Testing Portfolio Efficiency with Conditioning Information

Wayne E. Ferson
University of Southern California and NBER
Andrew F. Siegel*
University of Washington

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We develop asset pricing models' implications for portfolio efficiency when there is conditioning information in the form of a set of lagged instruments. A model of expected returns identifies a portfolio that should be minimum variance efficient with respect to the conditioning information. Our framework refines previous tests of portfolio efficiency by using a given set of conditioning information optimally. The optimal use of the lagged variables is economically important. With standard portfolio designs and lagged instruments, by using the instruments optimally we reject several efficiency hypotheses that are not otherwise rejected. The Sharpe ratios of a sample of hedge fund indexes appear consistent with the optimal use of conditioning information.

*Wayne E. Ferson is Professor of Finance at the Marshall School, University of Southern California, 701 Exposition Blvd., Los Angeles, CA 90089-1427, phone (213) 740-5615. Andrew F. Siegel is the Grant I. Butterbaugh Professor of Finance and Business Economics, also of Information Systems and Operations Management, and is Adjunct Professor of Statistics, University of Washington, Box 353200, Seattle, WA. 98195-3200 [phone: (206)543-4476, fax: 685-9392, e-mail asiegel@u.washington.edu]. This paper has benefited from comments of an anonymous referee and from Tom George, Jay Shanken and from workshops at the 2000 Pacific Northwest Finance Conference, the 2001 Western Finance Association, the Thirteenth Annual Conference on Financial Economics and Accounting/Fifth Maryland Finance Symposium (2002), the 2003 American Finance Association and at the following Universities: Arizona, Arizona State, the Copenhagen Business School, CUNY Baruch, INSEAD, Iowa, McGill, New York University, the Norwegian School of Economics, the Norwegian School of Business, Penn State, Toronto, Washington, and Wisconsin.

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We develop asset pricing models' implications for portfolio efficiency when there is conditioning information in the form of a set of lagged instruments. A model of expected returns identifies a portfolio that should be minimum variance efficient with respect to the conditioning information. Our framework refines previous tests of portfolio efficiency by using a given set of conditioning information optimally. The optimal use of the lagged variables is economically important. With standard portfolio designs and lagged instruments, by using the instruments optimally we reject several efficiency hypotheses that are not otherwise rejected. The Sharpe ratios of a sample of hedge fund indexes appear consistent with the optimal use of conditioning information.

Asset pricing models say that particular portfolios are minimum variance efficient, and testing the efficiency of a given portfolio has long been an important topic in empirical asset pricing.[1] Classical efficiency tests ask if a tested portfolio lies "significantly" inside a sample mean variance boundary. These studies form the boundary from fixed-weight combinations of the tested asset returns. However, many studies in asset pricing now use predetermined variables to model conditional expected returns, correlations and volatility, and portfolio weights may be functions of the predetermined variables. This paper develops tests of portfolio efficiency in a conditional setting.

Our contribution is a new framework for testing asset pricing theories in the presence of conditioning information. The framework uses "unconditional" efficiency as defined by Hansen and Richard (1987). An unconditionally efficient portfolio uses the conditioning information in the portfolio weight function, to minimize the unconditional variance of return for its unconditional mean. We refer to this as (minimum variance) *efficiency with respect to the information*, Z. By testing the implications of asset pricing models for efficiency with respect to Z, we use the conditioning information "optimally," allowing for nonlinear functions of the information. Our testing framework has other attractive properties as well.

The basic logic of our approach is as follows. A model of expected returns implies that for the right stochastic discount factor, m, $E(mR|Z) = 1$, where Z are observable lagged

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[1] The Capital Asset Pricing Model (CAPM, Sharpe, 1964) implies that a market portfolio should be mean variance efficient. Multiple-beta asset pricing models such as Merton (1973) imply that a combination of the factor portfolios is minimum variance efficient (Chamberlain, 1983; Grinblatt and Titman, 1987). The consumption CAPM implies that a maximum correlation portfolio for consumption is efficient (Breeden, 1979). More generally, any stochastic discount factor model implies that a maximum correlation portfolio for the stochastic discount factor is minimum variance efficient (e.g., Hansen and Richard, 1987). Classical efficiency tests are studied by Gibbons (1982), Jobson and Korkie (1982), Stambaugh (1982), MacKinlay (1987), Gibbons, Ross and Shanken (1989) and others.

instruments and R is the vector of gross (i.e., one plus the rate of) returns. Any specification for m implies that particular portfolio(s) should be efficient with respect to Z. We test an asset pricing model by testing the efficiency of the indicated portfolio(s) with respect to Z.

Our testing framework involves expanding the mean variance frontier through the use of nonlinear "dynamic strategies," as explained below. Therefore, a central empirical issue is whether such strategies can improve the unconditional Sharpe ratio. For example, some hedge funds claim large Sharpe ratios, and Fung and Hsieh (1997) find that hedge funds follow nonlinear strategies. We find that an equity market neutral hedge fund index delivers an average monthly Sharpe ratio of 0.76 during the 1995-2002 period. Static combinations of the 25 Fama-French portfolios formed on size and book/market can only achieve a (bias adjusted) Sharpe ratio of 0.31. However, by efficiently using standard lagged variables the Sharpe ratio is 1.05. Thus, the economic significance of our approach is potentially large. Hedge funds might appear to expand the mean variance boundary dramatically, but not when the boundary includes the nonlinear lagged variable strategies.

Previous studies also use conditioning information to expand the set of returns. For example, the "factors" or assets' returns may be multiplied by lagged instruments, as in Shanken (1990), Hansen and Jagannathan (1991), Cochrane (1996), Jagannathan and Wang (1996) or Ferson and Schadt (1996). This "multiplicative" approach corresponds to dynamic strategies whose portfolio weights are linear functions of the lagged instruments. However, Ferson and Siegel (2001) show that the portfolio weight functions that maximize the Sharpe ratio are not linear functions. They show (see their Figure 1) that the nonlinearities occur within statistically reasonable limits.

Recent evidence calls into question the usefulness of standard lagged instruments to predict asset returns, once bias and sampling errors are accounted for (e.g. Ghysels (1997), Goyal and Welch (2003, 2004), Simin (2006), Ferson, Sarkissian and Simin, 2003). However, these studies do not use the conditioning information optimally. We find that when similar variables are used in the optimal nonlinear strategy they do have information.

In the standard approach, with N asset returns and L lagged instruments, a $NL \times NL$ covariance matrix must be inverted. With our approach the matrices are $N \times N$, so larger problems with fewer time series can be handled. The main cost is the requirement to model the conditional means and covariance matrix of returns. We evaluate this cost below.

Another advantage of our approach is robustness. Asset pricing tests can be misspecified for various reasons. The econometrician can assume the wrong probability distribution, misspecify the moments of the returns, or the returns can be measured with error. It is well known that mean-variance portfolio solutions are especially sensitive to errors in estimating the mean (e.g. Michaud, 1989). Since mean-variance analysis is the foundation of asset pricing tests, errors in the means are particularly problematic. Our methods should be more robust to these problems than the classical approach.[2]

The rest of the paper is organized as follows. Section 1 further motivates and presents the main ideas. Section 2 develops the tests. The data are described in Section 3 and Section 4 presents the main empirical results. Section 5 concludes the paper.

[2] Abhyankanar, Basu and Stremme (2006) and Chiang (2007) study the out-of-sample performance of the optimal portfolio strategies that form the basis our tests and find that they perform better than the standard mean variance solutions.

1. Asset Pricing, Portfolio Efficiency and Conditioning Information

Most asset pricing models can be represented using the fundamental valuation equation:

$$E\{m_{t+1}R_{t+1}|Z_t\} = \underline{1}, \tag{1}$$

where R_{t+1} is an N-vector of test asset gross returns, Z_t is the *conditioning information*, a vector of observable variables at time t, m_{t+1} is the *stochastic discount factor* (SDF) implied by the model and $\underline{1}$ is an N-vector of ones. A common approach to testing an asset pricing model is to examine necessary conditions of (1). For example, multiplying both sides of Equation (1) by the elements of Z_t and then taking the unconditional expectations leads to a *multiplicative approach*:

$$E\{m_{t+1}(R_{t+1} \otimes Z_t)\} = E\{\underline{1} \otimes Z_t\}. \tag{2}$$

Equation (2) asks the stochastic discount factor to "price" the dynamic strategy payoffs, $R_{t+1} \otimes Z_t$, on average (or "unconditionally"), where $E\{\underline{1} \otimes Z_t\}$ are the average prices. The multiplicative approach captures only a portion of the information in Equation (1). By using "the right" functions of Z_t we can capture more of the information. Of course, if the choice of Z excludes important, unobserved information, this will result in a loss of power. In this paper we take the choice of Z as given.

Equation (1) is equivalent to Equation (3), holding for *all* bounded integrable functions $f(.)$:

$$E\{m_{t+1}[R_{t+1}f(Z_t)]\} = E\{\underline{1}f(Z_t)\}. \tag{3}$$

Equation (2) is a special case of (3), which may be seen by taking $f(Z_t)$ to be each of the instruments in turn and stacking the equations. Thus, Equation (2) asks the stochastic discount factor to price only a subset of the strategies implied by Equation (1) and the asset pricing model. Our tests use the following version of Equation (3):

$$E\{m_{t+1}x'(Z_t)R_{t+1}\} = \underline{1} \quad \forall x(Z_t): x'(Z_t)\underline{1} = 1. \tag{4}$$

Equation (4) uses all portfolio weight functions $x(Z)$ in place of the general functions in Equation (3), subject only to the restrictions that the weights are bounded integrable functions that sum to 1.0.[3]

By using all portfolio weights in Equation (4), our approach rejects asset pricing models that previous methods would not reject. While many asset pricing models are rejected in the literature, Lewellen, Nagel and Shanken (2007) argue that it may be too easy to find models that appear to "explain" some returns, like those of the Fama-French portfolios. Our approach appears to be powerful in that setting.

Ferson and Siegel (2001) provide the expressions from which we construct the tests. These describe the efficient frontier of all portfolio weight functions. The optimal weight function minimizes the unconditional variance of $x'(Z_t)R_{t+1}$ for its unconditional mean, μ_p,

[3] Equation (4) follows by multiplying (1) by the elements of the portfolio weight vector $x(Z)$ and summing, using the fact that the weights sum to 1.0, then taking the unconditional expectation. Because of the portfolio weight restriction, Equation (4) is an implication of but is not equivalent to (3). Equation (4) retains the dynamic asset allocation decisions allowed by (3)—moving funds from one asset to another based on conditioning information—but leaves out the opportunity to save more or less, altering the overall scale of the investment based on conditioning information. In equation (3), since both sides of the equation may be arbitrarily scaled by a constant, the unconditional expectation of the portfolio weights sum to 1.0 (Abhyankar, Basu and Stremme, 2006). Restricting to weights that almost always sum to 1.0 in Equation (4) allows us to work with portfolio returns and portfolio efficiency, as opposed to asset prices and payoffs. Working with prices and payoffs, it would be necessary in any event, to normalize the prices to achieve stationarity for empirical work.

over the functions $x(Z)$. The solution for the weights on the risky assets, in the presence of a risk free asset with return R_f, is given by Ferson and Siegel (2001) as: [4]

$$x(Z)' = \frac{\mu_p - R_f}{\zeta}[\mu(Z) - R_f \underline{1}]'\, Q, \tag{5}$$

where $$Q = \left[(\mu(Z) - R_f \underline{1})(\mu(Z) - R_f \underline{1})' + \Sigma(Z)\right]^{-1},$$

and $$\zeta = E\{\mu(Z) - R_f \underline{1})' Q(\mu(Z) - R_f \underline{1})\},$$

and $\underline{1}$ is an N-vector of ones. We posit a parametric model for the conditional mean vector, $\mu(Z_t)$, and the conditional covariance matrix, $\Sigma(Z_t)$. Note that even if the conditional mean function is linear in Z, the optimal weight is nonlinear.

Ferson and Siegel (2001) study the shape of the optimal weight function of Equation (5). They show that the portfolios are likely to be robust to extreme observations, because the nonlinear shape makes them conservative in the face of extreme realizations of Z_t. Ferson and Siegel (2003) apply the expressions to the Hansen-Jagannathan (1991) bounds and find robustness in that setting. Ferson, Siegel and Xu (2006) study modifications of the solutions to compute maximum correlation portfolios and find evidence of robustness. Bekaert and Liu (2004) argue that an approach like ours is inherently robust to misspecifying the conditional moments of returns. The intuition is that with the wrong moments $\mu(Z_t)$ and $\Sigma(Z_t)$, using the expression for the "optimal" $x(Z)$ is suboptimal. However, the solution still describes a valid portfolio strategy. The strategy will no longer expand the boundary to the maximum possible

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[4] Equation (5) applies when there is a fixed risk-free rate or a time-varying, conditionally risk-free rate. We have experimented with each interpretation and find that in our sample of returns and monthly Treasury bills, the two interpretations are virtually empirically indistinguishable.

extent. Thus the tests may sacrifice power, but remain valid with misspecified conditional moments. The key to obtaining the advantages of our approach is the relation of Equation (4) to minimum variance efficient portfolios.

1.1 Portfolio Efficiency with Respect to Conditioning Information

We first formally define efficiency with respect to the information, Z_t. Consider the set of all portfolios of the *N* test assets R_{t+1}, where the weights $x(Z_t)$ that determine the portfolio at time *t* are functions of the given information Z_t. The restrictions on the portfolio weight function are that the weights must sum to 1.0 (almost surely in Z_t), and that the expected value and second moments of the portfolio return are well defined. This set of portfolio returns determines a mean-standard deviation frontier, as shown by Hansen and Richard (1987). This frontier depicts the *unconditional* means versus the *unconditional* standard deviations of the portfolio returns. A portfolio is defined to be efficient with respect to the information Z_t, when it is on this mean standard deviation frontier.

Proposition 1. *(Hansen and Richard, 1987, Corollary 3.1) Given* N *test asset gross returns,* R_{t+1}, *a given portfolio with gross return* $R_{p,t+1}$ *is* ***minimum-variance efficient with respect to the information*** $\mathbf{Z_t}$ *if and only if Equation (6) (equivalently, Equation 7) is satisfied for all* $x(Z_t)$: $x'(Z_t)\underline{1} = 1$ *almost surely, where the relevant unconditional moments exist and are finite:*

$$Var\left(R_{p,t+1}\right) \le Var\left[x'\left(Z_t\right)R_{t+1}\right] \quad \text{if} \quad E\left(R_{p,t+1}\right) = E\left[x'\left(Z_t\right)R_{t+1}\right] \tag{6}$$

$$E\left[x'\left(Z_t\right)R_{t+1}\right] = \gamma_0 + \gamma_1 Cov\left[x'\left(Z_t\right)R_{t+1}; R_{p,t+1}\right]. \tag{7}$$

Equation (6) is the *definition* of minimum variance efficiency with respect to *Z*. It states that $R_{p,t+1}$ is on the minimum variance boundary formed by all possible portfolios that use the test assets and the conditioning information. Equation (7) states that the familiar expected return - covariance relation from Fama (1973) and Roll (1977) must hold using efficient–with-respect-to-*Z* portfolios. The expected returns on all the portfolio strategies are linear functions of their unconditional covariances with $R_{p,t+1}$. In Equation (7), the coefficients γ_0 and γ_1 are fixed scalars that do not depend on the functions $x(.)$ or the realizations of Z_t.

1.2 Asset Pricing Models and Efficiency with Respect to Information

Most asset pricing models specify a stochastic discount factor. In particular, linear factor models say that *m* is linear in one or more factors. Proposition 2 describes the simplest case of our framework, showing that when there is conditioning information, testing linear factor models amounts to testing for the efficiency of a portfolio of the factors with respect to the information.

Proposition 2. *Given* $\{R_{t+1}, Z_t\}$ *and a stochastic discount factor* m_{t+1} *such that Equation (4) holds, then if* $m_{t+1} = A + B'R_{B,t+1}$ *, where* $R_{B,t+1}$ *is a k-vector of benchmark factor returns, and* A *and* B *are a constant and a fixed* k-*vector, there exists a portfolio,* $R_{p,t+1} = w'R_{B,t+1}$, $w'\underline{1} = 1$*, where* $w \equiv B/(\underline{1}'B)$ *is a constant* k-*vector, and* $R_{p,t+1}$ *is efficient with respect to the information* Z_t.

<u>Proof</u>: See the Appendix for all proofs.

The intuition of Proposition 2 is the same as the classical case with no conditioning information, as the proof in the appendix illustrates. The difference is that in our framework the set of returns is expanded to all $x'(Z)R$.

We are interested in general stochastic discount factors, $m(X,\theta)$, where X is observable data and θ is a vector of parameters. We also wish to allow for time-varying weights in the efficient portfolio. This requires the definition of portfolios that are *maximum correlation with respect to Z.*

Definition. *A portfolio* R_P *is* ***maximum correlation for a random variable,*** **m*****, with respect to conditioning information Z****, iff:*

$$\rho^2\left(R_P,m\right)\geq\rho^2\left[x'(Z)R,m\right] \quad \forall x(Z):x'(Z)\underline{1}=1, \tag{8}$$

where $\rho^2(.,.)$ *is the squared unconditional correlation coefficient and we restrict to functions x for which the correlation exists.*

Proposition 3. *If a given* m *satisfies Equation (4), then a portfolio* R_P *that is maximum correlation for* m *with respect to* Z *must be minimum variance efficient with respect to* Z.

Proposition 2 is clearly a special case of Proposition 3, because a linear regression maximizes the squared correlation. If m_{t+1} is linear in $R_{B,t+1}$, a linear regression holds with no error. We use Proposition 3 in our tests as follows. Given a stochastic discount factor, m, we test the model by constructing a portfolio that is maximum correlation for this m with respect to Z, and we then test the implication that the portfolio is efficient with respect to Z.[5]

[5] To construct the maximum correlation portfolio for m with respect to Z, we form the portfolio weights using Equation (6) and the Corollary to Proposition 2 in Ferson, Siegel and Xu (2006).

With the preceding results we can consider a case where the model implies a stochastic discount factor that is linear in *k* factor-portfolios, allowing for time-varying weights.

Corollary. *Given* $\{R_{t+1}, Z_t\}$ *and a stochastic discount factor* m_{t+1} *such that Equation (4) holds, then if a maximum correlation portfolio for* m_{t+1} *with respect to* Z_t *has nonzero weights only on the* k*-vector of benchmark factor returns* $R_{B,t+1}$*, (a subset of* R_{t+1}*), then this portfolio is efficient with respect to Z, both in the full set of test asset returns and in the benchmark returns.*

The situation described in the Corollary is a "dynamic" version of mean variance "intersection," as developed by Huberman, Kandel and Stambaugh (1987). The Corollary follows because the factor portfolio in question satisfies the condition of Proposition 3, and so is efficient with respect to *Z,* in both the full set and the subset of assets. Thus, the full set and subset minimum variance boundaries must touch at the point defined by the maximum correlation portfolio. The Corollary does not say that *all* efficient combinations of the factor returns are efficient in the full set of returns. Other points on the subset boundary may be inside the full set boundary.

1.3 Conditional Efficiency

Previous studies test *conditional* efficiency given *Z*, where efficiency is defined in terms of the conditional means and variances.[6] Tests of conditional efficiency given *Z* may be handled

[6] Hansen and Hodrick (1983) and Gibbons and Ferson (1985) test versions of conditional efficiency given *Z*, assuming constant conditional betas. Campbell (1987) and Harvey (1989) test conditional efficiency restricting the form of a market price of risk, and Shanken (1990) and Ferson and Schadt (1996) restrict the form of time-varying conditional betas.

as a special case of our approach. If there is a (possibly, time-varying) combination of the k benchmark returns, R_B, that is conditionally efficient, there is an SDF, $m = A(Z) + B(Z)'R_B$. The coefficients are: $A(Z) = R_f^{-1} - E(R_B \mid Z)' Cov(R_B \mid Z)^{-1} [\underline{1} - R_f^{-1} E(R_B \mid Z)]$ and $B(Z) = Cov(R_B \mid Z)^{-1} [\underline{1} - R_f^{-1} E(R_B \mid Z)]$. When $k = 1$ we have a single-factor model, as in the conditional CAPM. (See Ferson and Jagannathan, 1996.) We test conditional efficiency by constructing the maximum correlation portfolio for the indicated m with respect to Z. This portfolio, call it R_p^*, should be efficient with respect to Z. Note that R_p^* will be different from R_B when the coefficients $A(Z)$ or $B(Z)$ are time varying functions of Z. Thus, for example, the conditional CAPM does not imply that the market portfolio is efficient with respect to Z. However, the conditional CAPM does identify a portfolio of the test assets that should be efficient with respect to Z, and this can be tested using our approach.

If we reject conditional efficiency, then we reject dynamic intersection a fortiori. This follows from the Hansen and Richard (1987) result that efficient-with-respect-to Z portfolios must be conditionally efficient. If there is no combination of the benchmark returns that is conditionally efficient, then no combination can be efficient with respect to Z, so there can be no dynamic intersection.

2. Testing Efficiency

Classical tests of efficiency involve restrictions on the intercepts of a system of time-series regressions. If r_t is the vector of N *excess* returns at time t, measured in excess of a risk-free or zero-beta return, and $r_{p,t}$ is the excess return on the tested portfolio, the regression is:

$$r_t = \alpha + \beta r_{p,t} + u_t; \quad t = 1,...,T, \tag{9}$$

where T is the number of time-series observations, β is the N-vector of betas and α is the N-vector of alphas. The portfolio r_p is minimum-variance efficient and has the given zero-beta return only if α =0.

Classical test statistics for the hypothesis that α =0 can be written in terms of squared Sharpe ratios (e.g., Jobson and Korkie, 1982). Consider the simplest case of the Wald Statistic:

$$W = T\hat{\alpha}'[Cov(\hat{\alpha})]^{-1}\hat{\alpha} = T\left(\frac{\hat{S}^2(r) - \hat{S}^2(r_p)}{1 + \hat{S}^2(r_p)}\right) \overset{a}{\sim} \chi^2(N) \tag{10}$$

where $\hat{\alpha}$ is the OLS or ML estimator of α and $\sqrt{T}\left(\hat{\alpha} - \alpha\right)$converges to a normal random vector with mean zero and covariance matrix, $Cov(\hat{\alpha})$. The term $\hat{S}^2\left(r_p\right)$ is the sample value of the squared Sharpe ratio of r_p, defined by $S^2\left(r_p\right) = \left[E\left(r_p\right)/\sigma\left(r_p\right)\right]^2$. The term $\hat{S}^2\left(r\right)$ is the sample value of the maximum squared Sharpe ratio that can be obtained by portfolios of the assets in r (including r_p):

$$S^2(r) = \max_x \left\{\frac{[E(x'r)]^2}{Var(x'r)}\right\}. \tag{11}$$

The Wald statistic has an asymptotic chi-squared distribution with N degrees of freedom under the null hypothesis[7].

Classical tests ignoring conditioning information restrict the maximization of Equation (11) to *fixed-weight* portfolios, where x is a constant. Efficient portfolios with respect to the information Z maximize the squared Sharpe ratio over *all portfolio weight functions*, $x(Z)$.

2.1 Empirical Strategy

We compare the classical approach with no conditioning information, the multiplicative approach, and the efficient use of the information. When we test the efficiency of a given portfolio, R_p, then $\hat{S}^2(R_p)$ in the test statistic is formed using the normal maximum likelihood estimators of the mean and variance. We use the one-month US Treasury bill return as the risk-free or zero-beta rate.

The squared Sharpe ratio of the boundary portfolio, $\hat{S}^2(R)$, differs according to the way conditioning information is used. When there is no conditioning information we use the fixed-weight solution to (11) evaluated at the maximum likelihood estimates. For the multiplicative approach we use the returns $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$, where R_{ft} is the one-month Treasury bill return for month t. We then proceed as in the fixed-weight case with the returns $\hat{R}_t$ in place of R_t. When the information is used efficiently, $\hat{S}^2(R)$ is formed using the

[7] When the Wald statistic in (10) is multiplied by $\left(\frac{T-N-1}{N(T-2)}\right)$, the result has an exact F distribution in finite samples under the assumption that the (r_t, r_{pt}) in (9) are normally distributed (e.g., Gibbons, Ross and Shanken, (1989).

sample mean and variance of $\hat{x}'(Z)R$ where $\hat{x}(Z)$ is the sample version of the efficient-with-respect-to-Z portfolio weight.

The solution for $\hat{x}(Z)$ is a function of the assumed parametric models for $\mu(Z_t) = E(R_{t+1}|Z_t)$ and $\Sigma(Z_t) = var(R_{t+1}|Z_t)$. In the simplest examples $\mu(Z_t)$ is the linear regression function for the returns on Z and $\Sigma(Z_t)$ is the covariance matrix of the residuals, which is held fixed over time. We also consider models with time-varying $\Sigma(Z_t)$.[8]

We evaluate the tests using simulations. To generate data consistent with the null hypothesis that a portfolio is efficient, we either restrict the return generating process to guarantee the portfolio's efficiency, or we replace its return with a portfolio that is efficient, based on the specification of the asset-return moments. We construct the null distribution of the test statistic by using the artificial data in the same way that we use the actual data to get the sample value of the statistic. The details are discussed in the Appendix.

We conduct experiments to assess the accuracy of our empirical *p*-values. With no lagged instruments and normality the exact finite sample *p*-values are known from the *F* distribution (footnote 7). We generate a random sample of normal returns from a population with mean and covariance matrix equal to our ML sample estimates. The tested portfolio, which is the SP500, is not efficient in this sample and the exact *p*-value from the *F* distribution is taken to be the "correct" *p*-value. We check whether our simulations generate similar *p*-values. We replace the SP500 with the portfolio that is efficient given the moments that generate the normal sample,

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[8] Previous approaches to conditional asset pricing directly specify ad-hoc functional forms for $x(Z)$. We use the optimal strategies, which are given functions of $\mu(Z)$ and $\Sigma(Z)$, but we must specify these functions. Pushing the selection of the functional forms closer to the data is an improvement because the functional forms of asset return moments can be evaluated independently of portfolio performance.

resample to generate 1,000 artificial samples, and compute the test statistic on each sample. The empirical *p*-value is the fraction of these 1,000 trials in which the simulated test statistic exceeds the value computed on the original normal sample. Averaging across 100 normal samples, we find that the empirical *p*-values and the GRS *p*-values are similar.[9] While similar p-values do not rule out the possibility that both approaches are inaccurate, we take some comfort from the similarity.

3. The Data

We use a standard set of lagged variables to model the conditioning information. These include: (1) the lagged value of a one-month Treasury bill yield (see Fama and Schwert, 1977); (2) the dividend yield of the market index (see Fama and French, 1988); (3) the spread between Moody's Baa and Aaa corporate bond yields (see Keim and Stambaugh, (1986) or Fama, 1990); (4) the spread between ten-year and one-year constant maturity Treasury bond

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[9] The *p*-values for 100 normal samples are summarized below:

	industries 1963-94	industries 1963-72	industries 1973-82	industries 1983-92	Size/BM 1963-94
Avg. GRS	0.042	0.021	0.055	0.031	0.000
Avg. empirical	0.041	0.039	0.082	0.054	0.000
Mean |GRS-empirical|	0.005	0.018	0.027	0.023	1.08E-5
Std (GRS-empirical)	0.008	0.020	0.025	0.024	9.87E-5

The standard deviation of a correct empirical p-value of 5%, when 1,000 trials are used in its construction, is $[.95(.05)/1000]^{1/2} = 0.007$. The standard deviation of the difference between two such *p*-values, assuming independent trials, is 0.010.

yields (see Fama and French, 1989) and (5); the difference between the one-month lagged returns of a three-month and a one-month Treasury bill (see Campbell, 1987).

We use two standard methods of grouping monthly common stock returns into portfolios. Twenty five value-weighted industry portfolios (from Harvey and Kirby, 1996) are used for the period February, 1963 to December, 1994.[10] Table 1 shows the SIC industry classifications for the 25 portfolios, and summary statistics of the returns. The second grouping follows Fama and French (1996). Stocks are placed into five groups according to their prior equity market capitalization, and independently into five groups on the basis of their ratios of book value to market value of equity per share. These are the same 25 portfolios used by Ferson and Harvey (1999), who provide details and summary statistics.

This project has matured over a length of time, providing the opportunity to investigate the results over a "hold-out" sample period, January, 1995 through December, 2002. We use 25 size x book-to-market and Industry portfolios from Kenneth French and update the other series with fresh data.[11] The hold-out sample results are interesting in view of recent evidence, cited above, that some of the lagged instruments may have lost their predictive power for stock returns since the 1990s. Table 1 reports the adjusted R-squares from regressing the industry returns on the lagged instruments over the 1963-1994 period and the 1995-2002 sample. The R-squares are substantially lower in the more recent period.

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[10] We are grateful to Campbell Harvey for providing these data.

[11] We use a subset of the 48 value-weighted industry portfolios provided by French to match the definitions in Table 1. We confirm that the matched industry returns produce similar summary statistics and regression R-squares on the lagged instruments as our original data, over the 1963-1994 period.

Regressions on the 25 size and book-to-market portfolios produces a similar result. The average adjusted R-squared over 1963-1994 is 10.5%, while over 1995-2002 it is only 1.4%.

4. Empirical Results

4.1 Inefficiency of the SP500 Relative to Industry Portfolios

Table 2 summarizes results for the 25 industry portfolios, where the tested portfolio, R_p, is the SP500. In Panel A there is no conditioning information. Substituting the maximum likelihood estimates of $\hat{S}^2(R_p)$ and $\hat{S}^2(R)$ into (10) gives the sample value of the test statistic. Referring to the asymptotic distribution, the right-tail p-value is 0.48 for 1964-94 and $0.14 - 0.39$ in the ten-year subperiods. These tests produce little evidence to reject the null hypothesis. During 1995-2002 the maximum Sharpe ratio is substantially higher, and so is the value of the test statistic: The asymptotic p-value is 0.001.

Panel A of Table 2 also reports 5% critical values and empirical p-values based on Monte Carlo simulation assuming normality, and based on a resampling approach that does not assume normality. In addition, we report p-values from the exact F statistic, under the assumption of normality. Consistent with Gibbons, Ross and Shanken (GRS, 1989) the Wald Test rejects too often when the asymptotic distribution is used, and when we correct for finite sample bias using simulation we find no evidence against the efficiency of the market index in the industry portfolios, at least when the conditioning information is ignored. The Monte Carlo and bootstrapped p-values are close to each other in every subperiod, suggesting that the departure from normality of the monthly industry returns is not severe. The p-values from the F distribution are also similar to the empirical p-values.

Panel B of Table 2 summarizes tests using the "multiplicative" returns, $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$. With 25 industry portfolios, the market return and five instruments plus a constant (L=6), there are 156 "returns." One disadvantage of the multiplicative approach is that the size of the system quickly becomes unwieldy. It is not possible to construct the Wald Test for the ten year subperiods, as the sample covariance matrix is singular.

Over the 1963-94 period the value of the Wald Test statistic using the multiplicative returns is 348.6. The asymptotic *p*-value is close to zero. However, we expect a finite-sample bias and the simulations confirm the bias. The empirical *p*-values reject efficiency at either the 2% (Monte Carlo) or 44% (resampling) levels. The Gibbons-Ross-Shanken *p*-value assuming normality is 3%. Thus, the results in the multiplicative case are sensitive to the data generating process. This makes sense, because even if R_t is approximately normal, the products of returns and the elements of Z_{t-1} are not normal. We therefore place more trust in the resampling results. Using the resampling scheme we find no evidence to reject the efficiency of the market index with the multiplicative approach.

Panel C uses the conditioning information Z optimally. With this approach results for the subperiods can be obtained. The empirical *p*-values are 0.5% or less in the full sample and each ten-year subperiod, and 4.4% or less during 1995-2002. Thus, we reject the hypothesis that the SP500 is efficient when using the conditioning information optimally. We even find marginal rejections during the 1995-2002 sample period, where Table 1 illustrates that the predictive power of the lagged instruments is low. The results based on the efficient-with-respect-to-Z frontier are also fairly robust to the method of simulation (Monte Carlo or resampling). This makes sense in view of the "robust" behavior of the portfolio weights that define the efficient-with-respect-to-Z boundary, as described by Ferson and Siegel (2001).

4.2 Size and Book/Market Porfolios

Studies that use portfolios grouped on firm size and book-to-market ratios find that a market index is not efficient (e.g. Fama and French, 1992). Table 3 presents results for these portfolios. In panel A there is no conditioning information. Consistent with previous studies, efficiency is rejected for the 1963-1994 period. However, in the 1995-2002 period, the efficiency of the market index is not rejected when the finite sample bias in the statistics is corrected. This is consistent with a weakening of the size and book-to-market effects during 1995-2002. In panel B the test assets are the multiplicative returns. The empirical p-value based on resampling is marginal, at 4%, over the 1963-1994 sample.

In panel C the test assets are all portfolios $x'\left(Z_{t-1}\right)R_t$. The resampling p-values are 0.3% or less, including the 1995-2002 subsample. Thus, efficiency can be rejected with our approach. Expanding the set of portfolio returns with the optimal nonlinear strategies changes the results, even in the size × book-to-market portfolio design.

4.3 Expanding the Mean Variance Boundary

The above evidence shows that the market index return lies "significantly" inside the mean-variance boundaries when the conditioning information is used optimally, but does not address directly the relations between the boundaries formed with versus without the information. Table 4 examines whether the use of conditioning information expands the mean variance boundary. For these tests we replace the market index with a portfolio of the test assets whose weights are proportional to $\Sigma^{-1}\mu$, where Σ is the unconditional covariance matrix and μ is the unconditional mean of the excess returns. This is a portfolio on the simulations' "population" mean-variance boundary with no conditioning information. We then test for the efficiency of this portfolio instead of the SP500. In panel A we use the multiplicative approach to expanding the boundary. The empirical *p*-values are 0.464 and 0.686, thus providing no evidence that the multiplicative approach expands the boundary.[12]

In Panel B of Table 4 the test assets are all portfolios $x'(Z_{t-1})R_t$. In the 1963-94 period the empirical *p*-values are 0.1% and 2.5%, showing that when the conditioning information is used optimally the mean variance boundary is expanded. However, during 1995-2002 we do not reject the null hypothesis. This is the weakest showing that our new approach makes. Note that the critical values are not drastically higher, but the sample values of the test statistic are lower during 1995-2002. This reflects the low explanatory power of the lagged variables during this period, as indicated in Table 1. The statistical noise involved in estimating the maximum Sharpe ratio for the 26 test assets in this experiment differs from that involving a smaller number of factors, so we may find rejections of other hypotheses during the 1995-2002 period.

4.4 Testing Static Combinations of the Fama-French Factors

The null hypothesis may be stated as $m = a + b_1R_m + b_2R_{HML} + b_3R_{SMB}$, where the coefficients are fixed over time. R_m is the gross return of the market index. R_{HML} is the one-month

[12] Using a fixed risk-free rate these tests may sacrifice power, as there may be other regions, corresponding to other values of the zero-beta rate, where the two boundaries are reliably distinct.

Treasury bill gross return plus the excess return of high book-to-market over low book-to-market stocks, and R_{SMB} is similarly constructed using small and large market-capitalization stocks. We replace the first and last portfolios in the industry or size × book-to-market design with the returns R_{HML} and R_{SMB}, to insure that the factor portfolios are a subset of the tested portfolio returns.

Table 5 presents the tests. Without conditioning information the only rejections occur for the industry portfolios. The GRS and empirical *p*-values produce similar conclusions. Fama and French (1997) also find that their factors don't explain average industry returns very well. In Panel B the multiplicative approach is used, and the empirical *p*-values strongly reject the model for 1963-94. This is consistent with studies such as Ferson and Harvey (1999) who find that the Fama-French factors do not explain time-varying expected returns over a similar sample period. We cannot examine the multiplicative approach over the holdout sample because the covariance matrices are too large to invert.

Panel C of Table 5 presents the tests relative to the efficient-with-respect-to-*Z* frontier. The tests confirm the value of using the conditioning information optimally. We observe strong rejections, both over 1963-1994 and in the 1995-2002 sample, and for both portfolio designs.

4.5 Testing Time-Varying Combinations of Factors

In Table 6 we use our framework to test the conditional efficiency of the market index (Panel A) and of time-varying combinations of the three Fama-French factors (Panel B). We reject both models over 1963-1994 in both portfolio designs. The bootstrapped *p*-values are 1.2% or less. We also reject both models in the 1995-2002 sample period with *p*-values of 1.8% or less. Thus, when the conditioning information is used optimally our tests strongly reject conditional versions of both the CAPM and the Fama-French three-factor model. If no time-varying combination of the factors is conditionally efficient, then no time-varying combination can be efficient with respect to Z. Thus, Table 6 rejects dynamic intersection a fortiori.[13]

4.6 A Hedge Fund Example

This section fleshes out the hedge fund example from the introduction. We use monthly returns for six hedge fund indexes from Credit Suisse/Tremont for the 1995-2002 period. Panel A of Table 7 presents the Sharpe ratios, which vary from -0.05 to 0.76 across the fund styles.

Fixed-weight combinations of the Fama-French portfolios and the CRSP value-weighted market index produce a Sharpe ratio of only 0.72 (Panel B). Using industry portfolios and the market, the maximum Sharpe ratio is 0.76 (Panel C). Sample Sharpe ratios are known to be biased when N is large relative to T. Using the correction in Ferson and Siegel (2003)[14] the Sharpe ratios are 0.31 and 0.38. The hedge funds appear to offer impressive Sharpe ratios in comparison. However, using the efficient-with-respect-to Z portfolio weights the Sharpe ratios are 1.39 and 1.36 before adjustment, and 1.05 and 1.02 after adjustment.

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[13] We explicitly test dynamic intersection and confirm that it is rejected in all the sample periods and portfolio designs.

[14] If the unadjusted squared Sharpe ratio is S, the adjusted squared ratio is $S(T - N - 2)/T - N/T$.

We test the null hypothesis that the hedge fund indexes offer no expansion of the mean variance opportunity set. This says that the alphas in regression (9) are jointly zero when $r_{p,t}$ is the efficient portfolio formed from the test assets, excluding the hedge funds. The test statistic is Equation (10), which now compares the maximum Sharpe ratios attainable with versus without the hedge funds. We strongly reject the null hypothesis using the fixed-weight benchmark, with empirical *p*-values of 1.2% or less. Using the lagged variables optimally the *p*-values range from 4.9% to 17.5%. Thus, while the hedge funds do expand the fixed-weight boundary, the tests do not reject the hypothesis that the hedge fund returns could have been generated with nonlinear strategies based on the lagged instruments.

5. Conclusions

We develop a framework for testing asset pricing models in the presence of lagged conditioning information. Our tests examine the (unconditional) minimum variance efficiency of a portfolio with respect to the conditioning information, a version of efficiency introduced by Hansen and Richard (1987). Asset pricing models identify portfolios that should be efficient with respect to the conditioning information, and by testing the efficiency of the portfolio, we test the asset pricing model. We illustrate the approach with versions of the Capital Asset Pricing model and the Fama-French (1996) factors.

Using a standard set of lagged instruments and test portfolios, the efficiency of all time-varying combinations of the Fama-French factors is rejected. In the same setting, the commonly-used "multiplicative" approach to conditioning information does not significantly expand the mean variance boundary, nor can it reject all the models. The predictive power of the lagged variables declines after 1995, but even during this period the optimal use of these variables is economically and statistically significant.

Our paper suggests opportunities for future research. We use the Treasury bill return as the risk-free rate. It should be interesting to apply our framework in a setting where the zero beta rate is a parameter to be estimated, perhaps by extending results in Kandel (1984). Some of our results use a maximum correlation, mimicking portfolio. It should be possible to study models in which the correlation is less than the maximum, as would be implied by missing assets, for example, perhaps by extending results in Kandel and Stambaugh (1989). Future applications of our framework should also consider alternative test statistics, test assets, asset pricing models and data generating processes. International asset pricing and portfolio performance evaluation where nonlinearities may be important, such as for hedge funds, could be especially interesting applications.

Appendix

Proof of Proposition 2. By the definition of covariance, $E\left[m_{t+1}x'\left(Z_t\right)R_{t+1}\right]=1$ implies

$$E\left[x'\left(Z_t\right)R_{t+1}\right]=\left\{1-Cov\left[m_{t+1},x'\left(Z_t\right)R_{t+1}\right]\right\}/E\left(m_{t+1}\right). \tag{12}$$

Now, using $m_{t+1} = A + B'\mathrm{R}_{B,t+1}$, we find that Equation (7) is satisfied, with $R_{p,t+1} = w'R_{B,t+1}$, $w \equiv B/(\underline{1}'B)$, $\gamma_0 = [A + B'E(R_{B,t+1})]^{-1}$, and $\gamma_1 = -\gamma_0\left(\underline{1}'B\right)$. ■

Proof of Proposition 3. Regress m on R_P using a simple regression: $m = a + bR_P + u$, where without loss of generality a and b are constants and $E(u) = E\left(uR_P\right) = 0$. If R_P is maximum correlation with respect to Z, then the error also satisfies: $E\left[ux'(Z)R\right] = 0 \;\; \forall x(Z): x'(Z)\underline{1} = 1$. If this were not true for some $x(Z)$, then $x'(Z)R$ enters an expanded regression with R_P and $x'(Z)R$ on the right-hand side. Since the regression maximizes the squared correlation, this would contradict the assumption that R_P is maximum correlation. Substitute the simple regression into (4) to obtain $E\left[\left(a+bR_P+u\right)x'(Z)R\right] = 1 = E\left[\left(a+bR_P\right)x'(Z)R\right]$ $\forall x(Z): x'(Z)\underline{1}=1$. Proposition 2 now establishes that R_P is efficient with respect to Z. ■

Evaluating the Tests by Simulation

Consider first a case with no conditioning information. For the Monte Carlo experiments we draw from a normal distribution with mean vector and covariance matrix set equal to the ML estimates for the sample period of the analysis. We replace the tested portfolio R_p by a portfolio whose weights maximize the Sharpe ratio at the ML estimates. The empirical 5% critical value is the value above which 5% of the 1,000 simulated statistics lie. The empirical *p*-value is the fraction of the 1,000 statistics that are larger than the value obtained in the original sample.

We also resample using a parametric bootstrap approach. A regression of the returns on the conditioning information defines the conditional mean function and the matrix of residuals defines the unexpected returns. We choose randomly selected rows, with replacement, from the matrix of residuals; the number of draws matches the length of the time series. We use the conditional mean functions, evaluated at the simulated *Z*, and add the independently resampled residuals to obtain the simulated returns.

We model Z_t as a vector AR(1) process, and the sample AR(1) coefficient matrix is a parameter of the simulations. We resample from the matrix of residuals of the AR(1) model and build the time series of the Z_t's recursively.

When the null hypothesis places a given portfolio on the efficient-with-respect-to-Z frontier, we replace the tested portfolio return with the time-varying combination of test assets that is ex ante efficient given the data generating process (Tables 2 through 4). When the null hypothesis specifies that a fixed weight combination of factors is efficient, we replace the first factor with the ex ante efficient portfolio (Table 5). When the null hypothesis specifies the conditional efficiency of a time-varying combination of the benchmark returns, R_B, we replace the conditional mean functions of the test assets with the expressions implied by the conditional beta pricing restriction: $\mu(Z) = \gamma_o + \Sigma_{j=1}^{k} \beta_j(Z) E[R_{Bj} - \gamma_0 | Z]$, where $\beta_j(Z)$ is the vector of conditional betas on the j-th benchmark return (tables 6 and 8). In Table 7 we set the alphas of the hedge funds on the efficient-with-respect-to Z portfolios equal to zero.

Conditional Heteroskedasticity

We evaluate the sensitivity of the tests to alternative specifications for conditional heteroskedasticity in the returns. The "artificial analyst" in the simulations estimates the test statistics as if the data were homoskedastic. The goal of these experiments is to see how our inferences, based on the statistics that ignore heteroskedasticity, might be affected by heteroskedasticity.

Since it may not be possible to agree on the right model for conditional heteroskedasticity, we use two alternative approaches. In the first approach (method A) the heteroskedasticity is driven by a factor, where the conditional betas on the common factor (the CRSP value-weighted stock index return) are linear functions of the lagged instruments. The conditional betas, $\beta(Z)$, are estimated by regressing the unexpected asset returns on the index return and the products of the index return with the lagged instruments. The time-varying beta is the regression coefficient on the index plus the coefficients on the product terms multiplied by the lagged instruments. The conditional covariance matrix is modeled as $\Sigma(Z) = \beta(Z)\beta'(Z)\sigma_f^2 + \Sigma_\mu$, where Σ_μ is the fixed covariance matrix of the factor model residuals and σ_f^2 is the fixed conditional variance of the common factor, estimated from the residuals of its linear regression on the lagged Z.

The second approach to modeling heteroskedasticity (method B) follows Davidian and Carroll (1987) and Ferson and Foerster (1994). The conditional standard deviations of the returns are assumed to be linear functions. To estimate this model the absolute residuals from the linear expected return models are regressed on the instruments. The fitted value, multiplied by $\sqrt{\pi/2}$, is the conditional standard deviation. The conditional covariances are modeled as the products of the standard deviations and the fixed conditional correlations, where the correlations are estimated from the residuals of the mean equations.

The models tested in Table 6 are evaluated under heteroskedastic data in Table 8. In panels A and and B the null distribution is generated by method A. Panels C and D use the linear conditional standard deviation approach, method B. The results of both approaches are similar. When testing conditional efficiency the specification of the stochastic discount factor changes under heteroskedasticity,[15] but the effect is small. We experiment by computing the

[15] Under conditional efficiency the stochastic discount factor is $A(Z) + B(Z)'R_B$, and the coefficients $A(Z)$ and $B(Z)$ change when the data generating process changes.

sample values of the various test statistics, either using the heteroskedastic structure in the calculations or ignoring it, and the sample values are not very sensitive to this choice.

References

Abhyankar, A., D. Basu, and A. Streme, 2007, "Portfolio Efficiency and Discount Factor Bounds with Conditioning Information: An Empirical Study," *Journal of Banking and Finance* (forthcoming, 2007).

Abhyankar, A., D. Basu, and A. Streme, 2006, "The optimal use of return predictability," working paper, Warwick Business School .

Bekaert, G, and J. Liu, 2004, "Conditioning Information and Variance Bounds on Pricing Kernels," *Review of Financial Studies* 17, 339-378.

Breeden, D.T., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics* 7, 265-296.

Campbell, John Y, 1987, "Stock returns and the Term Structure," Journal of Financial Economics 18, 373-399.

Chamberlain, G., 1983, "Funds, Factors and Diversification in Arbitrage Pricing Models," *Econometrica* 51, 1305-1324.

Chiang, Ethan, 2007, "Modern Portfolio Management with Information," working paper, Boston College.

Cochrane, J.H., 1996, "A Cross-Sectional Test of a Production Based Asset Pricing Model," working paper, *Journal of Political Economy*.

Davidian, M., and R.J. Carroll, 1987, "Variance Function Estimation," *Journal of the American Statistical Association*, 82, 1079-1091.

Fama, E.F., 1973, "A Note on the Market Model and the Two-Parameter Model," *Journal of Finance* 28, 1181-1185.

Fama, E.F., 1990, "Stock Returns, Expected Returns, and Real Activity," *Journal of Finance* 45, 1089-1108.

Fama, E.F., and K.R. French, 1988, "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics* 22, 3-25.

Fama, E.F., and K.R. French, 1996, "Multifactor Explanations of Asset Pricing Anomalies," *Journal of Finance* 51, 55-87.

Fama, E.F., and K.R. French, 1997, "Industry Costs of Equity," *Journal of Financial Economics* 43, 15-193.

Fama, E.F., and G.W. Schwert, 1977, "Asset Returns and Inflation," *Journal of Financial Economics* 5, 115-146.

Ferson, W.E., and S.R. Foerster, 1994, "Finite Sample Properties of the Generalized Method of Moments in Tests of Conditional Asset Pricing Models," *Journal of Financial Economics*, 36, 29-55.

Ferson, W.E., and C.R. Harvey, 1999, "Conditioning Variables and the Cross-Section of Stock Returns," *Journal of Finance* 54, 1325-1360.

Ferson, W.E., and R. Jagannathan, 1996, "Econometric Evaluation of Asset Pricing Models," In G.S. Maddala and C.R. Rao (eds.), *The Handbook of Statistics*, Volume 14 (Chapter One), Elsevier, New York.

Ferson, W. E., and R.W. Schadt, 1996, "Measuring Fund Strategy and Performance in Changing Economic Conditions," *Journal of Finance* 51, 425-462 .

Ferson, W.E., S. Sarkissian, and T. Simin, 2003, "Spurious Regressions in Financial Economics?" *Journal of Finance* 58, 1393-1414.

Ferson, W.E., and A.F. Siegel, 2001, "The Efficient Use of Conditioning Information in Portfolios," *Journal of Finance* 56, 967-982.

Ferson, W.E., and A.F. Siegel, 2003, "Stochastic Discount Factor Bounds with Conditioning Information," *Review of Financial Studies* 16, 567-595.

Ferson, W.E., and A.F. Siegel, 2007, "A note on the optimal orthogonal portfolio with conditioning information," working paper, University of Southern California.

Ferson, W.E., A.F. Siegel, and P. Xu, 2006, "Mimicking Portfolios with Conditioning Information," *Journal of Financial and Quantitative Analysis* 41, 607-635.

Fung, W. and D. Hsieh, 1997, Empirical characteristics of dynamic trading strategies: The case of hedge funds," *Review of Financial Studies* 10, 275-302.

Ghysels, E., 1997, "On Stable Factor Structures in the Pricing of Risk: Do Time-Varying Betas Help or Hurt?" *Journal of Finance* 53-549-573.

Gibbons, M.R., 1982, "Multivariate Tests of Financial Models," *Journal of Financial Economics* 10, 3-27.

Gibbons, M.R., and W.E. Ferson, 1985, "Testing Asset Pricing Models with Changing Expectations and an Unobservable Market Portfolio," *Journal of Financial Economics* 14, 217-236.

Gibbons, M.R., S.A. Ross, and J. Shanken, 1989, "A Test of the Efficiency of a Given Portfolio," *Econometrica* 57, 1121-1152.

Goyal, A., and I. Welch, 2003, "Predicting the Equity Premium with Dividend Ratios," *Management Science* 49, 639-654.

Goyal, A., and I. Welch, 2004, "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction," working paper, Emory University.

Grinblatt, M., and S. Titman, 1987, "The Relation Between Mean-Variance Efficiency and Arbitrage Pricing," *Journal of Business* 60, 97-112.

Hansen, L.P, and R.J. Hodrick, 1983, "Risk Aversion Speculation in Forward Foreign Exchange Markets: An Econometric Analysis of Linear Models," in J.A. Frenkel, (ed.) *Exchange Rates and International Macro Economics*, University of Chicago Press.

Hansen, L.P., and R. Jagannathan, 1991, "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy* 99, 225-262.

Hansen, L.P., and S.F. Richard, 1987, "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models," *Econometrica* 55, No. 3, 587-613.

Harvey, C.R., 1989, "Time-Varying Conditional Covariances in Tests of Asset Pricing Models," *Journal of Financial Economics* 24, 289-318.

Harvey, C.R., and C. Kirby, 1996, "Analytic Tests of Factor Pricing Models," working paper, Duke University, Durham, N.C.

Huberman, G., S. Kandel, and R. Stambaugh, 1987, "Mimicking Portfolios and Exact Arbitrage Pricing," *Journal of Finance* 42, No. 1, 1-9.

Jagannathan, R., and Z. Wang, 1996, "The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance* 51, 3-53.

Jobson, J.D., and B. Korkie, 1982, "Potential Performance and Tests of Portfolio Efficiency," *Journal of Financial Economics* 10, 433-466.

Kandel, S.A., 1984, "The Likelihood Ratio Test Statistic of Mean Variance Efficiency Without a Riskless Asset," *Journal of Financial Economics* 13, 575-592.

Kandel, S.A., and R.F. Stambaugh, 1989, "A Mean-Variance Framework for Tests of Asset Pricing Models," *Review of Financial Studies* 2, 125-156.

Keim, D.B., and R.F. Stambaugh, 1986, "Predicting Returns in the Bond and Stock Markets," *Journal of Financial Economics* 17, 357-390.

Lewellen, J., S. Nagel and J. Shanken, 2007, "A skeptical appraisal of asset pricing tests," working paper, Emory University.

MacKinlay, A.C., 1987, “On Multivariate Tests of the CAPM,” *Journal of Financial Economics* 18, 341-371.

MacKinlay, A.C., 1995, “Multifactor Models Do Not Explain Deviations From the CAPM,” *Journal of Financial Economics* 38, 3-28.

Merton, R., 1973, “An Intertemporal Capital Asset Pricing Model,” *Econometrica* 41, 867-887.

Michaud, R., 1989, The Markowitz Optimization Enigma: Is optimized optimal?” *Financial Analysts Journal* 24 (January/February).

Roll, R.R., 1985, “A Note on the Geometry of Shanken's CSR T^2 Test for Mean/Variance Efficiency,” *Journal of Financial Economics* 14, 349-357.

Roll, R., 1977, “A Critique of the Asset Pricing Theory's Tests - Part 1: On Past and Potential Testability of the Theory,” *Journal of Financial Economics* 4, 129-176.

Shanken, J., 1987, “Multivariate Proxies and Asset Pricing Relations: Living with the Roll Critique,” *Journal of Financial Economics* 18, 91-110.

Shanken, J., 1990, “Intertemporal Asset Pricing: An Empirical Investigation,” *Journal of Econometrics* 45, 99-120.

Sharpe, W.F., 1964, “Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk,” *Journal of Finance* 19, 425-442.

Simin, T., 2003, “The Poor Predictive Performance of Asset Pricing Models,” *Journal of Financial and Quantitative Analysis* (forthcoming).

Stambaugh, R.F., 1982, “On the Exclusion of Assets from Tests of the Two-Parameter Model,” *Journal of Financial Economics* 10, 235-268.

Treynor, J. and F. Black, 1973, “How to use Security Analysis to Improve Portfolio Selection, *Journal of Business*, 66-86.

Table 1. Monthly Industry Returns

	Industry	SIC codes	Mean	σ	ρ_1	R^2	$R^2_{HOLDOUT}$
1	Aerospace	372, 376	1.0107	0.0644	0.13	9.9	1.1
2	Transportation	40, 45	1.0094	0.0648	0.08	9.1	0.0
3	Banking	60	1.0086	0.0631	0.10	4.3	2.4
4	Building materials	24, 32	1.0097	0.0608	0.09	10.4	0.0
5	Chemicals/Plastics	281, 282, 286-289, 308	1.0094	0.0525	-0.01	8.0	2.5
6	Construction	15-17	1.0109	0.0760	0.16	10.2	0.0
7	Entertainment	365, 483, 484, 78	1.0135	0.0662	0.14	5.7	0.0
8	Food/Beverages	20	1.0117	0.0449	0.05	6.6	0.2
9	Healthcare	283, 384, 385, 80	1.0113	0.0524	0.01	2.4	0.0
10	Industrial Mach.	351-356	1.0089	0.0587	0.05	8.2	0.0
11	Insurance/Real Estate	63-65	1.0095	0.0581	0.15	6.4	2.3
12	Investments	62, 67	1.0097	0.0453	0.05	8.7	4.1
13	Metals	33	1.0075	0.0610	-0.02	4.5	0.2
14	Mining	10, 12, 14	1.0108	0.0535	0.01	7.2	0.3
15	Motor Vehicles	371, 551, 552	1.0095	0.0584	0.11	10.6	0.0
16	Paper	26	1.0095	0.0536	-0.02	6.9	2.4
17	Petroleum	13, 29	1.0102	0.0518	-0.02	4.4	0.0
18	Printing/Publishing	27	1.0114	0.0586	0.21	11.3	0.0
19	Professional Services	73, 87	1.0111	0.0693	0.13	8.4	2.8
20	Retailing	53, 56, 57, 59	1.0106	0.0597	0.15	8.7	3.7
21	Semiconductors	357, 367	1.0080	0.0559	0.08	9.0	0.0
22	Telecommunications	366, 381, 481, 482, 489	1.0085	0.0412	-0.05	5.4	8.8
23	Textiles/Apparel	22, 23	1.0100	0.0613	0.21	11.0	0.0
24	Utilities	49	1.0078	0.0392	0.02	6.8	4.3
25	Wholesaling	50, 51	1.0109	0.0614	0.13	10.7	0.0

Monthly returns on 25 portfolios of common stocks are from Harvey and Kirby (1996). The portfolios are value-weighted within each industry group, based on the SIC codes as shown. *Mean* is the sample mean of the gross (one plus rate of) return, σ is the sample standard deviation and ρ_1 is the first order autocorrelation of the monthly return. R^2 is the adjusted coefficient of determination in percent from the regression of the returns on the lagged instruments. The sample period is February of 1963 through December of 1994 (383 observations). $R^2_{HOLDOUT}$ is for the 1995-2002 holdout sample (96 observations). Negative adjusted *R*-squares are reported as 0.0.

Table 2: Tests of the Mean Variance Efficiency of the Standard and Poors 500 Stock Index.

Subperiod	63-72	73-82	83-92	63-94	95-02
Panel A: Test assets $R_{t,}$ no conditioning information:					
Wald Statistic	32.8	26.3	29.8	24.8	51.3
asymptotic *p*-value	0.14	0.39	0.23	0.48	0.001
GRS *p*-value	0.45	0.71	0.57	0.57	0.10
Monte Carlo 5% Critical Value	52.8	52.3	50.8	51.9	120.1
empirical *p*-value	0.43	0.71	0.58	0.59	0.44
Resampling 5% Critical Value	60.1	63.9	62.3	40.0	121.4
empirical *p*-value	0.52	0.81	0.65	0.58	0.49
Panel B: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:					
Wald Statistic	NA	NA	NA	348.6	NA
asymptotic *p*-value				0.00	
GRS *p*-value				0.03	
Monte Carlo 5% Critical Value				328.0	
empirical *p*-value				0.02	
Resampling 5% Critical Value				476.0	
empirical *p*-value				0.44	
Panel C: Test assets are all Portfolios $x'(Z_{t-1})R_t$:					
Test Statistic	203.3	188.6	165.0	161.8	148.2
Monte Carlo 5% Critical Value	125.7	121.6	121.6	133.3	139.9
empirical *p*-value	0.000	0.000	0.001	0.002	0.029
Resampling 5% Critical Value	117.3	130.6	121.6	118.8	144.9
empirical *p*-value	0.003	0.005	0.003	0.001	0.044

The monthly returns on 25 industry-sorted portfolios of common stocks are test assets for February 1963 through December 1994 (T=383 observations), and ten-year subperiods. A holdout sample from January, 1995 through December, 2002 (96 observations) is also shown. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA denotes not applicable, when the number of assets is larger than the number of time series observations. Asymptotic *p*-values are from the chi-squared distribution. GRS *p*-values are from the *F* distribution, after the test statistic is rescaled to have an exact *F* distribution assuming normality as in Gibbons, Ross, and Shanken (1989).

Table 3: Tests of the Mean Variance Efficiency of the Standard and Poors 500 Stock Index in Size and Book/Market Portfolios.

Sample	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: No conditioning information:				
Sample Statistic	83.0	74.1	24.8	51.3
GRS *p*-value	0.000	0.007	0.57	0.102
Resampling 5% Critical Value	45.1	131.5	40.0	121.4
Empirical *p*-value	0.000	0.277	0.58	0.49
Panel B: Test assets are $R_{ft}+(R_t-R_{ft})\otimes Z_{t-1}$**:**				
Sample Statistic	517.1	NA	348.6	NA
Resampling 5% Critical Value	508.8		476.0	
Empirical *p*-value	0.040		0.44	
Panel C: Test assets are all Portfolios $x'\left(Z_{t-1}\right)R_t$**:**				
Sample Statistic	272.7	210.4	161.8	148.2
Resampling 5% Critical Value	107.6	135.1	118.8	144.9
Empirical *p*-value	0.000	0.003	0.001	0.044

The size/BM returns are 25 portfolios of stocks sorted on market capitalization and book-to-market ratio, for the sample period July 1963 through December 1994 (T=378 observations). A holdout sample covers January 1995 through December, 2002 (96 observations). The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA indicates that the sample size does not allow the statistic to be calculated.

Table 4: Tests of the Hypothesis that Conditioning Information does not Expand the Mean Variance Boundary.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	356.3	NA	304.3	NA
Resampling 5% Critical Value	520.4		486.0	
Empirical p-value	0.464		0.686	
Panel B: Test assets are all Portfolios $x'(Z_{t-1})R_t$:				
Sample Statistic	155.8	77.7	128.8	63.8
Resampling 5% Critical Value	108.7	138.3	118.8	148.1
Empirical p-value	0.001	0.539	0.025	0.779

The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index return. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA indicates that the sample size does not allow the test statistic to be calculated.

Table 5: Tests of the Efficiency of Fixed-weight Combinations of the Fama-French Factors.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are R_t:				
Sample Statistic	35.0	49.5	43.0	55.5
GRS p-value	0.154	0.123	0.035	0.064
Resampling 5% Critical Value	41.6	64.0	39.2	61.2
empirical *p*-value	0.117	0.157	0.021	0.077
Panel B: Test assets are all Portfolios $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	521.9	NA	415.0	NA
Resampling 5% Critical Value	319.3	NA	313.7	NA
empirical *p*-value	0.000		0.000	
Panel C: Test assets are $x'\left(Z_{t-1}\right)R_t$:				
Sample Statistic	340.6	181.6	180.1	174.6
Resampling 5% Critical Value	70.5	128.0	75.6	118.4
empirical *p*-value	0.000	0.003	0.000	0.001

The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a value-weighted index. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratio and a value-weighted return. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors, respectively. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA indicates that the sample size does not allow the test statistic to be calculated.

Table 6: Tests of Conditional Efficiency.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index				
Sample Statistic	339.2	131.7	189.7	143.0
Resampling 5% Critical Value	116.1	83.4	98.8	86.4
Empirical *p*-value	0.000	0.011	0.002	0.011
Panel B: Conditional Efficiency of the Fama-French Factors				
Sample Statistic	347.3	138.7	147.5	142.1
Resampling 5% Critical Value	67.3	95.0	71.2	99.1
Empirical *p*-value	0.000	0.012	0.000	0.018

The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

Table 7: The Efficiency of Hedge Funds

	Sharpe Ratio	Adjusted Sharpe
Panel A: Hedge Fund Indexes		
Convertible Arbitrage	0.49	0.49
Dedicated Short Bias	-0.05	-0.05
Equity Market Neutral	0.76	0.76
Event Driven	0.32	0.31
Long/Short Equity	0.24	0.23
Hedge Fund Index	0.26	0.26
Panel B: Size and B/M Portfolios and Market Index		
Fixed-weight Benchmark:		
Sharpe ratios	0.72	0.31
Empirical *p*-value	0.002	0.001
Efficient wrt.*Z* benchmark		
Sharpe ratios	1.39	1.05
Empirical *p*-values	0.052	0.049
Panel C: Industry Portfolios and Market Index		
Fixed-weight Benchmark:		
Sharpe ratios	0.76	0.38
Empirical *p*-value	0.009	0.012
Efficient wrt.*Z* benchmark		
Sharpe ratios	1.36	1.02
Empirical p-values	0.171	0.175

The indexes of hedge funds are monthly total returns from Credit Suisse/Tremont. The industry portfolios are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

Table 8: The Impact of Conditional Heteroskedasticity.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index – Method A				
Sample Statistic	344.5	131.4	184.9	144.3
Resampling 5% Critical Value	48.8	80.3	56.8	84.0
Empirical p-value	0.000	0.010	0.000	0.005
Panel B: Conditional Efficiency of the Fama-French Factors – Method A				
Sample Statistic	357.2	145.4	145.5	148.5
Resampling 5% Critical Value	66.7	94.2	70.0	89.6
Empirical p-value	0.000	0.004	0.000	0.007
Panel C: Conditional Efficiency of the Market Index – Method B				
Sample Statistic	371.3	152.5	211.3	158.4
Resampling 5% Critical Value	84.6	89.8	79.2	86.9
empirical p-value	0.000	0.000	0.000	0.000
Panel D: Conditional Efficiency of the Fama-French Factors – Method B				
Sample Statistic	371.3	174.5	152.2	174.4
Resampling 5% Critical Value	79.9	117.5	90.5	108.7
empirical p-value	0.000	0.002	0.004	0.003

The simulated data incorporate conditional heteroskedasticity either through a factor model with conditional betas that are linear functions of the lagged instruments (method A) or through a model in which the conditional standard deviations are linear functions of the conditioning variables and the conditional correlations are constant over time (method B). The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.