Journal of Financial Economics 14 (1985) 217–236. North-Holland

TESTING ASSET PRICING MODELS WITH CHANGING EXPECTATIONS AND AN UNOBSERVABLE MARKET PORTFOLIO*

Michael R. GIBBONS

Stanford University, Stanford, CA 94305, USA

Wayne FERSON

University of Pennsylvania, Philadelphia, PA 19104, USA

Received April 1983, final version received October 1984

When the assumption of constant risk premiums is relaxed, financial valuation models may be tested, and risk measures estimated without specifying a market index or state variables. This is accomplished by examining the behavior of conditional expected returns. The approach is developed using a single risk premium asset pricing model as an example and then extended to models with multiple risk premiums. The methodology is illustrated using daily return data on the common stocks of the Dow Jones 30. The tests indicate that these returns are consistent with a single, time-varying risk premium.

1. Introduction

The past twenty years have witnessed numerous empirical examinations of the Capital Asset Pricing Model (CAPM); examples include Black, Jensen and Scholes (1972), Fama and MacBeth (1973), and Gibbons (1982). Like much empirical work on asset pricing models, these examples suffer from three methodological shortcomings: (1) expected returns are assumed to remain constant over some period of time, (2) the market portfolio of risky securities must be observable, and (3) evidence on the validity of more general asset pricing models [e.g., Merton (1973), Long (1974) or Breeden (1979)] is not provided.

While returns may contain a predictable component in an efficient market [see Fama (1976, p. 149) for an illustration], the empirical literature has not

*The authors would like to thank Richard Roll, Myron Scholes, G. William Schwert, and participants in seminars at Columbia University, New York University, Ohio State University, the University of California at Los Angeles, the University of Chicago, the University of Pennsylvania, Yale University, and the Western Finance Association for helpful comments on earlier drafts of this paper. Discussions with Robert Stambaugh have been especially helpful, as were the comments of Stephen Brown, the referee. Of course, errors or inadequacies are our own. The first author was a Batterymarch Fellow at the time this work was completed; the second author was Visiting Assistant Professor of Finance at the University of Chicago. Partial financial support from Batterymarch and the Stanford Program in Finance is gratefully acknowledged.

0304-405X/85/$3.30

fully explored the implications. For example, most tests of the CAPM have focused on the cross-sectional relation between unconditional expected returns and 'betas', but the underlying theory refers to moments conditional on available information. While these conditional expectations may change over time, empirical studies of asset pricing have not utilized this time series behavior when testing cross-sectional models of returns. The present paper represents a step in this direction, and it suggests that there are further opportunities to refine empirical methodology along these lines. By relaxing the assumption of constant risk premiums, tests of asset pricing models are developed which do not require identification of the market portfolio or state variables. The methodology is constructive because, having rejected a particular asset pricing model, the tests can indicate a more appropriate specification.

The cost of obtaining these advantages is the assumption of a model for conditional expectations. Given the problems associated with a market proxy [Roll (1977)] and empirical evidence which suggests changing expected returns, allowing non-constant risk premiums is an attractive approach. Moreover, the particular model of expectations employed (a linear regression model) is fairly general, robust to certain misspecifications, and amenable to statistical verification.

The second section of this paper explains the suggested methodology using the traditional [Sharpe (1964), Lintner (1965)] version of the CAPM as an example. Section 3 extends the approach to models with multiple risk premiums. In the fourth section, the procedure is illustrated by conducting tests on the individual common stocks of the Dow Jones 30. Section 5 concludes the paper and presents some suggestions for future research.

2. Testing the capital asset pricing model: An example

If Z_{t-1}^* is the available information at time $t-1$, the Sharpe–Lintner version of the CAPM states

$$\mathrm{E}(\tilde{r}_{it}|Z_{t-1}^*) = \beta_{im}\mathrm{E}(\tilde{r}_{mt}|Z_{t-1}^*), \tag{1}$$

where

$\tilde{R}_{it}$ ≡ return on asset i realized time t;
$\tilde{R}_{mt}$ ≡ return on the market portfolio of risky assets (not necessarily available to the econometrician);
R_{ft} ≡ riskless rate of interest (observable at time $t-1$);
$\tilde{r}_{it}$ ≡ $\tilde{R}_{it} - R_{ft}$ is the 'excess return' on asset i;
$\tilde{r}_{mt}$ ≡ $\tilde{R}_{mt} - R_{ft}$ is the excess return on the market portfolio;
β_{im} = conditional covariance between the return on asset i and the market portfolio divided by the conditional variance of the market portfolio return.

Eq. (1) specifies a linear relation between conditional expected returns on all securities and that of the market portfolio. Since (1) holds for all assets, it holds for asset $i = 1$:

$$\mathrm{E}(\tilde{r}_{1t}|\boldsymbol{Z}_{t-1}^{*}) = \beta_{1m}\mathrm{E}(\tilde{r}_{mt}|\boldsymbol{Z}_{t-1}^{*}). \tag{2}$$

Combining (1) and (2) and assuming $\beta_{1m} \neq 0$ yields, for all i,

$$\mathrm{E}(\tilde{r}_{it}|\boldsymbol{Z}_{t-1}^{*}) = (\beta_{im}/\beta_{1m})\mathrm{E}(\tilde{r}_{1t}|\boldsymbol{Z}_{t-1}^{*}). \tag{3}$$

Thus, the CAPM implies a linear relation across securities' conditional expected returns that does not involve the expected return on the market portfolio. If betas are constant, eq. (3) implies that shifts in expected excess returns must be proportional. If expected excess returns, given $\boldsymbol{Z}_{t-1}^{*}$, are also assumed to be linear: $\mathrm{E}(\tilde{r}_{it}|\boldsymbol{Z}_{t-1}^{*}) = \boldsymbol{\delta}_i^{*\prime}\boldsymbol{Z}_{t-1}^{*}$, then eq. (3) implies $\boldsymbol{\delta}_i^{*} = (\beta_{im}/\beta_{1m})\boldsymbol{\delta}_1^{*}$. Such a cross-sectional restriction is testable.[1]

Despite the appearance of the example, the approach does not require the full information set $\boldsymbol{Z}_{t-1}^{*}$. To see this, assume that expectations are rational (that is, mathematical conditional expectations), and consider a vector of information variables $\boldsymbol{Z}_{t-1}$, a subset of $\boldsymbol{Z}_{t-1}^{*}$. Assume also that ratios of betas are constant, given $\boldsymbol{Z}_{t-1}$. The 'law of iterated conditional expectations' and eq. (3) then imply[2]

$$\mathrm{E}(\tilde{r}_{it}|\boldsymbol{Z}_{t-1}) = (\beta_{im}/\beta_{1m})\mathrm{E}(\tilde{r}_{1t}|\boldsymbol{Z}_{t-1}), \quad \forall i, t. \tag{4}$$

Even if the econometrician overlooks some of the variables in $\boldsymbol{Z}_{t-1}^{*}$, eq. (4) indicates that a subset of information can still provide a legitimate, although perhaps less powerful, test of the CAPM.[3] Hence, an investigator may select variables for $\boldsymbol{Z}_{t-1}$ so that the model for conditional expectations appears to be well-specified.

This approach allows expected returns to vary but assumes risk measures of the securities examined are constant conditional on the test information $\boldsymbol{Z}_{t-1}$.

[1] Hall (1981) and Hansen and Singleton (1983) independently conduct tests of a consumption-based asset pricing model which are similar in spirit to the approach taken here. They assume stationary joint lognormality of consumption and asset prices and constant relative risk aversion. Ferson (1983) applies similar methodology to test consumption models with constant relative and absolute risk aversion.

[2] Eq. (3) implies eq. (4) because $\mathrm{E}\{\mathrm{E}(\tilde{r}_{it}|\boldsymbol{Z}_{t-1}^{*})|\boldsymbol{Z}_{t-1}\} = \mathrm{E}\{\tilde{r}_{it}|\boldsymbol{Z}_{t-1}\}$ and ratios of betas are assumed to be constant over $\boldsymbol{Z}_{t-1}$.

[3] Alternatively, standard regression theory for 'left-out variables' bias implies that the coefficients using $\boldsymbol{Z}_{t-1}$ are related to those using $\boldsymbol{Z}_{t-1}^{*}$ in such a way as to preserve the parameter restriction on a subset of information. This is shown in Gibbons and Ferson (1984, app. B), a working paper which differs from the present paper by the addition of two appendices.

The tests are not invalidated if the 'true' risk measures (given Z_{t-1}^*) are changing over time.[4] Since the tests build on the assumption that expected returns fluctuate, they represent a natural extension of the usual assumptions.

Most empirical work on the CAPM has assumed constant expected returns and covariances although the asset pricing theory does not require this stationarity.[5] Earlier studies recognized the importance of their stationarity assumptions, and these same studies found evidence of changing expected returns. Black, Jensen and Scholes (1972), for example, point out that constant betas justify grouping assets on beta estimates from a prior subperiod. Black, Jensen and Scholes also stressed that mean excess returns on the zero-beta portfolio seemed to be non-stationary over time. Fama and MacBeth (1973) also found substantial variation over time in estimates of market risk premiums.

Unconditional moments can be constant when conditional expectations are changing over time, so the existence of changing expected returns need not invalidate traditional approaches. However, tests using unconditional moments ignore information that may influence expected returns and such tests require a specification of the market portfolio. Consider, for example, a model with constant expected excess returns. If $\mathrm{E}(\tilde{r}_{it}|Z_{t-1}^*) = \mathrm{E}(\tilde{r}_i)$, then ratios of betas are just identified in eq. (4); they must equal ratios of unconditional mean excess returns. Such a condition does not place any restrictions on observable data without the market portfolio. When covariances with a market portfolio are specified, then the cross-sectional relation of unconditional excess returns has empirical content. If conditional expectations of returns in eq. (4) are changing over time, then the CAPM provides a testable restriction (given at least two elements of Z_{t-1}) even without observing the market portfolio. These restrictions form the basis of our tests.[6]

For the remainder of the paper conditional expectations of returns are assumed to be linear in the test information. That is,

$$\tilde{R}_{it} = \delta_i' Z_{t-1} + \tilde{u}_{it}, \qquad \forall t = 1, \ldots, T, \quad i = 0, \ldots, N, \tag{5}$$

[4] In particular, the 'true' market portfolio weights as well as the 'true' betas, may change over time. Conditional on the test information Z_{t-1}, expectations may shift with the mean of the market portfolio and the covariance matrix of returns may change over time. While the ratio of betas given Z_{t-1} is assumed constant, this assumption need not imply constant market weights.

[5] E.g., see Constantinides (1980).

[6] Cheng and Grauer (1980) have developed tests of the CAPM which do not rely on observing the market portfolio. Assuming that market prices are set period by period with the same mean returns and covariances implies that equilibrium asset prices are linearly related through time. This implication is testable using only three assets and three observation periods. Rosenberg and Ohlson (1976) also noted that stationarity of return distributions and portfolio separation place severe restrictions on asset price changes. Unfortunately, neither of these studies distinguish between the information set available to market participants and the information subset used in empirical work. As long as the return distributions given Z_{t-1}^* are changing over time, the restrictions developed in these two studies need not hold.

where

$\tilde{R}_{it}$ $\equiv$ return on asset i in period t;
$\boldsymbol{Z}_{t-1}$ $\equiv$ a vector of L variables available at $t-1$ which are used to form expectations of $\tilde{R}_{it}$;
$\boldsymbol{\delta}_i$ $\equiv$ an L vector of regression coefficients;
$\tilde{u}_{it}$ $\equiv$ forecast error in period t for asset i with the property that $\mathrm{E}(\tilde{u}_{it}|\boldsymbol{Z}_{t-1}) = 0$.

This specification is fairly general in that $\boldsymbol{Z}_{t-1}$ may include any transformation of a predetermined variable which results in a well-specified linear regression model. Furthermore, the statistical model need only apply to the assets under study, not the entire population of assets or the market portfolio.

Note that eq. (5) is not a multivariate analogue to a 'market model' because the regressors $\boldsymbol{Z}_{t-1}$ are not revealed contemporaneously with returns. Rather, $\boldsymbol{Z}_{t-1}$ is a subset of information about the distribution of time t returns that was available when prices were formed at time $t-1$. The elements of $\boldsymbol{Z}_{t-1}$ may be correlated with but are not necessarily equivalent to the state variables in a dynamic asset pricing model.

The linearity of conditional expectations in (5) seems to be a mild condition. For example, stationary joint normality of the returns $\tilde{R}_{it}$ and the information $\boldsymbol{Z}_{t-1}$ guarantees the specification. Multivariate normality of asset returns given available information motivates a derivation of the CAPM. Furthermore, Long (1974) relied on normality to derive his intertemporal model, and instantaneous normality is assumed in the continuous-time asset pricing model [e.g., Merton (1973), Breeden (1979)]. Most importantly, the specification of (5) is testable for a given $\boldsymbol{Z}_{t-1}$ by standard diagnostic tests for a multiple regression model.

To summarize the approach, consider the case of the CAPM. If the riskless rate R_{ft} is a component of $\boldsymbol{Z}_{t-1}$, then (5) implies the expectation model will also hold in excess returns,

$$\tilde{r}_{it} = \hat{\boldsymbol{\delta}}_i'\boldsymbol{Z}_{t-1} + \tilde{u}_{it}, \quad \forall i, t, \tag{6}$$

where the coefficients $\hat{\boldsymbol{\delta}}_i$ differ from the original $\boldsymbol{\delta}_i$ in eq. (5). Taking the conditional expectation of (6) implies

$$\mathrm{E}(\tilde{r}_{it}|\boldsymbol{Z}_{t-1}) = \hat{\boldsymbol{\delta}}_i'\boldsymbol{Z}_{t-1}, \quad \forall i, t. \tag{7}$$

Selecting a 'reference' asset $j = 1$ (without loss of generality if $\beta_1 \neq 0$) and substituting into both sides of (4) from (7) yields

$$(\hat{\boldsymbol{\delta}}_i - (\beta_{im}/\beta_{1m})\hat{\boldsymbol{\delta}}_1)'\boldsymbol{Z}_{t-1} = 0, \quad \text{for all realization of } \boldsymbol{Z}_{t-1} \text{ and } \forall i, \tag{8}$$

$$\Leftrightarrow \hat{\boldsymbol{\delta}}_i = (\beta_{im}/\beta_{1m})\hat{\boldsymbol{\delta}}_1, \quad \forall i. \tag{9}$$

Eqs. (6) and (9) together suggest the following test:

$$\tilde{r}_{it} = \hat{\boldsymbol{\delta}}_i' \boldsymbol{Z}_{t-1} + \tilde{u}_{it}, \quad \forall i = 0,\dots,N \quad \text{and} \quad t = 1,\dots,T,$$

$$\mathrm{H}_0\colon \quad \hat{\boldsymbol{\delta}}_i = c_{i1}\hat{\boldsymbol{\delta}}_1, \quad \forall i \neq 1, \tag{10}$$

$$\mathrm{H}_\mathrm{A}\colon \quad \hat{\boldsymbol{\delta}}_i \neq c_{i1}\hat{\boldsymbol{\delta}}_1.$$

Under the null hypothesis, the restriction (9) reduces the dimension of the parameter space of regression coefficients in (10) from $(N+1)L$ to $L+N$, where L is the number of variables in $\boldsymbol{Z}_{t-1}$.

If a researcher is willing to identify a portfolio as the market portfolio, the CAPM hypothesis implies an additional testable restriction in system (9) by equating the proportionality coefficients c_{i1} with ratios of market betas:

$$\begin{aligned} c_{i1} &= \beta_{im}/\beta_{1m} \\ &= \operatorname{cov}(\tilde{u}_{it}, \tilde{u}_{mt})/\operatorname{cov}(\tilde{u}_{1t}, \tilde{u}_{mt}), \quad \forall i, \end{aligned} \tag{11}$$

where $\tilde{u}_{mt}$ is the unexpected return, given $\boldsymbol{Z}_{t-1}$, on the 'market' index.[7] Restriction (11) may be imposed on the system (10) to test for conditional mean-variance efficiency of a particular portfolio m.

If L, the dimension of the information vector $\boldsymbol{Z}_{t-1}$, equals one, then H_0 in (10) does not provide an over-identifying restriction to test unless the proportionality coefficient c_{i1} is specified as in (11). When L is greater than one, (10) alone implies testable parameter restrictions across the regression equations [although the magnitude of the coefficients c_{i1} $(i \neq 1)$ are unrestricted].

3. Testing multiple-factor asset pricing models

The tests in section 2 may be extended to more general asset pricing models of the form

$$\mathrm{E}(\tilde{R}_{it}|\boldsymbol{Z}_{t-1}) = \mathrm{E}(\tilde{R}_{0t}|\boldsymbol{Z}_{t-1}) + \sum_{h=1}^{K} \beta_{ih}[\mathrm{E}(\tilde{R}_{ht} - \tilde{R}_{0t}|\boldsymbol{Z}_{t-1})], \quad \forall i, \tag{12}$$

where

$\tilde{R}_{ht} \equiv$ return on K 'hedge portfolios' $(h = 1,\dots,K)$ not necessarily observable by the econometrician;

[7] Because the ratios of betas given $\boldsymbol{Z}_{t-1}$ are assumed to be constant in (11), it is easy to show they are equal to the unconditional beta ratios.

β_{ih} ≡ a measure of risk proportional to the conditional covariance between asset i and hedge portfolio h;

$\tilde{R}_{0t}$ ≡ return on a portfolio that is uncorrelated with the hedge portfolios.

Eq. (12) is similar to the Merton (1973) and Cox, Ingersoll and Ross (1978) continuous time models; the Black (1972), Long (1974), Dybvig (1983) and Grinblatt and Titman (1983) discrete time models; and (with the equality holding approximately) the Ross (1976) arbitrage pricing model.[8]

The following proposition derives testable implications of the asset pricing model (12) given the specification of the regression model (5).

Proposition. Given eq. (5) as the model of conditional expectations, choose $K+1$ *observable portfolios whose coefficient vectors* δ_j $(j=0,\dots,K)$ *are not linearly dependent. The asset pricing hypothesis (12) implies the following restrictions on the remaining* $N-K$ *regression coefficients of (5):*

$$\boldsymbol{\delta}_i = \sum_{j=0}^{K} c_{ij}\boldsymbol{\delta}_j, \quad \forall i = K+1,\dots,N, \tag{13}$$

where

$$\sum_{j=0}^{K} c_{ij} = 1, \quad \forall i.$$

Proof. See the appendix.

The proposition shows that the 'K-factor' model (12) may be tested by straightforward generalization of the example in the last section. Eq. (13) provides testable restrictions on the parameters of the multivariate regression (5), provided $N+1$ assets (where $N > K$) and L (where $L > K$) predetermined instruments are available. The $K+1$ hedge portfolios of eq. (12) need not be observed because the regression coefficients of $(N-K)$ assets (i.e., $i = K+1,\dots,N$) are restricted in terms of the first $K+1$ reference assets.

The logic of the test is that the unrestricted regression (5) projects expectations onto a space of dimension L, while the equilibrium model implies that

[8]Note that eq. (12) is stated in terms of expectations and risk measures conditioned on Z_{t-1}, not the market information set Z_{t-1}^*. Such a model follows from the first-order condition of a representative agent's consumption–investment problem:

$$\mathrm{E}\{\tilde{m}_t(1+\tilde{R}_{it})|Z_{t-1}^*\} = 1, \quad \forall i, t,$$

where $\tilde{m}_t$ is a marginal rate of substitution of consumption between times $t-1$ and t. [See Breeden and Litzenberger (1978), Cox, Ingersoll and Ross (1978), Lucas (1978), Grossman and Shiller (1982) and Hansen and Singleton (1983) for examples.] Taking the expected value of the above equation given Z_{t-1} and performing some algebraic substitutions yields a single-consumption-beta model based on a subset of market information [see Hansen, Richard and Singleton (1981)] which may then be expanded to a multiple-beta model like eq. (12). [A demonstration of the latter result is available by request to the authors.]

expected returns are spanned by a basis of dimension $K+1$. Thus, each $\boldsymbol{\delta}_i$ $(i = K+1,\ldots,N)$ must be equal to a linear combination of $\boldsymbol{\delta}_0, \boldsymbol{\delta}_1, \ldots, \boldsymbol{\delta}_K$.

Once an econometrician picks the L elements for $\boldsymbol{Z}_{t-1}$, restriction (13) may be tested for different values of K. This provides a constructive test of asset pricing models in that particular values of K may be rejected in favor of other values.

The suggested methodology assumes that conditional risk measures are constant to focus on the implications of changing expected returns. This represents an intermediate position between using only the unconditional moments and the more difficult task of modeling non-stationarities of both expected returns and covariances. However, theory suggests that risk measures may change as functions of a set of state variables,[9] so the exact relation of a model like (12) with constant risk measures to the underlying theory is not clear. In particular, a test for the number of priced factors K is difficult to interpret because (12) can often be collapsed to a model like that of Breeden (1979), with a single changing consumption beta.[10]

A full treatment of the issues associated with non-stationarity is, of course, beyond the scope of this paper. One interpretation of a changing consumption beta is available, however, if the conditional covariance matrix of asset returns is assumed to be constant.[11] In this case, changes in consumption betas depend on changes in the composition of a 'consumption hedge portfolio'. Assuming a single consumption-beta model and the existence of a portfolio (the consumption hedge portfolio) whose payout is perfectly correlated with the aggregate marginal rate of substitution of consumption over time, it can be shown that the multiple-factor model (12) will hold with K constant risk measures if and only if the consumption hedge portfolio return may be obtained as a (possibly non-constant) combination of a riskless asset and K 'mutual funds' which maintain constant portfolio weights on the risky assets.[12] Because the changing

[9] Breeden (1979) and Cornell (1981) note that consumption betas are non-constant. Cornell emphasized that this instability in beta may make direct tests of Breeden's model difficult to accomplish.

[10] In general, the risk measures of a 'K-factor' model like (12) may also change through time. However, the case where (12) has constant risk measures while the consumption beta is changing is in the spirit of Cornell's (1981) comment.

[11] This assumption is convenient for a particular interpretation, but is not required for the methodology. Risk measures in (12) can be constant without precluding a changing conditional covariance matrix. For example, in Stambaugh's (1983) arbitrage pricing model the relevant risk measures are unconditional second moments, and expectations may change with information. MaCurdy (1981) presents econometric techniques which could be applied to test the restrictions (13) when the conditional covariance matrix of asset returns is changing over time. In the empirical work that follows, more familiar techniques are employed, which do assume a constant conditional covariance matrix.

[12] A proof of this result is available by request to the authors.

consumption beta is a linear combination of the K constant risk measures, β_{ih}, of (12), a rejection of the restrictions (13) can be interpreted as evidence that more than K constant risk measures are required to model the non-stationarity of the consumption beta. Of course, such an interpretation assumes that no violation of the maintained statistical assumptions causes the rejection of (13).

4. Empirical application to daily stock return data

This section illustrates the methodology using daily returns on common stocks for 1962–1980. There are several reasons for selecting daily as opposed to (say) monthly returns. Many financial equilibrium models rely on continuous time [e.g., Merton (1973), Breeden (1979)], so temporal aggregation bias can be reduced with data sampled over short time intervals. Daily data also provide many observations without requiring constant conditional betas over long periods of calendar time. Given the large sample sizes made possible with daily data, test statistics should closely approximate their asymptotic distributions. Therefore, concerns about the finite sample properties of alternative test statistics are not as relevant here as in studies that employ monthly returns. However, methodologies that require a market index present difficulties using daily data. Since market proxies include assets which trade infrequently, reliable measures of covariation between a market return and even a highly traded asset are difficult to obtain. Because the proposed methodology does not require the market portfolio, the infrequent trading problem [e.g., Scholes and Williams (1977)] can be circumvented if the assets under study are frequently traded. The following tests are conducted with the individual stocks of the Dow Jones 30; the trading frequency of these should be quite high. The first column of table 1 provides a list of the firms.

The tests require predetermined variables to model expectations of returns. Fama (1976, ch. 5) has emphasized that most tests involve a joint hypothesis of market efficiency and the conditions which characterize market equilibrium. An historically important example is the assumption that equilibrium prices are set so that expected returns are constant over time: $E(\tilde{R}_{it}|Z_{t-1}^*) = E(\tilde{R}_i)$. In this case, returns are uncorrelated with all predetermined variables and $\boldsymbol{\delta}_i = 0$ in eq. (5) [except for an intercept]. Assuming constant expected returns and finding $\boldsymbol{\delta}_i \neq 0$ in (5) are inconsistent with market efficiency. However, by assuming efficiency, statistical association of returns with a predetermined variable is evidence that expected returns are changing; the present study adopts this view. If the regression model (5) is well-specified, then any predetermined variable can legitimately be used in the tests. The instrumental variable need not represent an information event or a state variable as these are usually thought of; it is sufficient that changes in risk premiums and therefore ex post returns be correlated with the variable. Furthermore, an

Table 1

Results of unrestricted ordinary least squares regressions of daily stock returns for the Dow Jones 30 on a dummy variable for Monday and one lag of the CRSP value-weighted index, 1962–1980. Number of observations is 4595. The multivariate regression model has the form

$$\tilde{R}_{it} = \delta_{i0} + \delta_{iD} D_t + \delta_{iM} R_{Mt-1} + \tilde{u}_{it}, \quad \forall i = 0, \dots, 29, \quad \forall t = 1, \dots, T,$$

where $\tilde{R}_{it} \equiv$ return on asset i for period t, $D_t \equiv 1$ if day t is a Monday and $D_t \equiv 0$ otherwise, and $R_{Mt-1} \equiv$ return on the CRSP value-weighted index for period $t-1$.

Company	$\hat{\delta}_{i0} \times 10^3$	$t(\hat{\delta}_{i0})$	$\hat{\delta}_{iD} \times 10^3$	$t(\hat{\delta}_{iD})$	$\hat{\delta}_{iM}$	$t(\hat{\delta}_{iM})$	R^2	F[a]	$S^2 \times 10^4$[b]	$\hat{\rho}_1$[c]
Allied Chem. Corp.	0.870	3.16	−3.251	−5.20	0.385	11.80	0.03	80.87	2.800	0.00
Alcoa	0.625	2.35	−2.186	−3.62	0.278	8.83	0.02	44.28	2.623	0.07
American Brands	0.694	3.32	−1.418	−2.99	0.231	9.32	0.02	46.84	1.615	0.04
American Can	0.549	2.94	−2.250	−5.30	0.272	12.30	0.04	87.27	1.291	0.05
AT&T	0.493	3.34	−1.581	−4.71	0.120	6.88	0.01	33.53	0.807	0.02
Bethlehem Steel	0.598	2.21	−1.92	−3.13	0.165	5.17	0.01	17.61	2.705	0.06
Chrysler Corp.	0.733	1.78	−2.865	−3.07	0.039	0.80	0.00	4.94	6.260	0.03
Du Pont	0.486	2.29	−1.684	−3.49	0.111	4.42	0.01	15.27	1.671	0.02
Eastman Kodak	0.685	2.77	−1.295	−2.30	0.008	0.26	0.00	2.66	2.270	−0.01
Esmark	1.015	3.50	−2.731	−4.15	0.356	10.38	0.03	60.89	3.107	0.05
Exxon	0.794	4.30	−1.746	−4.17	0.217	9.93	0.02	56.44	1.259	0.07
General Electric	0.776	3.47	−2.692	−5.30	0.219	8.26	0.02	46.46	1.851	0.01
General Foods	0.391	1.76	−1.426	−2.83	0.302	11.51	0.03	69.00	1.819	0.01
General Motors	0.710	3.37	−2.447	−5.12	0.112	4.51	0.01	22.40	1.637	0.03
Goodyear Tire	0.729	2.89	−2.799	−4.89	0.254	8.50	0.02	46.52	2.350	−0.01
Inco	0.493	1.93	−1.822	−3.15	0.236	7.80	0.01	34.48	2.406	0.04
International Harvester	0.602	2.45	−2.156	−3.87	0.312	10.73	0.03	63.46	2.230	0.06
International Paper	0.692	2.64	−1.952	−3.28	0.271	8.72	0.02	42.31	2.540	0.03
Johns Manville	0.523	1.91	−1.651	−2.66	0.383	11.81	0.03	72.12	2.767	−0.00
3M	0.557	2.47	−1.550	−3.03	0.261	9.77	0.02	51.17	1.879	0.02
Owens Ill.	0.575	2.28	−1.907	−3.33	0.272	9.09	0.02	45.67	2.357	−0.01
Procter and Gamble	0.477	2.55	−1.300	−3.06	0.241	10.88	0.03	62.61	1.294	0.02
Sears Roebuck	0.620	2.91	−2.779	−5.74	0.218	8.62	0.02	51.75	1.681	0.02
Standard Oil (Cal.)	0.806	3.51	−1.500	−2.88	0.262	9.65	0.02	49.67	1.947	0.05
Texaco	0.837	3.61	−2.389	−4.54	0.179	6.51	0.01	30.31	1.991	−0.01
Union Carbide	0.424	1.88	−1.237	−2.41	0.287	10.72	0.03	59.38	1.886	0.02
U.S. Steel	0.657	2.55	−2.260	−3.87	0.142	4.65	0.01	17.60	2.452	0.03
United Technologies	0.776	2.57	−1.303	−1.90	0.261	7.28	0.01	27.77	3.382	0.05
Westinghouse	0.649	2.07	−1.282	−1.80	0.217	5.83	0.01	18.20	3.646	0.00
Woolworth	0.763	2.79	−2.740	−4.42	0.310	9.59	0.02	54.10	2.759	0.04

[a] F is the F-statistic for the hypothesis that $\delta_{iM} = \delta_{iD} = 0$ for a particular stock. Except for Eastman Kodak all F-statistics are significant at the five percent level.

[b] S^2 is the variance of regression residuals.

[c] $\hat{\rho}_1$ is the first-order autocorrelation of the residuals.

instrument may be measured with error provided the *measured* value is publicly available information.[13]

Researchers in finance are increasingly sensitive to changing expected returns. Merton (1980), for example, considers several models of changing mean returns. Fama and Schwert (1977) found autocorrelation in bond returns and correlation of stock returns with predetermined measures of expected inflation. Fama (1981, 1984) has presented evidence of predetermined variables that are correlated with stock returns and of predictable variation in Treasury bill risk premiums. Much of the evidence on 'anomalies' and seasonality in security returns is consistent with changing expectations.

Fama (1965) examined daily returns on the Dow Jones 30 common stocks for 1957–1962, finding statistically significant autocorrelations. The sample autocorrelations of the 30 stocks were predominantly of the same sign for a given lag; a pattern Fama suggested could result from autocorrelation of the market index, combined with contemporaneous association of each return with the index. Such behavior is expected if the CAPM holds over time with changing expected returns on the market portfolio.

Fisher (1966) pointed out that infrequent trading can induce spurious negative autocorrelation in the measured returns of individual assets, and positive autocorrelation in measured portfolio returns. If a serial dependence of measured returns is spurious, then the true expected returns may not be changing. However, consistent with results of Fama (1965) and Keim and Stambaugh (1984), the first-order autocorrelations of the individual Dow Jones 30 daily stock returns are predominantly positive. This is consistent with the conclusion of Smirlock and Starks (1983) that non-trading problems are unimportant for the Dow Jones 30.

As suggested by previous empirical evidence, a lagged return of a large portfolio of common stocks serves as one of the predetermined variables.[14] The portfolio contains some stocks which trade infrequently, and its measured return will no doubt contain error. However, since the lagged return is employed as an instrument and not as a 'market portfolio' proxy, such error will not be a problem if the regression model (5) is well-specified.

French (1980) and Gibbons and Hess (1981), among others, have noted that the day of the week helps to predict stock returns. In particular, the evidence suggests that mean stock returns on Monday are not only lower than on other days, but frequently negative.

[13]Given the maintained assumptions of the test if restriction (13) holds for the public information Z_{t-1}, then it must also hold for any subset including the known values of variables measured with error (see footnote 3).

[14]This index is the value-weighted index of common stocks listed on the New York Stock Exchange (provided by the Center for Research in Security Prices [CRSP] of the University of Chicago).

The shift in mean returns on Monday is still a puzzle, despite many recent attempts to explain it. Keim and Stambaugh (1984), for example, conduct tests with different measures of daily returns, including returns based on bid prices of actively traded over-the-counter stocks. They reject exchange specialist-related explanations of the Monday effect as well as other measurement error explanations. Smirlock and Starks (1983) find the Monday effect persists for the Dow Jones 30 whether daily returns are measured from close-to-close or from mid-day prices. Lakonishok and Levy (1982) conclude that adjusting for the settlement period of stock transactions does not explain the low mean returns on Monday.

Furthermore, a Monday effect is observed in other assets. Gibbons and Hess (1981), for example, observed a similar pattern in the Treasury bill market. Persistent negative returns for many assets on Mondays seem inconsistent with the CAPM and risk aversion because it suggests that expected returns on the market portfolio can be negative. Therefore, the use of a Monday effect variable should enhance the power of the tests to reject the 'single-factor' model [$K = 1$ in eq. (12)].[15] The second predetermined variable in the tests that follow is a dummy variable for the day of the week ($D_t = 1$ if the day is Monday, otherwise $D_t = 0$).

For the overall period from August 17, 1962 to December 31, 1980, table 1 shows the results of ordinary least squares (OLS) regression of each of the Dow Jones 30 stocks' daily returns on the lagged stock index ($R_{m,t-1}$), the Monday dummy (D_t), and an intercept. Consistent with findings of previous studies, the table shows that coefficients on the dummy variable for Monday as well as the lag of the CRSP Value-Weighted Index are almost always statistically significant. The t-ratios and the magnitudes of the coefficients are generally smaller for Monday than for the lagged index returns, and the portion of the variation in returns left unexplained is very high. The coefficient of determination is always less than five percent. Given the unpredictable variation usually associated with daily stock returns, small R-squares are to be expected.

However, the statistics in table 1 suggest that Monday and a lag of a market index are useful predictors. The coefficient estimates appear to be quite precise, indicating that tests of constraints on the coefficients would be expected to

[15] If the equilibrium model has $K > 1$, then finding negative conditional expected returns on the market portfolio need not be inconsistent with risk aversion. Consider, for example, a two-factor Merton model with wealth and a single state variable. Using Breeden's (1979) symbology, the expression for the conditional mean excess return on the market is

$$\mu_m - r = (w/T^M)\sigma_m^2 + V_{ms}(C_s/C_w T^M).$$

Monday, of course, cannot be interpreted as a state variable but it is possible that the risk premium for a state variable is correlated with a Monday dummy. If the partial derivative of consumption with respect to the state variable is positive, the partial of consumption with respect to wealth is positive and the covariance of the market return with the state variable is sufficiently negative; then $\mu_m - r < 0$.

Table 2

F-statistics for the significance of the pre-determined variables in forecasting the returns on the individual firms of the Dow Jones 30, 1962–1980. The multivariate regression model has the form

$$\tilde{R}_{it} = \delta_{i0} + \delta_{iD} D_t + \delta_{iM} R_{Mt-1} + \tilde{u}_{it}, \quad \forall i = 0,\dots,29, \quad \forall t = 1,\dots,T,$$

where $\tilde{R}_{it} \equiv$ return on asset i for period t, $D_t \equiv 1$ if day t is a Monday and $D_t \equiv 0$ otherwise, and $R_{Mt-1} \equiv$ return on the CRSP value-weighted index for period $t-1$.

Subperiod	No. of obs.	H_0: $\delta_{0D} = \delta_{1D} = \cdots = \delta_{29D} = 0$ F^b	p^b	H_0: $\delta_{0M} = \delta_{1M} = \cdots = \delta_{29M} = 0$ F^b	p^b	H_0. $\delta_{0D} = \delta_{1D} = \cdots = \delta_{29D} = 0$ $\delta_{0M} = \delta_{1M} = \cdots = \delta_{29M} = 0$ F^b	p^b
8/17/62–3/13/67	1148	1.931	0.0020	5.935	–[a]	5.911	–[a]
3/14/67–11/15/71	1149	2.075	0.0006	8.185	–[a]	8.114	–[a]
11/16/71–6/8/76	1149	2.005	0.0011	8.593	–[a]	8.484	–[a]
6/9/75–12/31/80	1149	1.382	0.0833	8.838	–[a]	8.807	–[a]

[a]The null hypothesis can be rejected at a significance level of less than 0.0001.
[b]F-statistics and p-values.

have some power. Note, however, that the magnitudes of the coefficients appear to differ across securities in a roughly similar fashion for both of the predetermined variables. This is consistent with Fama's (1965) earlier observations and suggests that the behavior of conditional expected returns may reveal a dominant single factor. Table 2 further confirms the importance of the predetermined variables by examining their joint significance across the 30 securities.[16]

So far, only the validity of the statistical model has been examined. Conditional on this specification, implications of the financial model (12) may be tested. Recall that $K+1$ is the number of risk premiums when the 'zero-beta' portfolio is counted. For K greater than two, there are no testable restrictions since there are three predetermined variables in this application.

Eq. (13) (for $K=1$ and $K=2$) provides the null hypotheses which are examined with likelihood ratio statistics. The likelihood ratio compares the generalized variance of the multivariate regression model (5) with and without the restrictions (13). The null hypothesis is rejected if imposing the restriction

[16]In most textbook discussions on testing the general linear hypothesis for a multivariate regression model, the F-test requires multivariate normality, which may not be appropriate for daily data. Statistical inference about regression coefficients is still possible based on asymptotic theory if there are departures from normality. MaCurdy (1981) discusses the mild regularity conditions under which the results in tables 2 and 3 can be justified by quasi-maximum likelihood methods.

Table 3

Likelihood ratio test of the restriction on the coefficients in table 1 implied by a one-factor asset pricing model (plus a zero-beta portfolio) and a two-factor asset pricing model (plus a zero-beta portfolio). The data consist of daily returns on Dow Jones 30, 1962–1980.

Subperiod	No. of obs.	H_0: One-factor model[a] χ^2_{56}	H_0: One-factor model[a] p-value	H_0: Two-factor model[b] χ^2_{27}	H_0: Two-factor model[b] p-value
8/17/62–3/13/67	1148	52.81	0.596	15.20	0.967
3/14/67–[d] 11/15/71	1149	56.19	0.468	18.27	0.895
11/16/71–[d] 6/8/76	1149	56.19	0.468	18.27	0.895
6/9/76–12/31/80	1149	61.93	0.273	–[c]	–[c]
Overall[e]		227.18	0.428	51.74	0.995

[a]The one-factor restriction is that

$$\boldsymbol{\delta}_i = \sum_{j=0}^{1} c_{ij}\boldsymbol{\delta}_j, \quad \forall i = 2,\dots,N, \quad \text{where} \quad \sum_{j=0}^{1} c_{ij} = 1, \quad \forall i,$$

and where $\boldsymbol{\delta}_i$ is the vector of regression coefficients for company i.

[b]The two-factor restriction is that

$$\boldsymbol{\delta}_i = \sum_{j=0}^{2} c_{ij}\boldsymbol{\delta}_j, \quad \forall i = 3,\dots,N, \quad \text{where} \quad \sum_{j=0}^{2} c_{ij} = 1, \quad \forall i,$$

and where $\boldsymbol{\delta}_i$ is the vector of regression coefficients for company i. See the discussion of eq. (13) in the text for an explanation of this restriction.

[c]Program did not converge.

[d]The results for the second and third subperiods would differ slightly if further decimal positions were reported.

[e]The overall results sum up the independent chi-square statistics across the subperiods. The degrees of freedom in this case is the sum of the degrees of freedom in each subperiod.

significantly increases the residual variance of the system. Gibbons (1982) uses a similar approach for a different hypothesis about asset pricing models.

Table 3 summarizes the results of the tests. Estimation without the restriction was accomplished by ordinary least squares while estimation subject to the restriction utilized a modified Gauss–Newton algorithm.[17] The restrictions of a single-factor model with a zero-beta return are rejected with right-tail p-values

[17]All estimation was performed using algorithms supplied by SAS Institute, Inc.

in the 27% to 60% range for each subperiod and for the overall period. Table 3 suggests that the 'two-factor' model leaves the data virtually unrestricted.[18]

These results differ from Gibbons (1982), who employed a likelihood ratio statistic for a different test of the CAPM. There are several possible explanations for this discrepancy. Stambaugh (1982) suggests that the finite sample properties of the likelihood ratio test (LRT) statistic may overstate the rejection of the CAPM reported by Gibbons. Based on large numbers of daily return observations, the p-values for the LRT in table 3 are comparable in magnitude to Stambaugh's results which use the Lagrange multiplier statistic.

However, sample size is not the only rationale for results that differ from Gibbons (1982). Previous tests, including those of Gibbons, assume a specific index is the market portfolio, do not model changing expected returns and employ temporally aggregated returns data (typically using a monthly sampling interval). The CAPM could have been rejected in past work due to one or more of these problems, but the results in table 3 are not affected by these potential misspecifications.

Sample design could be yet another explanation for the discrepancy with Gibbons' (1982) results. For example, expected returns on the Dow Jones 30 stocks may be better explained by a single-factor model than would the returns on a broader sample of assets. Tests of the CAPM have commonly employed portfolio grouping procedures to ensure cross-sectional dispersion of betas. Stambaugh's (1982) results emphasize the importance of dispersion in the test assets in the context of the CAPM; this should be an important consideration for tests of other asset pricing hypotheses as well. One test of the cross-sectional dispersion of expected returns in the sample is to examine the hypothesis that $(\delta_{iD}, \delta_{iM}, \delta_{i0})$ is the same across the assets.[19] If the coefficients are equal, then conditional expected returns on the sample of assets are identical functions of the instruments, and a single-factor model (with identical risk measures) could not be rejected.

The F-test of the null hypothesis that $(\delta_{iD}, \delta_{iM}, \delta_{i0})$ is equal for each of the Dow Jones 30 produced p-values of 0.0051 or less in each subperiod. This suggests that inadequate dispersion within the sample is not a serious problem and reinforces the acceptance of the single-factor model.

As a further check, table 4 reports results of the Gibbons (1982) likelihood ratio test of the Black (1972) CAPM, using daily data and the Dow Jones 30 stocks. This test does not model changes in expected returns and it requires a specification of a market portfolio (in this case, the CRSP value-weighted

[18] In the fourth subperiod the two-factor model did not converge. This was the period in which the tests on the unrestricted regression did not reject the hypothesis that the coefficient of the dummy variable for Monday was equal to zero for all stocks.

[19] We are grateful to Bill Schwert and the referee for suggesting this test.

Table 4

Likelihood ratio rests of the CAPM with a zero-beta asset using the method of Gibbons (1982). The test assets are the Dow Jones 30 and the CRSP value-weighted index is the market portfolio proxy. Daily returns from 1962–1980 are employed. The regression model has the form

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\varepsilon}_{it}, \quad \forall i = 0,\dots,29, \quad \forall t = 1,\dots,T,$$

where $\tilde{R}_{it}$ = return on asset i for period t, and $\tilde{R}_{mt}$ = return on the CRSP value-weighted index for period t. The likelihood ratio tests the hypothesis

$$\alpha_i = \gamma_0(1 - \beta_i), \quad \forall i = 0,\dots,29.$$

Subperiod	No. of obs.	Likelihood ratio test statistic $\chi^2(29)$	Likelihood ratio test statistic p-value
8/17/62–8/13/67	1148	16.07	0.975
3/14/67–11/15/61	1149	20.68	0.871
11/16/71–6/8/76	1149	28.73	0.480
6/9/76–12/31/80	1148	32.14	0.313

[a]The number of observations in the last subperiod is one less than in table 3 because a lead value was created in this data set for purposes of computing Scholes–Williams estimates of betas (see footnote 20).

index).[20] Despite these differences, the results of table 4 reinforce those in table 3. The null hypothesis is not rejected in any subperiod, and the p-values in tables 3 and 4 display a similar pattern across subperiods (largest in the first subperiod and smallest in the fourth). A comparison of the p-values indicates that the present methodology produces a slightly stronger rejection than does the Gibbons approach. This should not be surprising because the restriction of table 4 is an implication of the tests of table 3 when the market portfolio is observed and unconditional means exist.

Compared with other CAPM tests, these results reinforce some of Stambaugh's (1982) earlier conclusions. In particular, the results are consistent with a lack of sensitivity to the specification of the market portfolio and with the hypothesis that the LRT employed in Gibbons's original tests was biased against the null hypothesis in small samples.

[20]Because infrequent trading of some of the stocks in the CRSP index will bias direct estimates of betas, we also conducted a test using a modification of Gibbons' restriction for Scholes–Williams (1977) beta estimates. The results of this test were similar to those reported in table 4.

5. Conclusion

Asset pricing models can be estimated and tested without observing the market portfolio or state variables. Avoiding a specification of these is a by-product of relaxing the assumption that risk premiums are constant. While changing risk premiums does require a model for conditional expected returns, a regression model permits standard specification tests and is robust to missing information. The methodology was applied to daily stock return data, and a single-factor asset pricing hypothesis could not be rejected in any of four equal subperiods from 1962 to 1980. Replicating the tests with Gibbons (1982) methodology produced similar results.

Empirical studies in financial economics have typically studied cross-sectional relations among the unconditional moments of asset returns. While some evidence on securities' time series characteristics exists, there have been few attempts to integrate models of changing expectations with the cross-sectional implications of asset pricing models. Yet applications of modern financial theory use such information to form conditional moments, and the models themselves are usually conceived in these terms. Existing methodology can be refined by focussing on conditional moments of asset returns; the present study is but a first step in this direction. Two examples of potential applications and extensions of this approach are offered to suggest the possibilities.

In the context of the CAPM, systematic risk has typically been measured by an asset's covariation with a proxy for the market portfolio. Roll (1977) pointed out that the ranking of these 'risks' need not be the same as if measured relative to the true market. The suggested methodology avoids this problem by using the behavior of expected returns over time to estimate ratios of betas without observing the market portfolio. These security risk rankings are free from Roll's criticism.

New tests of the Arbitrage Pricing Model [Ross (1976)] or exact factor pricing [Dybvig (1983), Grinblatt and Titman (1983)] could be developed in this framework. The statistical specification is general enough to allow a factor structure for the covariance matrix of unexpected returns. Tests could examine expected returns to see if they are spanned by a basis of a given dimension, corresponding to the factor structure. Since the testable restrictions derive from the dynamic properties of returns and do not depend on explicit identification of the underlying factors, such a methodology may be especially appropriate for this application.

Appendix: Proof of the proposition

To simplify notation, (12) may be written as

$$\mu_{it} = \sum_{h=0}^{K} \beta_{ih}\mu_{ht}, \tag{14}$$

where

$$\beta_{i0} \equiv 1 - \sum_{h=1}^{K} \beta_{ih} \quad \text{and} \quad \mu_{it} \equiv \mathrm{E}(\tilde{R}_{it}|Z_{t-1}).$$

Substitute from eq. (5) for the conditional means in (14) to obtain

$$\left[\boldsymbol{\delta}_i' - \sum_{h=0}^{K} \beta_{ih}\boldsymbol{\delta}_h'\right] Z_{t-1} = 0 \quad \text{for all realizations of } Z_{t-1},$$

$$\Leftrightarrow \boldsymbol{\delta}_i = \sum_{h=0}^{K} \beta_{ih}\boldsymbol{\delta}_h.$$

Writing this in matrix notation,

$$\dot{\boldsymbol{\delta}}_A = B_{AH}\dot{\boldsymbol{\delta}}_H, \tag{15}$$

where $\dot{\boldsymbol{\delta}}_A \equiv \{\boldsymbol{\delta}_i'\}_i$ is an $(N+1)\times L$ matrix of regression coefficients for the assets on the instruments Z_{t-1}, $B_{AH} \equiv \{\beta_{ih}\}_{ih}$ is the $(N+1)\times(K+1)$ matrix of asset risk measures, and $\dot{\boldsymbol{\delta}}_H \equiv \{\boldsymbol{\delta}_h'\}_h$ is a $(K+1)\times L$ matrix of regression coefficients for the unobservable hedge portfolios. Choose $K+1$ observable 'reference' assets $(j=0,\ldots,K)$ and denote the submatrix of $\dot{\boldsymbol{\delta}}_A$ containing their regression coefficients as: $\{\boldsymbol{\delta}_j'\}_j \equiv \dot{\boldsymbol{\delta}}_r = B_{rH}\dot{\boldsymbol{\delta}}_H$, where B_{rH} is the appropriate submatrix of B_{AH}. The reference assets may be chosen in any fashion as long as B_{rH} is non-singular. Premultiplying the last expression by B_{rH}^{-1} and substituting for $\dot{\boldsymbol{\delta}}_H$ into eq. (15) yields

$$\dot{\boldsymbol{\delta}}_A = \left(B_{AH}B_{rH}^{-1}\right)\dot{\boldsymbol{\delta}}_r,$$

or equivalently,

$$\boldsymbol{\delta}_i = \sum_{j=0}^{K} c_{ij}\boldsymbol{\delta}_j, \tag{16}$$

where

$$\{c_{ij}\}_{ij} \equiv \dot{c} \equiv B_{AH}B_{rH}^{-1}. \tag{17}$$

Eq. (16) is identical to eq. (13) in the text. If the relevant hedging portfolios or state variables are specified, then eq. (17) imposes overidentifying restrictions, in addition to those of (16), on the parameters of a regression model like (5) which includes these hedging portfolios. [Eq. (17) is the multifactor extension of eq. (11).] Rewriting the definition from (17) yields

$$\dot{c}B_{rH} = B_{AH}.$$

Since $\sum_{h=0}^{K}\beta_{ih} \equiv 1$ for all i,

$$B_{AH}\mathbf{1} = \mathbf{1},$$

where the vectors of ones, $\mathbf{1}$, are assumed to be of the appropriate lengths.

Thus,

$$B_{AH}\mathbf{1} = \dot{c}(B_{rH}\mathbf{1}) = \dot{c}\mathbf{1} = \mathbf{1},$$

that is,

$$\sum_{j=0}^{K} c_{ij} = 1, \quad \forall i. \qquad \text{Q.E.D.}$$

References

Black, F., 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45, 444–454.

Black, F., M. Jensen and M. Scholes, 1972, The capital asset pricing model: Some empirical tests, in: M. Jensen, ed., Studies in the theory of capital markets (Praeger, New York) 79–121.

Breeden, D., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, Journal of Financial Economics 7, 265–296.

Breeden, D. and R. Litzenberger, 1978, Prices of state contingent claims implicit in options prices, Journal of Business 51, 621–651.

Cheng, P. and R. Grauer, 1980, An alternative test of the capital asset pricing model, American Economic Review 70, 660–671.

Constantinides, G., 1980, Admissible uncertainty in the intertemporal asset pricing model, Journal of Financial Economics 8, 71–86.

Cornell, B., 1981, The consumption based asset pricing model: A note on potential tests and applications, Journal of Financial Economics 9, 103–108.

Cox, J., J. Ingersoll and S. Ross, 1978, A theory of the term structure of interest rates, Unpublished working paper no. 468 (Graduate School of Business, Stanford University, Stanford, CA) forthcoming in Econometrica.

Dybvig, P., An explicit bound on individual assets' deviations from APT pricing in a finite economy, Journal of Financial Economics 12, 438–496.

Fama, E., 1965, The behavior of stock market prices, Journal of Business 38, 34–105.

Fama, E., 1976, Foundations of finance (Basic Books, New York).

Fama, E., 1981, Stock returns, real activity, inflation and money, American Economic Review 71, 545–565.

Fama, E., 1984, Information in the term structure, Journal of Financial Economics 13, 509–528.

Fama, E. and J. MacBeth, 1973, Risk, return and equilibrium: Empirical tests, Journal of Political Economy 81, 607–636.

Fama, E. and G. Schwert, 1977, Asset returns and inflation, Journal of Financial Economics 5, 115–146.

Ferson, W., 1983, Expected real interest rates and consumption in efficient financial markets: Empirical tests, Journal of Financial and Quantitative Analysis 18, 477–498.

Fisher, L., 1966, Some new stock market indices, Journal of Business 29, 191–255.
French, K., 1980, Stock returns and the weekend effect, Journal of Financial Economics 8, 55–70.
Gibbons, M., 1982, Multivariate tests of financial models: A new approach, Journal of Financial Economics 10, 3–27.
Gibbons, M. and W. Ferson, 1984, Tests of asset pricing models with changing expectations and an unobservable market portfolio, Unpublished working paper no. 684R (Graduate School of Business, Stanford University, Stanford, CA).
Gibbons, M. and P. Hess, 1981, Day of the week effects and asset returns, Journal of Business 4, 579–596.
Grinblatt, M. and S. Titman, 1983, Factor pricing in a finite economy, Journal of Financial Economics 12, 497–508.
Grossman, S. and R. Shiller, 1982, Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information, Journal of Financial Economics 10, 195–210.
Hall, R., 1981, Intertemporal substitution in consumption, Unpublished working paper (National Bureau of Economic Research, Stanford, CA).
Hansen, L. and K. Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of stock market returns, Journal of Political Economy 91, 249–265.
Hansen, L., S. Richard and K. Singleton, 1981, Econometric implications of the capital asset pricing model, Unpublished working paper (Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, PA).
Keim, D. and R. Stambaugh, 1984, A further investigation of the weekend effect in stock returns, Journal of Finance 39, 819–834.
Lakonishok, J. and M. Levi, Weekend effects on stock returns: A note, Journal of Finance 37, 883–889.
Lintner, J., 1965, The valuation of risk assets and the selection of risky investment in stock portfolios and capital budgets, Review of Economics and Statistics 47, 13–37.
Long, J., 1974, Stock prices, inflation, and the term structure of interest rates, Journal of Financial Economics 1, 131–170.
Lucas, R., 1978, Asset prices in an exchange economy, Econometrica 46, 1429–1446.
MaCurdy, T., 1981, Asymptotic properties of quasi-maximum likelihood estimators and test statistics, Technical paper no. 14 (National Bureau of Economic Research, Stanford, CA).
Merton, R., 1973, An intertemporal capital asset pricing model, Econometrica 41, 867–887.
Merton, R., 1980, On estimating the expected return on the market: An exploratory investigation, Journal of Financial Economics 8, 323–362.
Roll, R., 1977, A critique of the asset pricing theory's tests – Part 1: On past and potential testability of the theory, Journal of Financial Economics 4, 129–176.
Rosenberg, B. and J. Ohlson, 1976, The stationary distribution of returns and portfolio separation in capital markets: A fundamental contradiction, Journal of Financial and Quantitative Analysis 11, 393–402.
Ross, S., 1976, The arbitrage theory of capital asset pricing, Journal of Economic Theory 13, 341–360.
Rubinstein, M., 1976, The valuation of uncertain income streams and the pricing of options, Bell Journal of Economics and Management Science 7, 407–425.
Scholes, M. and J. Williams, 1977, Estimating betas from nonsynchronous data, Journal of Financial Economics 5, 309–328.
Sharpe, W., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance 19, 425–442.
Smirlock, M. and L. Starks, 1983, Day of the week effects in stock returns: Some intraday evidence, Unpublished working paper (The Wharton School, University of Pennsylvania, Philadelphia, PA).
Stambaugh, R., 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, Journal of Financial Economics 10, 237–268.
Stambaugh, R., 1983, Arbitrage pricing with information, Journal of Financial Economics 12, 357–370.
Zellner, A., 1962, An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias, Journal of the American Statistical Association 57, 348–368.