mechanism, recent work has suggested that advertising effectiveness, coupled with consumers' expectations regarding quality, can determine which revenue source firms should focus on in attempting to enhance profitability (Halbheer et al. 2014). Moreover, it has been shown that the appeal of the free option is influenced by the effectiveness of word of mouth (Boudreau et al. 2017, Niculescu and Wu 2014), and that referral considerations affect efforts to move a user from free to paying (Lee et al. 2015). These research efforts, however, do not consider the role of customer retention or long-term customer value in general in the market equilibrium that emerges.

As the consumers in our setting learn about the product, our work addresses also the demand-side literature on consumers' learning (Ackerberg 2003; Erdem et al. 1999; Erdem and Keane 1996; Iyengar et al. 2007). Our learning structure is not on specific attributes of the product, but rather on the idiosyncratic utility that the consumer can expect from experiencing the product, similar in spirit to Ackerberg (2003).

Our work is also relevant to the literature branch that addresses customer relationships, and in particular the role of customer retention (Ascarza et al 2017; Gupta et al. 2004; Libai et al. 2009; Schweidel et al. 2008), and how optimal retention efforts may differ based on market characteristics (Musalem and Joshi 2009; Shin and Sudhir 2012, Subramanian et al. 2013). In the context of online behavior, researchers have considered the antecedents of users' tendencies to stick with a website (Lin 2007; Li et al. 2006) or specifically with an app (Hsu and Lin 2016; Kim et al. 2015), as well as free digital product adoption's effect on retention rates (Datta et al. 2015). Yet the dynamics of how retention affects monetization in free digital markets have not been examined.

3. The Model

We consider a digital product primarily designed for mobile devices, for which such products are largely distributed as applications on either Google's Play Store or Apple's iTunes. In the rest of the paper, for simplicity, we refer to our product or service as an "app". The term "free" app is somewhat of a misnomer, as the consumer pays for the app in one of two ways: either she pays a one-time price to download the app or special features in the app, or else she pays with her time by viewing advertising. However, for convenience, we will refer to the ad-supported version as "free". A user downloading the app can choose instead of downloading the free version to purchase a version in which no advertisements are displayed (Tåg 2009).

The three main unique characteristics of the model are (1) time: a two-period model, (2) uncertainty about the match with the app, and (3) depreciation: the match-utility in the second period is lower than in the first. As discussed in the introduction, these features are motivated by empirical regularities. In each period, the user can choose whether to use the app or not, and which version of the app to use (free, or paid). Users' decisions in the second period are affected by their choices in the first period in two ways: Firstly, if the consumer chooses to use the app in the first period, his exact utility match with the product is revealed, and he does not face any uncertainty in the second period. Secondly, if he paid for the app in the first period, he can use the ad-free version in the second period with no additional fees. The firm sets the levels of price and advertising, given the consumers' heterogeneity and decision rules.

3.1 Consumers' Utility

We begin by describing the factors affecting consumers' utility such as uncertainty, advertising, and costs; then proceed with the formal utility maximization problem; and conclude with the resultant demand and firm maximization.

Uncertainty. Users are heterogeneous in their match with the app, and they face uncertainty with respect to the exact fit of the app to their preferences. In other words, although consumers have some prior knowledge, they remain uncertain about the idiosyncratic utility the consumer can expect from using the app. Once the consumer uses it, all uncertainty is resolved. The framework we use is a two-stage game: Specifically, we assume that while consumers experience depreciation in their utility from using the app, it is only partial. In other words, consumers might enjoy using the app beyond the first period. The degree of utility that might be still experienced in the second period, termed here the "stickiness factor", plays a key role in this framework.

Product fit uncertainty is a major impediment to online markets in general (Hong and Pavlou 2014). Consumers' uncertainty about apps is amplified since the majority of apps have been developed by small businesses with unknown reputation (Arora, Hofstede and Mahajan 2017). Market reports suggest that given the absence of monetary cost to download apps, consumer learning often occurs post adoption rather than pre adoption, as in classical consumer markets, so that users are more comfortable than ever before downloading an app and giving it a try (Klotzbach 2016). Yet consumers are not entirely in the dark with respect to their match with any specific app. For example, some consumers know a-priori that they prefer "health and fitness apps" over "food and drink apps", or "social games" over "brain games". However, since each app is unique, even after reading an app's description, consumers are uncertain about the fit between their preferences and the app.

To capture this type of uncertainty, we let nature determine the match parameter in two stages: First, each individual is assigned with an α (below we assume that α is distributed uniformly in the population with support 0,1). Second, nature either adds or subtracts ε to α (with probability of 1/2 to each option). This process is common knowledge, and each individual knows her own α. However, the consumer does not know whether nature added or subtracted ε in her case. Accordingly, from her perspective, the match parameter is a random variable, denoted by $\tilde{\alpha}$ with the following distribution:

$$(1) \quad \tilde{\alpha} = \begin{cases} \alpha - \epsilon & \text{with probability } 1/2 \\ \alpha + \epsilon & \text{with probability } 1/2 \end{cases}$$

where $0 \leq \alpha \leq 1,\ 0 \leq \varepsilon \leq 1$.

Thus, the parameter ε represents the degree of uncertainty the individual is facing with respect to her true base utility[2]. This uncertainty is completely resolved when the consumer is using the app, irrespective of whether she uses its free or paid version[3]. The source of uncertainty is that the potential downloader of a mobile game does not know the level of complexity, fun, and engagement that she, personally, will enjoy. Once she uses the app and realizes her engagement, or lack thereof, this uncertainty is resolved.

The intensity of advertising. The free version of the app is bundled with advertising. Exposure to advertising is annoying to the consumer, and creates discomfort and irritation, and thus the intensity of advertising, γ, depreciates the consumer's utility (Wilbur 2008). Advertising creates inconvenience for the consumer in that it effectively reduces the amount of content that the consumer can enjoy from the app, in addition to inflicting other costs, such as distraction and opportunity cost of time (Tåg 2009; Vratonjic et al. 2013). Indeed, previous work has empirically demonstrated that consumer annoyance associated with online advertising has a negative effect on consumer utility and resultant retention (Goldstein et al. 2014). While firms tend to have

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[2] Notice that for individuals with low α, the experience utility might be negative, which is quite possible in the real world.

[3] This type of learning is consistent with the experiential component of a consumption good described in Ackerberg (2003).

limited control over ad content, which is typically outsourced to ad exchanges that match ads to apps (e.g., Google's Admob, Flurry, Apple's iAd), the firm can select the level of advertising intensity. Since the base utility is bounded, we assume that $0 \leq \gamma \leq 1$.

Paid version cost. Consumers may choose to eliminate the annoyance of ads, for a price. The firm offers a paid version of the app for a price $p \geq 0$. If consumers choose to pay this price, the ads are removed and the consumers benefit from the full utility of the app. Thus price is a decision variable of the firm.

App holding costs. Although consumers are exposed to a large number of free apps, they download only a small subset, and use an even smaller subset. It is costly for a consumer to maintain another app in her current collection of apps, because of system limitations (such as memory capacity and screen capacity), cognitive limitations (such as depletion), constant notifications by installed apps, cost of battery use, privacy issues, effect on the smartphone speed, and others (Perez 2014; Smith 2016). For example, it is reported that users tend to delete apps in particular because they ask them for information or send annoying notifications, because apps collect personal data, and because they of phone memory usage (Upbin 2013). It is no wonder that due to these costs users are in fact encouraged by experts to delete many of their apps (Mossberg 2016), and that app users have indeed become "quick to uninstall" (BI Intelligence 2016). We consider these costs by having consumer utility decrease by the per-period cost of holding and using the app, *c.* The holding costs are thus subtracted from the base utility at each period. Since the base utility is bounded, we assume that $0 \leq c \leq 1$.

The stickiness factor. For various reasons the match utility experienced by the individual might be lower in the second period than in the first. For example, hype in the early days of an app is likely to wilt over time. Consider the widely successful gaming app "Candy Crush": some of the utility experienced by users in the months preceding the launch was related to the social aspects of playing the game (i.e., receiving bonuses in the game from your friends). After a while, these social aspects almost vanished and the experience utility decreased.

In the industry instead of talking about the depreciation, the focus is on the level of utility that is retained in the future. This level is referred to as “stickiness”. To be consistent with the language used by practitioners we adopt this term. Accordingly, we define the stickiness of an app as consumers’ tendency to continue and retain its value the future (Lin 2007), or to be precise, the fraction of the match-utility, $\tilde{\alpha}$, that might still be experienced in the second period. This fraction is denoted by δ, where $0 \leq \delta \leq 1$, and labeled the “stickiness factor”. The higher the value of δ, the higher the consumer’s utility in the second period. Thus, stickiness can affect the decision whether to stay for the second period. Furthermore, with forward-looking consumers (as in our setting), this parameter also affects first-period choices.

Our model considers stickiness to be a characteristic of the app. Various app-based factors may influence an app’s stickiness level, including the quality of the app and its ability to create consumer commitment and trust (Li et al. 2006). Yet market information suggests that stickiness may be highly dependent on the specific type of app (Klotzbach 2016). Some categories and subcategories are clearly stickier than others (e.g., the discussion above on more fun vs. more utilitarian apps), which may be a factor in the app’s long-range utility, variety-seeking and boredom on the part of the target market, and the quality of the alternatives. Firms that target a certain category or subcategory may thus be able to obtain an assessment of the range of stickiness they can expect to see, and they may use information from similar freemium apps to assess the extent to which consumers are expected to continue using a given app (Hadiji et al. 2014).

Setting the individual utility. The formal setting is as follows: In each period, denoted by $t = 1,2$, the individual faces three alternatives, denoted by j: (1) not to use the app ($j = 0$); (2) to use the free version of the app ($j = a$); or (3) to use its paid version ($j = p$). In each period, the individual makes a choice, denoted by C_t from among these 3 options. In other words, $C_t = j$ when the individual chooses alternative j at time t. The utility of the individual from these options $u_{j,t}$ is:

$$(2) \qquad \begin{cases} u_{0,t} = 0 \\ u_{a,t} = \tilde{\alpha}(1-\gamma)\delta_t - c \\ u_{p,t} = \tilde{\alpha}\delta_t - p[1 - I\{t = 2 \cap C_1 = p\}] - c \end{cases}$$

where $\tilde{\alpha}$ is the match-utility, γ is the intensity of advertising, δ_t is the app stickiness parameter (equals 1 in the first period and $\delta < 1$ in the second period), c is the cost associated with using the app, and p is the price of the paid version. The indicator function I captures the fact that the price is paid only once. Next we calculate the individual's expected value – i.e., the sum of her expected utility in both periods.

3.2 Consumer's Expected Utility

When forming her expectations, the individual takes into account the consequences of her choices in the first period on her state in the second. His actions in the first period have two types of effects on the states of the second: First, using the app resolves the uncertainty of $\tilde{\alpha}$, and the individual knows whether his base utility is $\alpha - \varepsilon$, or $\alpha + \varepsilon$. This happens irrespective of whether he uses the app's free or paid version. Second, paying for the app in the first period makes it free for use (without ads) in the second. Thus, to calculate the user's lifetime expected utility, we first need to describe her second-period utilities and choices (conditioned on his first-period choice).

We begin by considering the case in which the user chooses the paid version in the first period. In such a case, she has two options in the second period: (a) continue to use the paid version or (b) not use the app at all. The option of using the free version is not appealing to her, as it contains ads, which decrease her utility, and she has already paid for the ad-free version.

Her utility would be either $(\alpha - \varepsilon)\delta - c$, or $(\alpha + \varepsilon)\delta - c$ if she continues using the paid version, depending on the realization of $\tilde{\alpha}$, which will be known to her at that point, or else zero if she does not continue using the app. Therefore, the choice between using the app or not, which maximizes her utility, can be characterized as follows:

(i) if $\alpha < \frac{c}{\delta} - \varepsilon$, she will not use the app in the second period,

(ii) if $\frac{c}{\delta} - \varepsilon < \alpha < \frac{c}{\delta} + \varepsilon$, she will use it only if her realized base utility is high, and

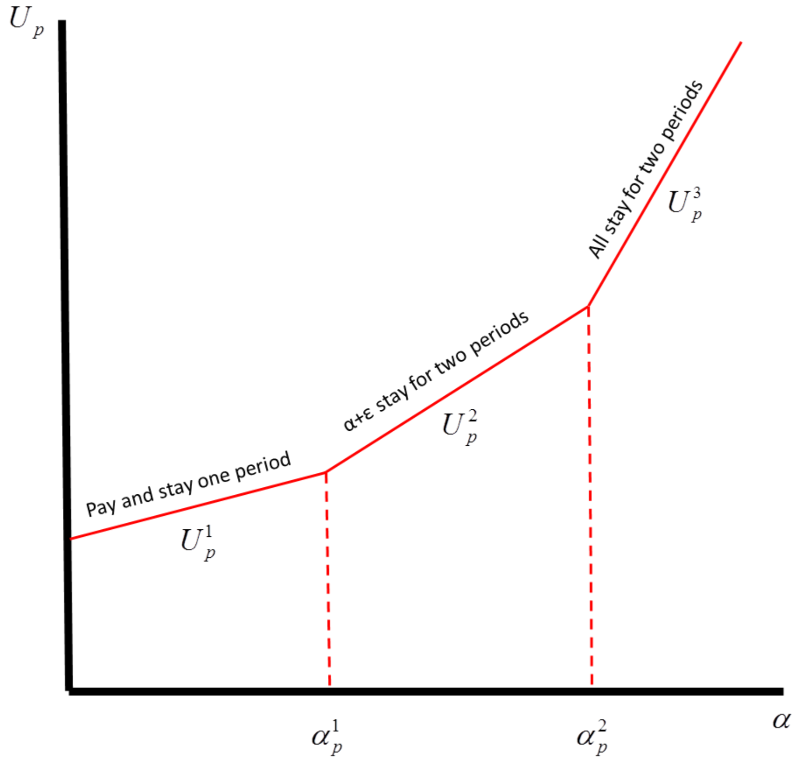
(iii) if $\frac{c}{\delta} + \varepsilon < \alpha$, she will use the app in the second period irrespective of the realization of her base utility.

We denote these break points, as they will become handy later, as follows: $\alpha_p^1 = \frac{c}{\delta} - \varepsilon$ and $\alpha_p^2 = \frac{c}{\delta} + \varepsilon$. Accordingly, her lifetime expected utility if she pays for the app in the first period is:

(3) $$U_p(\alpha) = \alpha - c - p + \frac{1}{2}(\alpha\delta - c)I\{\alpha_p^1 < \alpha < \alpha_p^2\} + (\alpha\delta - c)I\{\alpha_p^2 < \alpha\}$$

Figure 1 depicts the lifetime utility for each level of the base utility α.

Figure 1: Expected lifetime utility when choosing the paid option in the first period



Next, we consider her lifetime expected utility if she chooses the free version in the first period. In such a case, she has three options in the second period: a) stop using the app, in which case her utility in the second period is zero; b) keep on using the free version, gaining second-period utility of $(\alpha-\varepsilon)(1-\gamma)\delta-c$ or $(\alpha+\varepsilon)(1-\gamma)\delta-c$ depending on the realization of $\tilde{\alpha}$; or c) pay for the premium version with utility of $(\alpha+\varepsilon)\delta-c-p$[4].

While it is clear that $(\alpha+\varepsilon)(1-\gamma)\delta-c>(\alpha-\varepsilon)(1-\gamma)\delta-c$, it is unknown whether $(\alpha+\varepsilon)\delta-c-p$ is greater than both these expressions, greater only than the one with the low base utility, or smaller than both. As a result, the structure of the lifetime expected utility (i.e., the break points between the various parts of the utility) has three versions. While in

[4] It is immediate to show that if the realization of her base utility is low, s/he will not pay for the premium version in the second period, and thus in (c) above we consider high base utility only.

Appendix A we present and describe all three structures, here we focus on one of them: the one in which the price of the paid version is relatively high. In such a case:

(i) if $\alpha < \frac{c}{\delta(1-\gamma)} - \varepsilon$, the consumer will not use the app in the second period,

(ii) if $\frac{c}{\delta(1-\gamma)} - \varepsilon < \alpha < \frac{c}{\delta(1-\gamma)} + \varepsilon$, she will use the free version of the app only if her realized base utility is high,

(iii) if $\frac{c}{\delta(1-\gamma)} + \varepsilon < \alpha < \frac{p}{\delta\gamma} - \varepsilon$, she will use the free version of the app in the second period irrespective of the realization of her base utility, and

(iv) if $\alpha < \frac{p}{\delta\gamma} - \varepsilon$, she will use the free version when her base utility is low, and buy the paid version if her base utility is high.

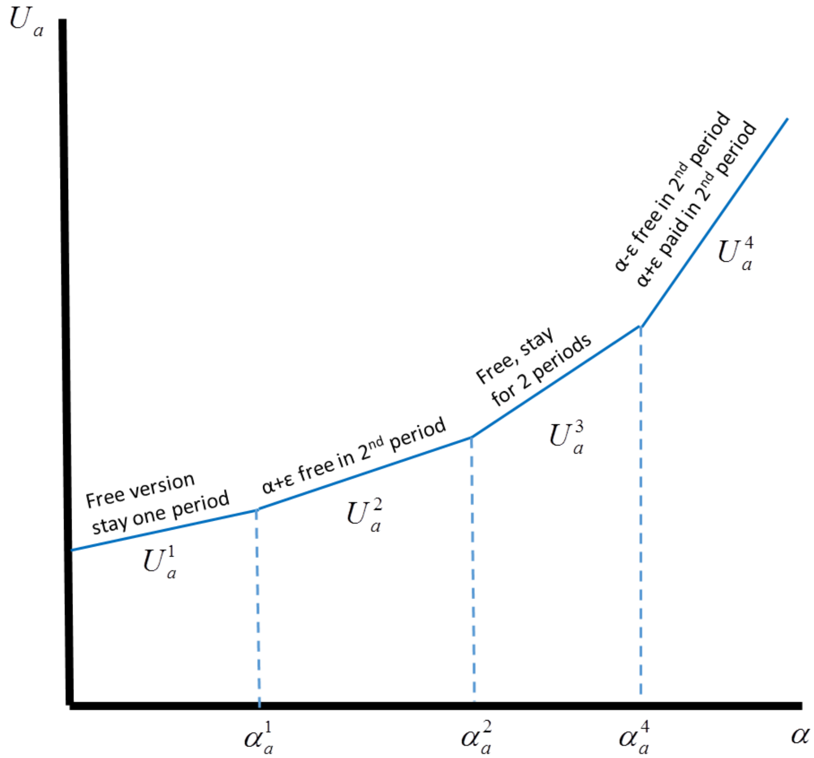
We denote these break points as the following: $\alpha_a^1 = \frac{c}{\delta(1-\gamma)} - \varepsilon$, $\alpha_a^2 = \frac{c}{\delta(1-\gamma)} + \varepsilon$, and $\alpha_a^4 = \frac{p}{\delta\gamma} - \varepsilon$ (α_a^3 is defined in Appendix A, where we present the various scenarios that depend on the relative values of α_a^j). Accordingly, in this case, her lifetime expected utility if she selects the free version in the first period is:

$$(4) \qquad U_a(\alpha) = \alpha(1-\gamma) - c + \frac{1}{2}\big((\alpha - \varepsilon)(1-\gamma)\delta - c\big)I\{\alpha_a^2 < \alpha\}$$

$$+ \frac{1}{2}\big((\alpha + \varepsilon)(1-\gamma)\delta - c\big)I\{\, \alpha_a^1 < \alpha < \alpha_a^4\} + \frac{1}{2}\big((\alpha + \varepsilon)\delta - c - p\big)I\{\alpha_a^4 < \alpha\}$$

Figure 2 depicts the utility level for each level of the base utility α.

Figure 2: Expected lifetime utility when choosing the free option in the first period



Finally, consider the case in which the consumer does not use the app in the first period. It is easy to show that in such a case, her lifetime expected utility is zero.[5]

3.3 Consumer's Choice and Demand

The individual's decision in the first period is quite simple – given her base utility $\tilde{\alpha}$, she selects the option that yields the highest utility. In other words,

(5) $$C_t = \arg\max_{j\epsilon\{0,a,p\}} U_j\,(\tilde{\alpha})$$

So far we have focused on the decision of one individual with a specific α and ε, but the firm faces a large number of individuals with heterogeneous levels of base utility. To account for

[5] If the consumer does not use the app in the first period, her expected second-period utilities from using the free and paid versions are $\alpha(1-\gamma)\delta - c$ and $\alpha\delta - c - p$ respectively. In both cases, these values are lower than her expected utility in the first period (and definitely lower than her associated lifetime expected utility). As this individual finds it optimal to not use the app in the first period, her expected utility from using the app in the second period is negative. Thus, s/he would choose to continue abstaining from using the app.

that, we assume, as mentioned above, that α comes from a uniform distribution at interval $[0,1]$.[6] Next, we wish to calculate the resulting demand. Specifically, we aim to find the share of individuals in each option and in each period, denoted by π_{jt}. In other words, we seek to identify the subset of utilities α who find it optimal to choose option *j* ($j \epsilon \{0, a, p\}$), in each period *t*. The following two Lemmas and the proceeding Proposition are useful in the solution of the first-period shares. The proofs are in Appendices B and C for Lemmas 1 and 2 respectively.

Lemma 1: $U_a(0)$ is always greater than $U_p(0)$.

Lemma 2: $\frac{\partial U_a(\alpha)}{\partial \alpha} < \frac{\partial U_p(\alpha)}{\partial \alpha}$

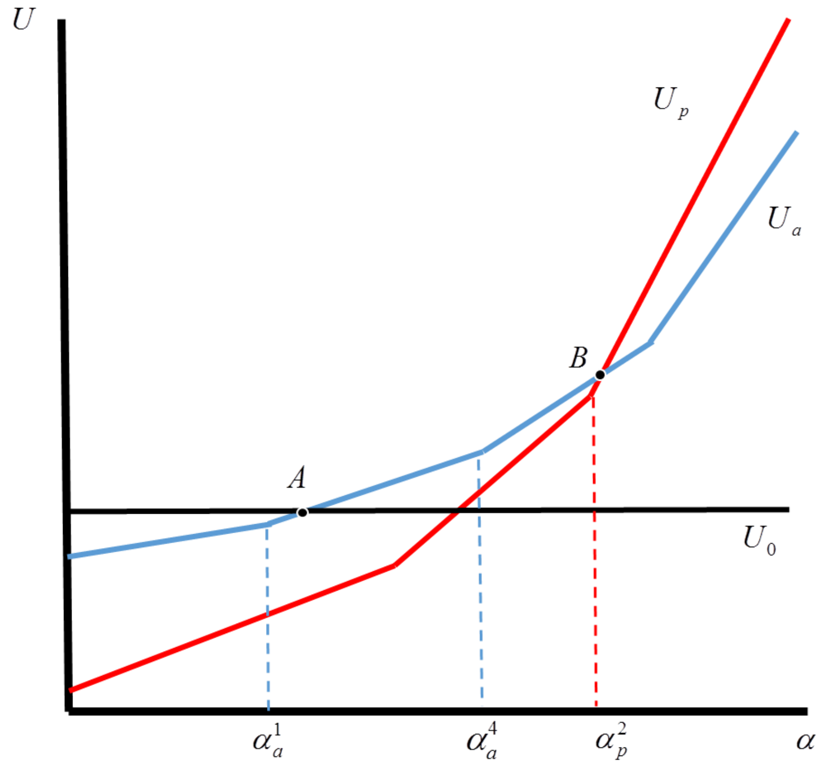
These two Lemmas lead to the following Proposition:

Proposition: There exists at most one intersection between U_a andU_p.

Following the Lemmas and Proposition, we obtain that (a) if there is no intersection, the paid version is never selected, and (b) if there is intersection, the free version yields higher utility for those with low match-utility, while the paid version yields higher utility for those with high match-utility. Of course, the decision to use the app depends not only on the relationship between U_a and U_p, but also upon whether they yield a positive expected utility. Otherwise, it is optimal for the individuals to refrain from using the app at all. To get a sense of these relationships, consider Figure 3.

[6] This means that the support of the actual base utility, i.e., $\tilde{\alpha}$ is $[-\varepsilon, 1 + \varepsilon]$. In other words, it is possible that the individual will learn that using the app yields negative utility, i.e., $\tilde{\alpha} - \varepsilon < 0$, as can occur in many real-world cases.

Figure 3: Utility of free app (U_a), paid app (U_p), and nonparticipation (U_0)



In this case, consumers with low levels of match-utility ($\alpha \leq A$) do not download the app; consumers with intermediate levels of utility ($A \leq \alpha \leq B$) use the free version in the first period, and consumers with high match-utility ($B \leq \alpha$) use the paid app. It is immediate to see that if the line that represents U_0 had been higher, the market share of the free version could have been zero in the first period. Specifically, if instead of crossing U_a at A, it would have crossed it at B, the market would be split between those who do not use the app and those who use the paid version.

It is easy to show how in this model, the following cases are possible: (1) no one uses the app; (2) those with low α do not use the app, and the rest buy the ad-free version; (3) those with low α do not use the app, intermediate α use the free version, and those with high α use the paid version; (4) the market is split between those who use the free version (low α) and those who use the paid version (high α); (5) the market is split between non-users (low α) and users of the free version (high α); and (6) everyone is using the free version.

The Lemmas and Proposition enable us to calculate, for any value of $\{\gamma, \delta, \varepsilon, p, c\}$, the market shares of the first period, i.e., π_{j1} $(j\epsilon\{0, a, p\})$. The market shares of the second period are more straightforward. They are given by the following:

$$(6) \qquad \begin{cases} \pi_{02} = \pi_{01} + \pi_{a1}\alpha_1 + \pi_{p1}\alpha_2 \\ \pi_{a2} = \pi_{a1}\alpha_3 \\ \pi_{p2} = \pi_{a1}\alpha_4 + \pi_{p1}\alpha_5 \end{cases}$$

(i) α_1 is the share of individuals whose $\tilde{\alpha}$, given this realization, satisfies: $\tilde{\alpha} < \frac{c}{\delta(1-\gamma)}$

(ii) α_2 is the share of individuals whose $\tilde{\alpha}$ satisfies the condition $\tilde{\alpha} < \frac{c}{\delta}$

(iii) α_3 is the share of individuals whose $\tilde{\alpha}$ satisfies $\frac{p}{\delta\gamma} > \tilde{\alpha} > \frac{c}{\delta(1-\gamma)}$

(note that it is possible that $\frac{p}{\delta\gamma} < \frac{c}{\delta(1-\gamma)}$, and in such a case, $\alpha_3 = 0$)

(iv) α_4 is the share of individuals whose $\tilde{\alpha}$ satisfies the condition $\tilde{\alpha} > \frac{p}{\delta\gamma}$

(v) α_5 is the share of individuals whose $\tilde{\alpha}$ satisfies the condition $\tilde{\alpha} > \frac{c}{\delta}$

Now that we have the share of individuals in each option in each period, i.e., π_{jt}, we can solve the profit function of the firm and find the optimal levels of price and ad intensity. This is done in the following subsection.

3.4 The Firm's Objective Function

The firm chooses the price and level of advertising intensity so as to maximize its profits from the two periods. We normalize the overall market potential to 1. Note that in our setting, a consumer with low match-utility α will not download the app, yet the market potential includes these consumers as well. Therefore, the choices of advertising intensity and pricing affect not only the proportion of paying buyers to free downloaders, but primary demand as well. In addition, since the price is expressed in monetary terms, e.g., \$ or €, we need a parameter, denoted by k, to convert advertising intensity γ to the same monetary term. The advertising contribution parameter k thus represents the monetary payment received by the firm from advertisers per unit of ad intensity. The objective function of the firm is thus:[7]

(7) $$\Pi(p, \gamma; \varepsilon, \delta, c, k) = p(\pi_{p1} + \pi_{p2}) + k\gamma(\pi_{a1} + \pi_{a2})$$

The profit is a function of (i) the two decision variables of the firm – price, and advertising intensity, and (ii) the model's parameters – ε, δ, c and k. Following the common practices and contractual setting in the real app markets, we assume that price and advertising intensity do not change in the second period. Specifically, in the real app market, the price and intensity are embedded in the software and are rarely changed. For example, the business press reports that changing the in-app purchase price of an app is a complicated, resource-intensive process that can intimidate developers (Ogg 2013). The complexity of price and advertising changes is driven also by apps' relatively short life cycles, and by the fact that advertising contracts are established in advance with third-party entities.

4. Equilibrium Computations

Given the analysis in the previous subsection, it is immediate to solve the pair of price and advertising intensity that maximizes the firm's profit (p^*, γ^*) for any set of parameters. While we have the exact solution for any set of parameters, there is no *closed-form solution* for p and γ as a function of ε, δ, c and k – i.e., there is no closed-form solution for the functions

[7] Consumers who buy the premium version might or might not stay for the second period, yet attrition of these consumers does not affect the profit function, as the paying consumers in our model do not generate any revenues in a later period. Thus, consumers pay for the app in the second period only if they switched from the free version to the paid one in that period.

$p^*(\varepsilon, \delta, c, k)$ and $\gamma^*(\varepsilon, \delta, c, k)$. The lack of such a solution is due directly to the existence of break points in the lifetime expected utility function, U_p and U_a. That said, one of the main interests of this study is to understand the effects of stickiness and uncertainty on both price and advertising intensity, and also on share of profits from the paid vs. free versions of the app. In other words, we seek to be able to answer questions such as whether advertising intensity increases or decreases with stickiness.

Despite the absence of a closed-form solution for the functions $p^*(\varepsilon, \delta, c, k)$ and $\gamma^*(\varepsilon, \delta, c, k)$, we can solve these optimal values for any set of parameters, and so study these relationships numerically. To conduct this analysis, we solved the market equilibrium for 11 levels (from zero to one) on each of the four parameters, using a full factorial design resulting in 11^4 = 14,641 cases. When we examine these numerical relationships, we find that in most cases, price and advertising generally increase stickiness (and uncertainty). However, this monotonicity breaks down for some specific values of the model's parameters. For example, for ε = 0.8, c = 0.1, and k = 0.5. We find (see Figure 4a) a general monotonic behavior of price and advertising as stickiness increases. However, if we consider a much higher cost, c = 0.8, i.e., ε = 0.8, c = 0.8, k = 0.5, we find (see Figure 4b) a non-monotonic behavior.

To get a sense of the intuition behind these results, we need to gain an understanding of the driving forces in this model. In all cases, individuals with low α are more likely than others to refrain from using the product, those with high α are more likely to choose the paid version, and those in the middle are more likely to choose the free version.

Figure 4a: Effect of stickiness on price and advertising ($\varepsilon = 0.8$, $c = 0.1$, $k = 0.5$)

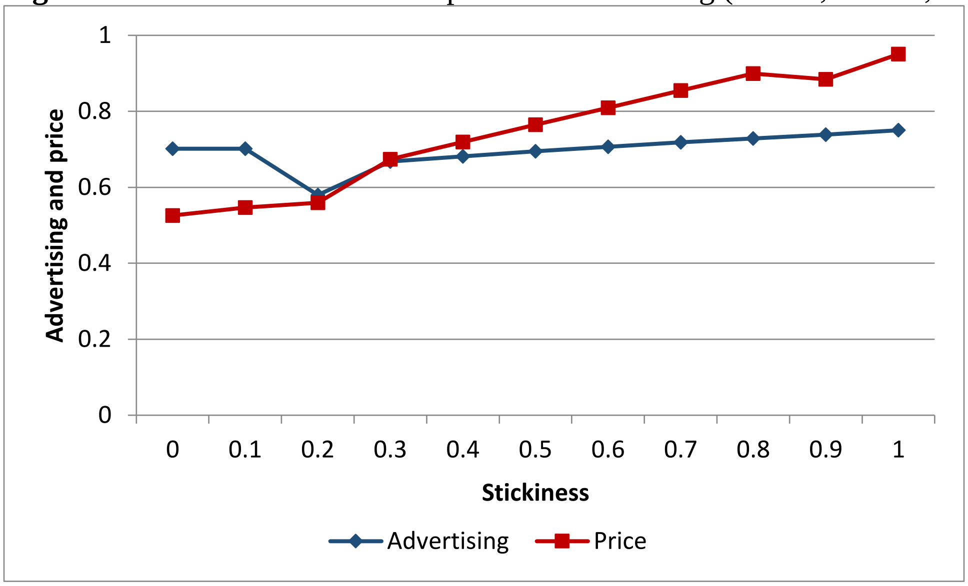
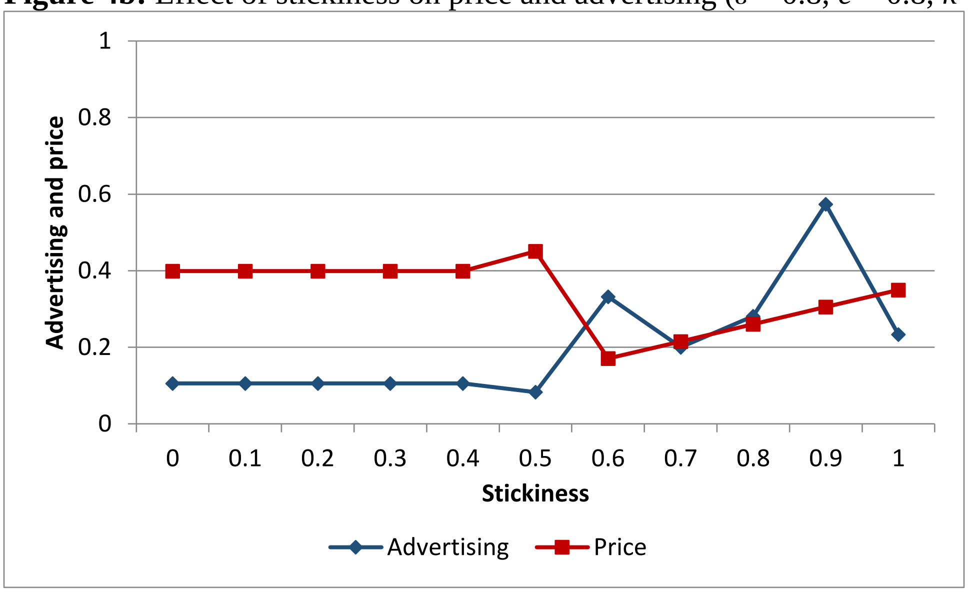


Figure 4b: Effect of stickiness on price and advertising ($\varepsilon = 0.8$, $c = 0.8$, $k = 0.5$)



Let's start by considering the case of a one-period model – i.e., the case of $\delta = 0$. In such a case, the effects of the decision variables of the firm – price, and ad intensity – are simple: As price rises, the share of non-users does not change, but the split of users between the paid and free versions skews toward free. The higher the ad intensity, the share of non-users and those who use the paid version increases. When choosing the optimal price and ad intensity, the firm balances these effects. When there are two periods, i.e., the case of $\delta > 0$, there are two major changes: From the consumers' perspective, the product is more appealing, as they can enjoy it also in the second period. From the firm's perspective, there is another source of income: sales of ads in the second period. These new forces might have opposing effects on the choices of price and ad intensity.

Consider the choice of price. When stickiness increases, as the product is becoming more appealing and consumers' willingness to pay increases, the firm tends to charge higher prices. However, the paid version is now becoming more appealing also to consumers who did not consider it in the past and downloaded the free version. To capture these potential consumers, the firm tends to lower the price of the paid version. Of course, the final decision depends on the relative sizes of these two market shares, and also on the initial price and ad intensity. A similar pattern exists regarding the choice of ad intensity: On the one hand, given that the product is more appealing, the firm can afford to adopt a higher ad intensity strategy. On the other hand, the free version can now become appealing to consumers who earlier did not consider using it at all; and to capture them, the firm might opt to lower the ad intensity.

With these patterns in mind, let's return to Figure 4 and focus on the effect of stickiness on price. As we will illustrate shortly, price generally increases with stickiness level, and thus Figure 4a represents its effect better than does 4b. That said, understanding the drop in price that we see in the middle of Figure 4b can shed some light on the dynamics in the model. In Figure 4b, the cost of using the app is very high, $c = 0.8$, and thus most of the individuals do not use the product. This is especially the case when δ is low (and thus the individual cannot really enjoy the product in the second period). As δ increases, some of these individuals are beginning to find the app appealing, and are drawn to the free version. When δ is around 0.5, the mass of consumers who use the free version builds up sufficiently, and the firm finds it optimal to shift some of them to the paid version. To achieve this, it does two things: It lowers the price of the paid version, and increases the "price" of the free version (i.e., ad intensity). This is the reason for the drop in price that we see in this figure.

Now compare Figure 4b with Figure 4a. The first difference that comes across is that price and ad intensity are much higher in 4a. The reason is simple: The cost of using the app is much lower in 4a, and thus the firm can charge higher rates. The second difference is that the price never drops in 4a. The logic behind this result relates to the discussion in the previous paragraph: The main reason to lower prices is to move consumers from the free version to the paid version. However, in 4a, there is already a mass of consumers who buy the paid version, and the aforementioned stratagem is therefore irrelevant.

While the discussion above sheds some light only on two specific cases (Figures 4a and 4b), and is far from comprehensive, it should give a sense of the driving forces in the model, and provide a rationale for the fact that price does not have to move monotonically with stickiness level. Similar arguments can be made with respect to the effect of uncertainty. Now that we have established that the effect of stickiness depends on the other parameters of the model, we wish to identify its common role, e.g., which is more frequent: an increase, or a decrease in price when stickiness increases?

We do this in two ways: First, we average the effect over all parameters' values in our numerical calculations, i.e., over all 14,641 cases. Second, we regress our variables of interest (e.g., price and the share of revenues from the paid versus the free version) over all the parameters of the model (e.g., stickiness and cost). These results are reported in the next section.

5. Main Results

Table 1 presents the outcome of OLS regressions where the explanatory variables are stickiness δ and the other three parameters of the model, which are used as controls (ε, c, and k), while the DVs are given in the heading of the table.

Table 1: Effects of model's parameters on price, advertising, and profits[8]

	Profits	**Price**	**Advertising intensity**	**Share of profits from paid version**
ε Uncertainty	0.068	0.005^{ns}	0.065	0.076
δ Stickiness	0.174	0.046	0.083	0.165
c Holding cost	-0.730	-0.378	-0.790	0.053
k Advertising contribution	0.240	0.287	-0.252	-0.682
Adj. R^2	58.2%	21.7%	69.1%	50.4%
AIC	-128	25,520	-10,902	4,750
BIC	-83	25,565	-10,858	4,795

All coefficients are standardized and significant at the 0.001 level except for the one marked with *ns*.

Recall that higher stickiness implies that the individuals can enjoy higher levels of utility from the app also in the second period. Thus, it is not surprising that higher stickiness leads to higher price and higher advertising intensity, as well as higher profits.

The estimates imply that the two decision variables of the firm, price and ad intensity, increase with stickiness level. In other words, while we find some specific cases (i.e., specific values of ε, c and k) for which p^* and γ^* might decrease in δ, these are the exceptions to the

[8] in 2,229 cases out of a total of 11^4 = 14,641 cases, the firm did not find it optimal to enter the market (i.e., with the optimal choice of price and advertising, profits turned out to be zero), and thus we have N = 12,412 for all of the following regressions and graphs.

rule. Controlling for the various values of ε, c and k, while regressing the optimal price and ad intensity on δ, we find a positive coefficient. Also, the same holds for the effect of ε on γ^*.

The rationale behind the increasing effect of stickiness δ on p^* is the following: The higher the stickiness, the higher the utility the individual can derive from the app in the second period, ergo her willingness to pay increases. The rationale behind the increasing effect of ε on p^* is a bit different: At first glance, it looks as if the role of ε is symmetric, as it appears as $\alpha - \varepsilon$ or $\alpha + \varepsilon$ in the utility of the consumer. This is misleading, as if the realization of the utility is $\alpha - \varepsilon$, then the consumer might not use the app at all in the second period, while if the realization is $\alpha + \varepsilon$, the consumer might either continue to use the free app, or switch to the paid version. Therefore, an increase in uncertainty has an asymmetric effect; indeed, it pushes some consumers out of the market, but on the other hand it shifts enough consumers from free to paid version in the second period.

It also implies that the effect of uncertainty will be smaller than that of stickiness: While an increase in stickiness implies that the utility of the app in the second period is larger – ergo the increased demand and profits – an increase in uncertainty increases both the number of consumers who stop using the product as well as the number of consumers who switch to the paid version. These conflicting effects of uncertainty are reflected in the fact that uncertainty follows the direction of stickiness, yet by a lower magnitude (see Table 1). If one wants to consider the effect of uncertainty on the share of profits from the monetization tools, we will also see a similar pattern, yet with a lower magnitude.

5.1 The interactions between the monetization tools

We start with a question of special interest to practitioners: While some firms are offering both versions of the app, paid and free, others offer only one, and there are questions in the industry whether the two should co-exist (Natanson 2016). From a straightforward economic point of view, disregarding program operation costs, having the ability to use two monetization tools should improve profits over using one tool only. Thus, the only way our analysis can provide some insights to practitioners is by (a) shedding light on the dynamics resulting from having the two versions versus only one, and (b) identifying the conditions under which having two versions is especially appealing.

We begin by solving three cases: a) the joint free/paid versions are available; b) only the free version is available; and c) only the paid version is available. We then compare the three results in terms of price, advertising, and profits. While average advertising and price for the joint case are 0.50 and 0.70 respectively, in the separate cases, these are much lower, given by 0.41 and 0.414 respectively. This points to a clear price discrimination in the joint case, leading obviously to higher profits. In other words, by having a high γ, the firm is pushing the high-α individuals away from the free version and into the paid version; while having a high p ensures that the medium-α individuals are pushed away from the paid version and into the free version.

If only the paid version is available, prices have to be lower in order to attract more users, especially some medium-α individuals. The reason for the lower γ in the case of the free version only (versus offering the two versions) is more interesting: Consider the case of a firm that has only the free version and is about to add the paid version. In order to gain the most from having both options, it must make the paid version especially appealing for the high-α individuals, and one way of doing so is by making the free version less appealing via higher ad intensity. Indeed, it is likely to lose some users – those with a relatively lower α are now likely to avoid the app altogether – but our analysis shows that this risk is worth taking.

Next, we wish to identify the conditions under which offering both free and paid is especially appealing to the firm. We do this by looking at the correlation between the stickiness level and the difference in profits between a joint program and one-tool programs. We find a positive correlation between stickiness and the difference in profits between joint and free version only ($r = 0.24$; $p < 0.01$), and a negative correlation between stickiness and profits difference between joint and paid only program ($r = 0.21$; $p < 0.01$). What we find here is that when stickiness is high, the gain in profit from having both options versus offering the paid version only is relatively low. In other words, when stickiness is high, the appeal of adding a free version is the lowest. Thus, taking into account cost considerations, this would be one of the cases in which it might be better for the firm to refrain from adding a free version. The following statement summarizes this result.

Result 1: The gain in the firm's profits due to price discrimination when both versions exist decreases in stickiness when compared to the paid-only version, and increases in stickiness when compared with the free-only version.

Given that the stickiness level seems to differ systematically across app categories, as suggested in the introduction, this result has real-world implications. Recall that the data seems to suggest that stickiness is relatively low for hedonic apps such as games and relatively high for utilitarian apps such as finance. Accordingly, our results suggests that it might be optimal to avoid the free version for utilitarian apps and the pay version for trendy hedonic apps.

In the next subsection we dig deeper in the mechanism that leads to this result and show, for example, that as stickiness increases, the share of profits from the paid version of the app increases. As a result, the appeal of the free-only option decreases.

5.2 Stickiness and the profitability of monetization tools

The last column in Table 1 assist us in digging deeper into the impact of stickiness on the monetization of apps. As just noted, it turns out that the higher the stickiness, the higher the share of profit from the paid version. To get a better sense of this result, consider Figure 5. In Figure 5a, we plot the relative shares of the paid and free versions over stickiness. It is evident that as stickiness increases, the relative share in profits of the paid version goes up, and, of course, that of the free version goes down. In Figure 5b, we see the breakdown of the profit into these two monetization tools. It turns out that the profit from the paid version goes up sizably with stickiness, yet that of the free version goes up and then dips.

Figure 5a: Effect of stickiness on **share of profits** from free and paid versions of the app

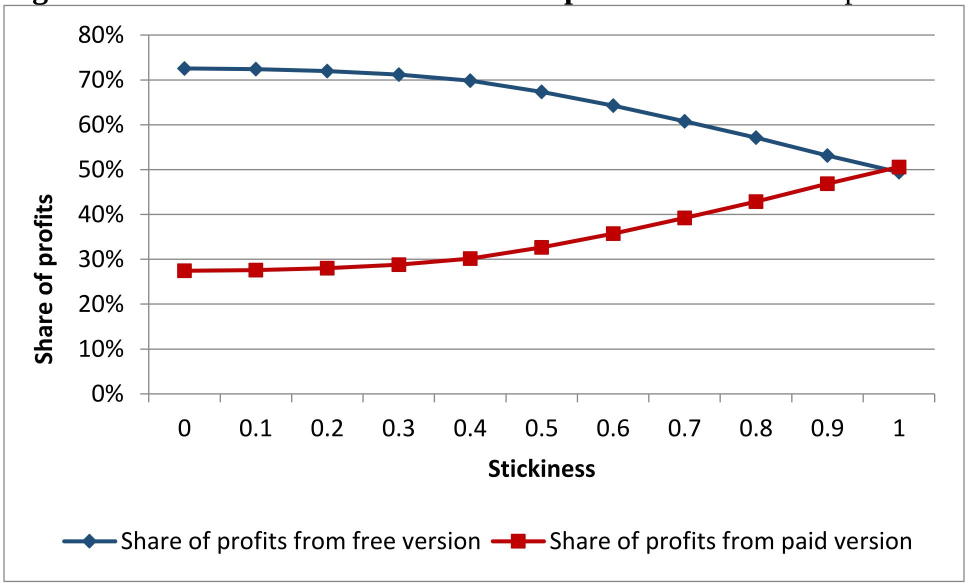
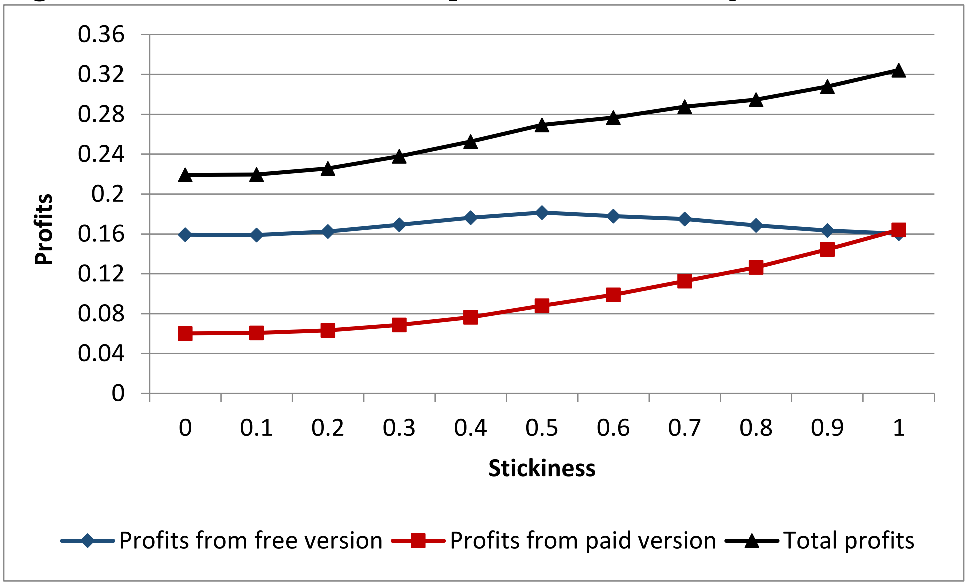


Figure 5b: Effect of stickiness on **profits** from free and paid version of the app



To understand the rationale of these results, recall that individuals who do not use the app are characterized by low match-utility; those who use the free version have a medium level of α, and those who buy the paid version have a high level. As pointed out above, when stickiness increases, consumers' willingness to pay increases as well. This pushes some of the non-users toward the free version, and some of those who are using the free version toward the paid version. As a result, if price and ad intensity do not change, the share of the non-users decreases, while the share of individuals who use the paid version increase. At the same time, the share of those using the free version might either increase or decrease. Indeed, it is likely that for low levels of stickiness, the mass of individuals who move from being non-users to users of the free version would be higher than of those who are moving from the free to the paid; while for high stickiness, the opposite will happen. And this might explain the rise and then the fall in the profit from the free version.

However, in the optimum, prices and ad intensity do change and intervene in this process. Specifically, as we have shown above, when stickiness levels increase, both variables, p^* and γ^*, increase (as the firm captures some of the increase in consumers' willingness to pay). Figure 5 demonstrates that, even when the increase in p^* and γ^* are taken into account, the profit from the paid version increases with δ, while the role of advertising as a source of profit increases, and then decreases. This is summarized in the following results:

Result 2a. With an increase in stickiness, share of profits decreases in the free version, and increases in the paid version.

Result 2b. Profits from the paid version of the app increase with stickiness, while profits from the free version first increase, and then decrease with stickiness.

Level of uncertainty, i.e., ε, also contributes to the firm's profit, but to a lesser degree, as demonstrated in the second column of Table 1. As mentioned above, a higher ε means a higher value from using the app in the first period, and thus higher willingness to pay. Furthermore, a higher ε also leads a larger proportion of individuals using the free version in the first period to switch to the paid version in the second. These two effects (higher willingness to pay in the first period, and larger proportion of switchers to the paid version in the second period) contribute to the profit of the firm. However, higher ε also has some negative effect on profit, as a larger proportion of users of the free version stop using the app in the second period. As a result, the contribution of ε to profit is lower than that of δ. To further understand the interactions between stickiness and uncertainty, observe Table 2, which shows the firm's profitability at various levels of stickiness and uncertainty.

Table 2: Profits as a function of stickiness and uncertainty levels

		Stickiness										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Uncertainty	0	0.18	0.18	0.18	0.18	0.19	0.20	0.21	0.22	0.24	0.25	0.27
	0.1	0.18	0.18	0.18	0.18	0.19	0.20	0.21	0.23	0.24	0.25	0.27
	0.2	0.18	0.18	0.18	0.18	0.19	0.20	0.21	0.23	0.24	0.26	0.27
	0.3	0.18	0.18	0.18	0.18	0.19	0.20	0.22	0.23	0.24	0.26	0.27
	0.4	0.18	0.18	0.18	0.19	0.20	0.21	0.22	0.24	0.25	0.27	0.28
	0.5	0.18	0.18	0.18	0.19	0.20	0.21	0.23	0.24	0.26	0.28	0.29
	0.6	0.18	0.18	0.18	0.19	0.20	0.22	0.23	0.25	0.27	0.29	0.31
	0.7	0.18	0.18	0.18	0.19	0.21	0.23	0.24	0.26	0.28	0.30	0.33
	0.8	0.18	0.18	0.18	0.20	0.21	0.23	0.25	0.27	0.29	0.32	0.35
	0.9	0.18	0.18	0.19	0.20	0.22	0.24	0.26	0.28	0.31	0.34	0.37
	1	0.18	0.18	0.19	0.20	0.22	0.24	0.27	0.30	0.33	0.36	0.40

If the game lasts only a single period (zero stickiness), or if stickiness is low, it is easy to see that there is no effect (or little effect) of uncertainty: Consumers download the app, and whatever they learn in the first period cannot be used, as there is no second period; or else the benefit in the second period is negligible. In particular, we show that stickiness can affect the difference between the cases: When analyzing elasticities in Table 2, we find that the interaction between stickiness and uncertainty is non-symmetric: Stickiness has a pronounced effect on sensitivity of profits to uncertainty, while uncertainty has a smaller effect on sensitivity of profits to stickiness.

5.3 The role of advertising's contribution

In a one-period model, i.e., $\delta = 0$, the only contribution of the free version to profits is via advertising revenues. In this subsection, we show that in a two-period model, i.e., $\delta > 0$, the free version can increase profits in another way. To illustrate this point, consider the case of $k = 0$: In a one-period model, the firm's objective function (Equation 7) is not a function of ad intensity at all, and the optimal decision of the firm is to offer a paid version only, as a free version cannot increase its profits in any way, and thus will not be offered.

However, in a model with stickiness, the firm's objective function depends on ad intensity even when $k = 0$. Furthermore, *the free version with ads can yield profits even if advertisers are not paying for these ads*.

In a two-period model, the free version contributes to the profit of the firm not just via advertising revenues, but also by acting as sort of a "sample". Specifically, some individuals who would otherwise not use the app (as the paid version is too costly for them) download and use the free version to learn about their fit with the app. Some of these are likely to switch in the second period to the paid version. Thus, even if advertisers do not pay for ads, the firm can benefit from offering a free version with ads. However, offering this version also has some costs: Some individuals who would otherwise buy the paid version, might be attracted to the free version. Thus, the optimal level of advertising balances between two incentives.

To examine this idea – that the free version with ads can yield profits even if advertisers are not paying for these ads – we focused our analysis on all cases in which $k = 0$. Indeed, we find in our numerical cases that when $k = 0$, it is still optimal for the firm to offer the free version, and the optimal level of advertising therein is $0 < \gamma^* < 1$. The firm has two sources of income: the consumers who downloaded the paid version in the first period; and the consumers who chose to download the free version in the first period, then switch to the paid version in the second period. The firm does not realize any revenues from the latter segment in the first period (as $k = 0$), but does realize some profits from these consumers in the second period.

This situation might bear some similarity to producing an imperfect product (e.g., Fornell and Wernerfelt 1988), except that the firm constructs this imperfect product knowing full well that it will not obtain any direct benefit from it, its main motivation being to get some consumers to download it for the sake of learning about its fit to their preferences. The firm makes money only from the paid version of the app, despite setting an optimal level of advertising for the free version, from which it earns no revenues. This is summarized in the next statement:

> **Result 3.** When advertising's contribution to the firm's profits is zero, the firm still advertises, despite the fact that it sees no revenues from these ads.

We conclude this subsection by noting another interesting finding on advertising intensity: In the fourth column of Table 1, we find that an increase in *k* leads to a decrease in γ^*. Recall that the advertising contribution parameter *k* converts the advertising intensity variable (γ) into monetary terms compatible with price, such as $. In other words, we find that when advertisers pay more for advertising, the firm decreases its advertising intensity. This result is not as strange as it sounds first. When *k* increases, the firm faces two conflicting forces: On the one hand, it wishes to increase ad intensity in order to directly benefit from the higher pay. On the other hand, an increase in ad intensity (which annoys consumers) would decrease the share of individuals who will choose the free version, and thus might eventually decrease the income from advertising. Thus the final decision of the firm of whether to increase or decrease ad intensity depends upon the model's elasticities, and in our case, it is optimal to decrease ad intensity.

6. Discussion, limitations and conclusions

When looking at apps markets, managers and consumers may see two broad types of apps. Some apps are by nature short ranged – consumers' utility may be higher early on, yet soon they can expect to move to the next one. Many games and other hedonic apps may fit this type. Other apps are longer term in nature - if they fit consumers, they may stick with them for a while. Business, functional and other utilitarian apps may better fit this category. What we showed here is that this distinction may lead to a fundamental difference in optimal management of the monetization process due to the difference in the tendency to stay.

We focused here on the tension between the main monetization sources: paying consumers versus paying advertisers. While the tension between the monetization options is recognized as fundamental to app success (Pozin 2014), in-depth answers as to the drivers of the use of each are lacking. Furthermore, to the best of our knowledge, no previous studies have analyzed this issue in a customer duration dynamic setting. We show that such a setting, i.e., the combination of stickiness and uncertainty, leads to some important insights on price levels, advertising intensity, profitability, and profitability sources. We summarize our findings in the following points.

- We consider the dilemma faced by some firms about whether to offer two versions of the app (free and paid) or only one version. Of course, when operating costs are ignored, offering two versions is more profitable. However, our analysis can identify the cases in which the additional profit is minimal. In such cases, taking costs into account, offering just one version might have more consumer appeal. We find that when the stickiness level is high, the firm might benefit by focusing on the paid version and avoiding the free version.
- We show that as stickiness increases, prices and ad intensity increase, and firms earn more of their profits from paying customers than they do from advertising. Specifically, we find that when stickiness increases, the firm's earning from the paid version increases monotonically. However, its earnings from advertising first increase, then drop. Accordingly, the share of profit accruing from the paid version increases with the stickiness level throughout.
- We demonstrate that in a dynamic setting, a firm can profit from offering a free version with ads even if advertisers are not paying for those ads. In other words, the firm benefits from offering a version of the app that includes ads even if the consumers are not paying for the app and the advertisers are not paying for the ads. The logic behind this result is the following: In a dynamic setting, the free version acts as a sort of "sample" that enables consumers to learn their fit with the app. The firm would rather add ads to this version

(even if they are not paid for) in order to keep the consumers who have high valuation of the app away from the "sample" and buying the paid version.

- Our analysis elucidates the combined role of uncertainty and stickiness in creating the market dynamics of apps. Specifically, the role of uncertainty is complex, as overall higher uncertainty, and in particular under high levels of stickiness, can lead to higher profits. This finding departs from the conventional wisdom regarding digital products, which says that marketers should seek to decrease the level of the uncertainty of fit to individuals (Matt and Hess 2016). However, for free digital products where product trial is part of the business cycle, and where the firm profits from advertising during trial, higher uncertainty reflects also the ability to attract customers who otherwise would not have downloaded the app, as they do so if they believe the upside is large enough to bear the costs of trying the product in the first place.

Our attempt to capture a complex market situation with a relatively parsimonious model clearly has a number of limitations. One issue is the lack of explicit modeling of competitive activity, beyond the app holding cost, which takes into account alternative space, time, and competitive apps. The added complexity thereof is beyond our scope here. It should also be mentioned that it is not trivial to model explicit competition, due to the complex meaning of what constitutes a "competitor" in this market. Many free products compete in a general sense for users' attention and time, and not necessarily with products to which they are very similar. However, there may be specific categories in which clear, direct competitors emerge.

We examined a specific freemium model in which the premium version of the product had no advertising. There are more complex free business models in which, for example, people purchase in-app additions that change the product's utility in various ways. Modeling each of these scenarios would add complexity that we felt is unneeded at this stage, yet can of course be done in future explorations. In the same vein, we assumed a fixed price for the paid version. This assumption is supported by market observations and interviews with managers, which suggest that given the short time periods, the paid version price is largely fixed. Changing this assumption adds significant complexity to the model, yet can be an interesting avenue for future research.

More generally, we believe our results to have broad implications beyond the specific freemium model discussed here. In recent years, much attention has been devoted to how the online ecosystem has led to fundamental changes in business activities such as advertising (Goldfarb 2014), competitive strategy (Casadesus-Masanell and Zhu 2010), and product pricing and design (Kannan 2013). Most analyses of these topics have not explicitly taken into account the role of retention in the emergence of market strategies. Yet in online markets, retention is not only important, but is also more easily traceable than in most offline businesses. We suggest that more attention should be paid to this issue as a basic part of analysis of firms' optimal behavior in these emerging markets.

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Appendices

Appendix A: Three scenarios for the utility of a free app

We define the following: lifetime expected utility from the three options[9]:

$$\text{(A1)}\quad \begin{cases} U_0 = 0 \\ U_a = \alpha(1-\gamma) - c + \tfrac{1}{2}\max(0, a) + \tfrac{1}{2}\max(0, b, d) \\ U_p = \alpha - c - p + \tfrac{1}{2}\max(0, e) + \tfrac{1}{2}\max(0, f) \end{cases}$$

where U_0, U_a, and U_p are the expected utility from the no-use, use free, and use paid options, respectively; and

$$\text{(A2)}\quad \begin{cases} a = (\alpha - \varepsilon)\cdot(1-\gamma)\cdot\delta - c \\ b = (\alpha + \varepsilon)\cdot(1-\gamma)\cdot\delta - c \\ d = (\alpha + \varepsilon)\cdot\delta - c - p \\ e = (\alpha - \varepsilon)\cdot\delta - c \\ f = (\alpha + \varepsilon)\cdot\delta - c \end{cases}$$

Let α_a^1 be the threshold when expression (*b*) of Equation A2 equals zero $(\alpha_a^1 = \frac{c}{(1-\gamma)\delta} - \varepsilon)$.

Similarly α_a^2 - the threshold when A2 (*a*) equals zero ($\alpha_a^2 = \frac{c}{(1-\gamma)\delta} + \varepsilon$),

α_a^3 as the threshold when A2 (*d*) equals zero ($\alpha_a^3 = \frac{c+p}{\delta} - \varepsilon$).

We define the following utility equations:

U_a^1 - the utility when *a, b, d* < 0 , $U_a^1 = \alpha(1-\gamma) - c$

U_a^2 - the utility when *b* > 0 and *d, a* < 0 , $U_a^2 = \alpha(1-\gamma) - c + \frac{1}{2}[(\alpha+\varepsilon)(1-\gamma)\delta - c]$

U_a^3 - the utility when *b, a* > 0 and *d* < 0 , $U_a^3 = \alpha(1-\gamma)(1+\delta) - 2c$

U_a^4 - the utility when *d* > *b* > 0 and *a* > 0

$U_a^4 = \alpha(1-\gamma) - c + \frac{1}{2}[(\alpha-\varepsilon)(1-\gamma)\delta - c] + \frac{1}{2}[(\alpha+\varepsilon)\delta - c - p]$

U_a^5 - the utility when *d* > 0 and *b, a* < 0 , $U_a^5 = \alpha(1-\gamma) - c + \frac{1}{2}[(\alpha+\varepsilon)\delta - c - p]$

---

[9] Note that Equations A1 and A2 are equivalent to Equations 2 and 3; however, we use the extended version here, as the multiple expressions in Equation A2 allow for a smoother discussion of this appendix.

The relative locations of α_a^1, α_a^2, and α_a^3 define the possible scenarios for U_a. Since $\alpha_a^2 > \alpha_a^1$ for all α, the only three possible scenarios are defined when: $\alpha_a^3 > \alpha_a^2 > \alpha_a^1$, $\alpha_a^2 > \alpha_a^3 > \alpha_a^1$, and $\alpha_a^2 > \alpha_a^1 > \alpha_a^3$. As a result of the following scenario analyses, the scenarios also differ as per the relative size of the price: high, intermediate, and low.

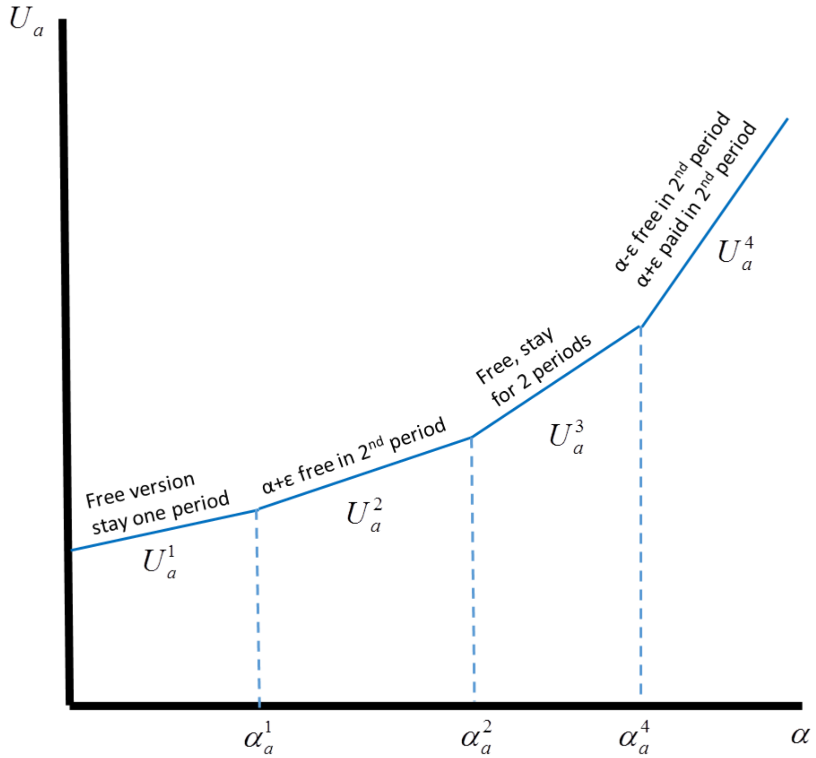
Scenario 1: $\alpha_a^3 > \alpha_a^2 > \alpha_a^1$ - high price

In the first scenario, price is relatively high, so only customers with high α will consider moving to the paid version in the second period. In the second period, as α increases, first the consumers with positive realization of ε will use the free version (A2 (*b*)), then those with a negative realization of ε will also use the free version (A2 (*a*)), and finally, those with positive realization of ε will move to the paid version (A2 (*d*)). In this scenario, $\alpha_a^3 > \alpha_a^2 > \alpha_a^1$. Note that a positive value for break point α_a^1 requires $c > \varepsilon(1-\gamma)\delta$, or we begin with section U_a^2 rather than with U_a^1. Furthermore, once α is large enough so that A2 (*b*) is greater than zero, the consumer will move to the paid version only when her utility from paying is greater than her utility from the free version, where A2 (*d*) = A2 (*b*), given by $\alpha_a^4 = \frac{p}{\gamma\delta} - \varepsilon$, and not in threshold α_a^3 (in this case $\alpha_a^4 > \alpha_a^3$, or $p > \frac{c}{1-\gamma}$). In addition, for this scenario to hold, we need $\alpha_a^4 > \alpha_a^2 > \alpha_a^1$; while $\alpha_a^2 > \alpha_a^1$ is always true, $\alpha_a^4 > \alpha_a^2$ is described by the following limitation, defining the "large" p: $p_{Large} > \frac{2\varepsilon(1-\gamma)\delta\gamma + c\gamma}{1-\gamma}$. Also note that if ε is large enough, α_a^1 is negative, and this scenario begins with the second segment of Figure A1 rather than the first segment.

The slopes of the utility segments of Figure A1 are given by the following equations:

$$\text{(A3)} \quad \frac{\partial U_a}{\partial \alpha} = \begin{Bmatrix} 1-\gamma \text{ for } \alpha < \alpha_a^1 \\ (1-\gamma)(1+\frac{\delta}{2}) \text{ for } \alpha_a^1 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\delta) \text{ for } \alpha_a^2 < \alpha < \alpha_a^4 \\ (1-\gamma)(1+\frac{\delta}{2}) + \frac{\delta}{2} \text{ for } \alpha_a^4 < \alpha \end{Bmatrix}$$

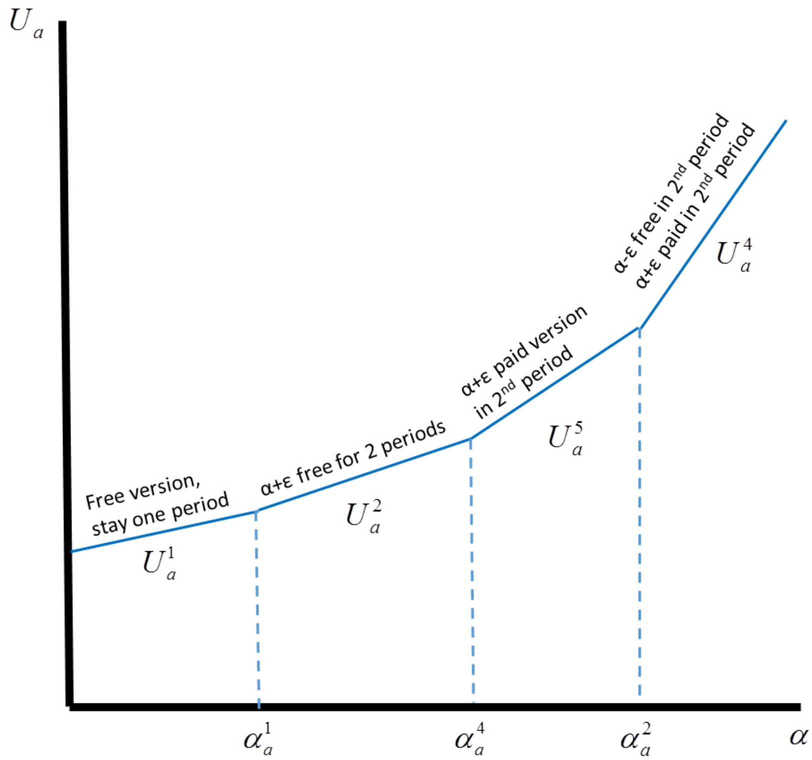
Figure A1: Utility of free app, high price scenario



Scenario 2: $\alpha_a^2 > \alpha_a^3 > \alpha_a^1$ - intermediate price

In Scenario 2, price is at an intermediate level, so customers with lower α than in Scenario 1 will consider moving to a paid version in the second period. In the second period, as α increases, first the consumers with positive realization of ε will use the free version (A2 (*b*)), then those with positive realization of ε will move to the paid version (A2 (*d*)), and finally, those with a negative realization of ε will use the free version (A2 (*a*)). In this scenario, $\alpha_a^2 > \alpha_a^3 > \alpha_a^1$. Note that a positive value for break point α_a^1 requires $c > \varepsilon(1-\gamma)\delta$, or we start with section U_a^2 rather than with U_a^1. Similar to Scenario 1, the threshold for moving from free to paid version is given by α_a^4. In addition, for this scenario to hold, we need $\alpha_a^2 > \alpha_a^4 > \alpha_a^1$; while $\alpha_a^2 > \alpha_a^1$ is always true, we need to limit α_a^4 to be in $\alpha_a^2 > \alpha_a^4 > \alpha_a^1$. $\alpha_a^2 > \alpha_a^4$ is described by the following limitation (the opposite of Scenario 1's limit): $p < \frac{2\varepsilon(1-\gamma)\delta\gamma + c\gamma}{1-\gamma}$, and $\alpha_a^4 > \alpha_a^1$ is described by the following: $p > \frac{c\gamma}{1-\gamma}$. Thus, the intermediate p is defined by: $\frac{c\gamma}{1-\gamma} < p_{Inter} < \frac{2\varepsilon(1-\gamma)\delta\gamma + c\gamma}{1-\gamma}$

Figure A2: Utility of free app, intermediate price scenario



The slopes of Figure A2's utility segments are given by the following equations:

(A4) $$\frac{\partial U_a}{\partial \alpha} = \begin{Bmatrix} 1-\gamma \text{ for } \alpha < \alpha_a^1 \\ (1-\gamma)(1+\frac{\delta}{2}) \text{ for } \alpha_a^1 < \alpha < \alpha_a^4 \\ (1-\gamma)+\frac{\delta}{2} \text{ for } \alpha_a^4 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\frac{\delta}{2})+\frac{\delta}{2} \text{ for } \alpha_a^2 < \alpha \end{Bmatrix}$$

Scenario 3: $\alpha_a^2 > \alpha_a^1 > \alpha_a^3$ **-** low price

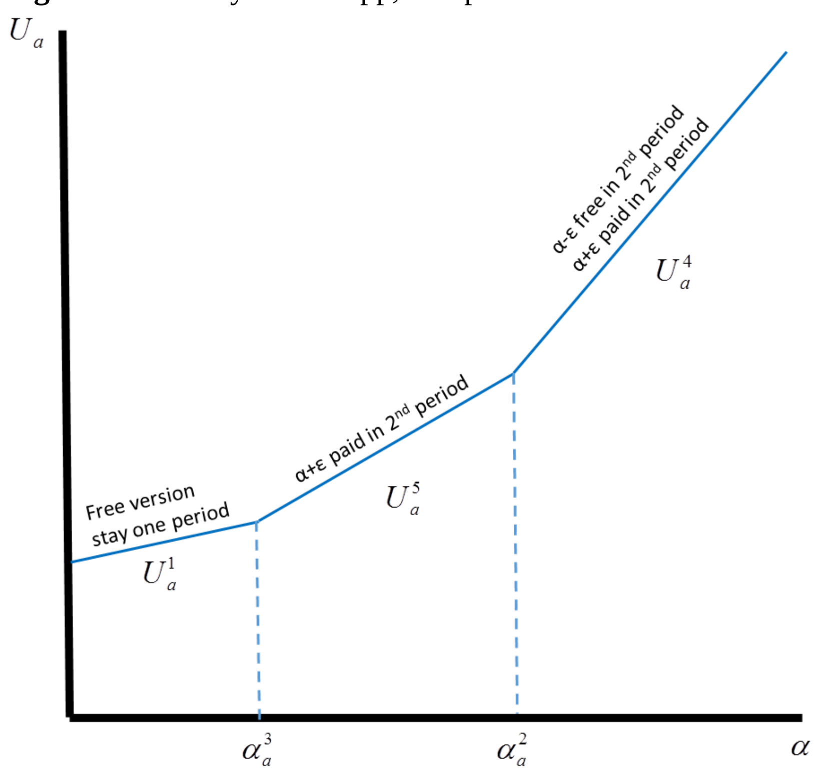
In the third scenario, price is relatively low, so customers with lower α than in Scenario 2 will consider moving to the paid version in the second period before even considering the free version. In the second period, as α increases, first the consumers with positive realization of ε will move to the paid version (A2 (*d*)), and then those with a negative realization of ε will use the free version (A2 (*a*)). In this scenario, $\alpha_a^2 > \alpha_a^1 > \alpha_a^3$. Recall that $\alpha_a^3 = \frac{c+p}{\delta} - \varepsilon$. Also, once α is large enough that A2 (*d*) exceeds A2 (*b*), this inequality will hold for any larger α. Thus, in this scenario, α_a^1 is not used, as consumers with a positive realization of ε will use the paid rather than the free version in the second period.

The slopes of the utility segments of Figure A2 are given by the following equations:

$$\text{(A5)} \quad \frac{\partial U_a}{\partial \alpha} = \left\{ \begin{array}{c} 1-\gamma \ \text{ for } \ \alpha < \alpha_a^3 \\ (1-\gamma) + \frac{\delta}{2} \ \text{ for } \ \alpha_a^3 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\frac{\delta}{2}) + \frac{\delta}{2} \ \text{ for } \ \alpha_a^2 < \alpha \end{array} \right\}$$

Note that a positive value for break point α_a^1 requires $c + p > \varepsilon\delta$, or we start with section U_a^5 rather than with U_a^1. Note that for this scenario to hold, we need $\alpha_a^1 > \alpha_a^3$, which is described by the following limitation, defining the "small" p: $p_{Small} < \frac{c\gamma}{1-\gamma}$.

Figure A3: Utility of free app, low price scenario



Appendix B: Proofs of Lemma 1

In this Appendix, we show that for all p, γ, δ, ε, and c, when $\alpha = 0$, $U_a(0) > U_p(0)$.

Notice that when $\alpha = 0$, from Equation 2 we get:

$$U_a(0) = -c + \tfrac{1}{2}\max\left[0, \varepsilon(1-\gamma)\delta - c, \varepsilon\delta - c - p\right]$$
$$U_p(0) = -c - p + \tfrac{1}{2}\max\left[0, \varepsilon\delta - c\right]$$

There are two cases to consider:

Case B1: $p > \varepsilon\delta\gamma$ (i.e., $\varepsilon(1-\gamma)\delta - c > \varepsilon\delta - c - p$)

In this case, we get that:

$$U_a(0) - U_p(0) = \begin{Bmatrix} p - \tfrac{1}{2}\varepsilon\gamma\delta \text{ for } c < \varepsilon(1-\gamma)\delta \\ p - \tfrac{1}{2}\varepsilon\delta + \tfrac{1}{2}\mathrm{c} \text{ for } \varepsilon(1-\gamma)\delta < c < \varepsilon\delta \\ p \text{ for } \varepsilon\delta < c \end{Bmatrix}$$

The first line is non-negative, since in this case (i.e., *Case B1*), we know that $p > \varepsilon\delta\gamma$, and thus certainly $p > \tfrac{1}{2}\varepsilon\delta\gamma$. The third line is non-negative, as $p \geq 0$.

For the second line, as $c > \varepsilon(1-\gamma)\delta$, we know that

$p - \tfrac{1}{2}\varepsilon\delta + \tfrac{1}{2}\mathrm{c} > p - \tfrac{1}{2}\varepsilon\delta + \tfrac{1}{2}\varepsilon(1\text{-}\gamma)\delta = p - \tfrac{1}{2}\varepsilon\gamma\delta > 0$ (because in *Case B1*, $p > \varepsilon\delta\gamma$).

Case B2: $p < \varepsilon\delta\gamma$

In this case, we get that:

$$U_a(0) - U_p(0) = \begin{Bmatrix} \tfrac{1}{2}p \text{ for } c < \varepsilon\delta - p \\ p - \tfrac{1}{2}\varepsilon\delta + \tfrac{1}{2}\mathrm{c} \text{ for } \varepsilon\delta - p < c < \varepsilon\delta \\ p \text{ for } \varepsilon\delta < c \end{Bmatrix}$$

The first and third lines are non-negative, as $p \geq 0$.

The second line is for $c > \varepsilon\delta - p$, thus we know that $p > \varepsilon\delta - c$, and as $c < \varepsilon\delta$, we can write that $p > \varepsilon\delta - c > 0$, and it is immediate that $p > \tfrac{1}{2}(\varepsilon\delta - c)$, which leads to $p - \tfrac{1}{2}\varepsilon\delta + \tfrac{1}{2}\mathrm{c} > 0$.

Notice that $U_a(0) = U_p(0)$ only when $p = 0$, as if $p > 0$, then $U_a(0) > U_p(0)$.

Appendix C: Proofs of Lemma 2

In this Appendix, we show that for all *p*, *γ*, *δ*, *ε*, and *c*; and for each *α*, the slope of U_p is greater than that of U_a.

Recall the slopes of U_p: $$\frac{\partial U_p}{\partial \alpha} = \begin{Bmatrix} 1 \text{ for } \alpha < \alpha_p^1 \\ 1+\frac{\delta}{2} \text{ for } \alpha_p^1 < \alpha < \alpha_p^2 \\ 1+\delta \text{ for } \alpha_p^2 < \alpha \end{Bmatrix}$$

We now compare these slopes to that of the three scenarios of U_a from Appendix A.

Scenario 1: $\alpha_a^3 > \alpha_a^2 > \alpha_a^1$ - high price

The slopes of U_a in scenario 1 are: $$\frac{\partial U_a}{\partial \alpha} = \begin{Bmatrix} 1-\gamma \text{ for } \alpha < \alpha_a^1 \\ (1-\gamma)(1+\frac{\delta}{2}) \text{ for } \alpha_a^1 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\delta) \text{ for } \alpha_a^2 < \alpha < \alpha_a^4 \\ (1-\gamma)(1+\frac{\delta}{2})+\frac{\delta}{2} \text{ for } \alpha_a^4 < \alpha \end{Bmatrix}.$$

Since $\alpha_p^1 < \alpha_a^1$, when $\alpha < \alpha_p^1$, the slope of U_p = 1 and the slope of U_a = $1-\gamma$. As $\alpha_p^2 < \alpha_a^2$, when $\alpha_p^1 < \alpha < \alpha_p^2$, the slope of U_p = $(1+\frac{\delta}{2})$ and the slope of $U_a \le (1-\gamma)(1+\frac{\delta}{2})$, that is, it can be either $(1-\gamma)$ or $(1-\gamma)(1+\frac{\delta}{2})$. Finally, when $\alpha_p^2 < \alpha$, the slope of U_p = $1+\delta$, which is the highest value of all of the slopes above.

Scenario 2: $\alpha_a^2 > \alpha_a^3 > \alpha_a^1$ - intermediate price

The slopes of U_a in scenario 2 are : $$\frac{\partial U_a}{\partial \alpha} = \begin{Bmatrix} 1-\gamma \text{ for } \alpha < \alpha_a^1 \\ (1-\gamma)(1+\frac{\delta}{2}) \text{ for } \alpha_a^1 < \alpha < \alpha_a^4 \\ (1-\gamma)+\frac{\delta}{2} \text{ for } \alpha_a^4 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\frac{\delta}{2})+\frac{\delta}{2} \text{ for } \alpha_a^2 < \alpha \end{Bmatrix}.$$

When $\alpha < \alpha_p^1$, the slope of U_p = 1 and the slope of U_a = $1-\gamma$ (recall that $\alpha_p^1 < \alpha_a^1$). As $\alpha_p^2 < \alpha_a^2$, when $\alpha_p^1 < \alpha < \alpha_p^2$, the slope of U_p = $(1+\frac{\delta}{2})$ and the slope of $U_a \le (1-\gamma)+\frac{\delta}{2}$. Finally, when $\alpha_p^2 < \alpha$, the slope of U_p = $1+\delta$, which is the highest value of all of the slopes above.

Scenario 3: $\alpha_a^2 > \alpha_a^1 > \alpha_a^3$ - low price

The slopes of U_a in scenario 3 are: $\frac{\partial U_a}{\partial \alpha} = \begin{cases} 1-\gamma \text{ for } \alpha < \alpha_a^3 \\ (1-\gamma)+\frac{\delta}{2} \text{ for } \alpha_a^3 < \alpha < \alpha_a^2 \\ (1-\gamma)(1+\frac{\delta}{2})+\frac{\delta}{2} \text{ for } \alpha_a^2 < \alpha \end{cases}$

When $\alpha < \alpha_p^1$, the slope of U_p = 1 and the slope of U_a = $1-\gamma$ (since $\alpha_p^1 < \alpha_a^3$). When $\alpha_p^1 < \alpha < \alpha_p^2$, the slope of U_p = $(1+\frac{\delta}{2})$ and the slope of $U_a \le (1-\gamma)+\frac{\delta}{2}$, as $\alpha_p^2 < \alpha_a^2$. Finally, when $\alpha_p^2 < \alpha$, the slope of U_p = $1+\delta$, which is the highest value of all of the slopes above.

Thus, in all of the three cases, we get $\frac{\partial U_a}{\partial \alpha} < \frac{\partial U_p}{\partial \alpha}$ for all α, leading to the conclusion that if U_a and U_p intersect, it happens only once.