A Cost Variance Investigation Using Belief Functions

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Abstract

Using belief functions, this paper develops a model of the situation of a management team trying to decide if a cost process is in-control, or out-of-control and, thus, in need of investigation. Belief functions allow accounting for uncertainty and information about the cost processes, extending traditional probability theory approaches. The purpose of this paper is to build and investigate the ramifications of that model. In addition, an example is used to illustrate the process.

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1. Introduction

A production system, such as a manufacturing system, periodically is investigated to determine if the process is in control or out of control. Decision making information typically is gathered from production and accounting information systems to determine the system status. Then that decision making information is analyzed, often by multiple decision makers, and the control status of the system is determined, along with whether or not the process is to be investigated. Unfortunately, determination of the control status is not a deterministic problem. As a result, there has been substantial analysis of the cost variance problem and a number of models have been proposed and developed to solve those cost variance problems. Researchers have employed probability models and fuzzy set models to investigate cost processes. The purpose of this research is to extend the cost variance model to a model based on belief functions.

This Paper

This paper proceeds in the following manner. Section 2 reviews some of the previous literature associated with cost variance analysis models. Section 3 briefly reviews decision making using belief functions. Section 4 reviews and summarizes some other applications of belief functions. Sections 5 and 6 discuss in detail an example designed to illustrate the use of belief functions in the cost variance problem. Finally, section 7 summarizes the paper and reviews some potential extensions.

2. Cost Variance Analysis

The basic cost variance investigation model is for a single period. In that model, it is assumed that the cost variance being controlled is either in control (ic) or out of control (oc). Management needs to determine if they should investigate a process based on a signal or a set of signals coming out of the process. That signal is typically set based on production information and accounting cost information. We also assume that there are multiple actors involved in the process, so that we need to aggregate their perspectives to generate a single overall strategy.

Lipe (1993) has discussed a number of issues of cost variance analysis, including framing of the problem and the effects of outcomes. Thus far, the primary mathematical models for cost variance analysis have been probabilistically-based or fuzzy set based. Kaplan and Atkinson (1989) and Hirsch (1988) discuss the probability model in detail. The basic finding of that research is that cost variance processes have a critical value, where if the probability of the process being out of control exceeds that amount then it is cost beneficial to investigate the process. Zebda (1984) and Lin and O'Leary (1989) analyzed cost variance processes using fuzzy sets, extending the cost variance model from the classic probability-based analysis. Some of that research involved aggregating information from multiple actors to develop a single strategy. Oz and Reinstein (1995) contrast probability-based models and fuzzy set approaches.

There are some basic differences between probability, fuzzy set and belief functions and their application in the cost variance process. Using the probabilistic version, a process is either in control or out of control, with probabilities associated with state. With fuzzy sets, a process can be "almost" in control or "almost" out of control. We will see with belief functions that a process can be assessed as in control, out of control or it can be assessed that there is uncertainty as to whether the process is in control or out of control.

3. Decision Making with Belief Functions

The Dempster-Shafer theory of belief functions was formed in 1960s and 70s (Shafer, 1976). Due to the difference between belief functions and probability framework in dealing with the uncertainties, belief functions framework are often regarded as more intuitive.

Background: Belief Functions

A frame Θ, is defined to be an exhaustive and mutually exclusive set of possible answers (a_i) to a particular question. If the question is a yes-no question, then there are only two answers, "yes" or "no."

Let $\Theta = (\{a_1, a_2, a_3\})$. Uncertainties are represented through m-values, which are equivalent to probabilities, but more general. The m-values are assigned to a subset of the elements of the frame.

For each Θ, let $m(\{a_i\})$, $m(\{a_i,a_j\})$, $m(\{a_i, a_j, a_k\})$, where $\sum_{A \subseteq \Theta} m(A) = 1$. That is, the m values relate to each answer and interactions between answers. Define the belief function for A as $Bel(A) = \sum_{B \subseteq A} m(B)$. We set $Bel(\Theta) = 1$, and $Bel(\acute{\emptyset}) = 0$.

The so-called "plausibility" for A is $Pl(A) = 1 - Bel(\sim A)$. Plausibility is the degree to which A is plausible in light of the evidence. It also can be thought of as the degree to which we do not disbelieve A.

Differences between Probability Theory and Belief Functions

The basic difference between m-values and probabilities is that m values are assigned to subsets of the frame, whereas, probabilities are assigned to individual elements of the frame. Thus, probability theory and belief functions differ because probability is assigned to events, e.g., "in control" or "out of control." However, with belief functions, there also can be weight assigned as "uncommitted" to the process being either in control or out of control. Finally, $Bel(\{a\}) + Bel(\{\sim a\}) \leq 1$, whereas in probability theory, $P(a) + P(\sim a) = 1$, where P(a) represents the probability of A.

Obtaining m values

There are two basic ways to obtain m-values for a frame. First, the decision maker could directly assign values based on their judgment. Second, there may be some relationship between a frame where the weights have already been established and the current frame. In this last case, the existing information might be used to set the weights.

Dempster's Rule of Combination

The basic rule for combining independent items of evidence, while using belief functions, is Dempster's rule. Probability theory's "Bayes' rule" is similar to Dempster's rule, and is a special case of Dempster's rule. We will use this rule to combine belief functions, for example, across multiple decision makers.

Next assume that we have multiple independent sets of Θ_j (e.g., multiple decision makers), where for each Θ_j the beliefs are denoted as m_j. Assume that the cardinality of Θ_j is 2. Let the combination of two pieces of evidence be denoted as m'. In that case

$$m'(A) = (\sum \{m_1(b_1)\, m_2(b_2) \mid B_1 \cap B_2 = A, A \neq \acute{\O}\})/ K.$$

In the case of combining belief functions from two sources I and O, the function could look as follows:

$$m'(I) = [m_1(I)*m_2(I) + m_1(I)*m_2(I,O) + m_2(I)*m_1(I,O)] / K$$

K is a constant, for normalization, and

$$K = 1 - \sum \{m_1(b_1)\, m_2(b_2) \mid B_1 \cap B_2 = \acute{\O}\}.$$

K is a term that is used to capture the "conflict" between two items of evidence. If K = 1 then there is no conflict. If K = 0 then the two items of evidence are in conflict with each other and cannot be combined.

For the case of combination of more than 2 pieces of evidence see Shafer (1976).

Statistical Information and Belief Information

Belief information can be combined with statistical information. If there are two sets of evidence, statistical and belief, then one approach to combining the two is to use Dempster's rule, treating the statistical data as one frame and the belief data as another frame. Similarly, combining probability and belief information can be done using Dempster's rule. For a further discussion see Srivastiva (1996).

4. Applications of Belief Functions

The purpose of this section is to briefly review some of the primary applications of belief functions in business.

Applications of Belief Functions to Business Decisions

There have been a number of applications to information systems. Xie and Phoha (2002) applied belief functions and Dempster's rule in order to cluster groups of web users, in order to facilitate development of profiles. Srivastiva and Mock (1999-2000) used belief functions to develop a model of WebTrust Assurance for electronic commerce. Srivastava and Datta (2002) investigated the use of belief functions for the process of mergers and acquisitions. Other applications are discussed in Srivastava and Liu (2003) and in Shenoy and Srivastava (2003).

Applications of Belief Functions to Auditing

Many of the applications of belief functions have been developed for auditing. As a result, there has been a focus on using belief functions to analyze a number of issues (Srivastava 1993, 1995a, 1997), including the following:

- The auditor's judgment process
- The strength of audit evidence
- The structure of audit evidence
- The risk of fraud
- Determining which opinion to give.

Srivastava (1995b) investigated auditor behavior, through value judgments using belief functions. A belief function model is developed that uses empirical data and predicts observed behavior discussed in the behavioral literature associated with "framing decisions" (e.g., Tversky and Kahneman 1986). Srivastiva and Mock (2000) reviewed how belief functions relate to behavioral research, such as the audit judgment process.

Shafer and Srivastava (1990) and Srivastava and Shafer (1992) investigated the basic nature and structure of audit evidence. In particular, they were concerned with how to model uncertainties involved in audit evidence, and understanding the structure of audit evidence.

Applications of Belief Functions to Management Accounting

Unfortunately, there has not been the same level of investigation of applications of belief functions in management accounting or other areas, as there has been in auditing. This paper addresses the use of belief functions to one of the primary problems associated with management accounting over the years. However, it is possible to extrapolate about it use. Most applications based on probability theory could be extended to use with belief functions. Further, Dempster's rule would facilitate combination of belief estimates across teams of users when a single estimate is necessary for decision making.

5. Microprocessor Chip Production Process

Sections 5 and 6 illustrate cost variance analysis using belief functions. The case study shows how to use belief functions from multiple decision makers in a semiconductor manufacturer to aggregate their assessments to derive a single estimate as to the belief that the process is in control.

Irvine Semiconductor Company is a firm that produces a super speed nanosecond microprocessor. The product goes through the following four-stage manufacturing process:

1. Fabricating

   The fabricating stage has three steps. First, raw materials, i.e., blank wafers are insulated with an oxide film. Second, wafers are coated with a soft, light-sensitive plastic called photoresist. Third, wafers are masked by a glass plate (called a reticle) and flooded with ultraviolet light to pattern each layer on the water. These three steps are repeated to builds the necessary layers on the wafer. An array of transistors made up of various connected layers is called integrated circuits or chips.

   The management accountant collects the wafer material and fabrication processing costs and the water fabrication yield rate information, and calculates the unit cost per wafer production.

2. Probing

   In the probing stage, an electrical performance test of integrated circuits is performed. The functioning chips are moved to the assembly stage.

   The management accountant collects the probing production cost and the probing yield rate information, and calculates both probe production cost per wafer and per chip.

3. Assembling

   The assembling stage has four steps. First, each wafer is cut into chips with a diamond saw. Second, the good chips are placed in a cavity of a ceramic package. Third, the bonding pads from the chips are connected by thin aluminum wires into leads of the package. Fourth, the package is sealed, with a metal lid placed over the exposed chips in the package.

   The management accountant collects the assembly product cost and the assembly yield rate information, and calculates the unit cost per chip.

4. Testing

   The packaged semiconductor device is tested to ensure all electrical specifications of the integrated circuits are met.

The management accountant collects the testing cost, test yield rate, and packaging cost information, and calculates the unit product cost per chip.

6. Belief functions and cost variance investigations

The company set the standard cost for each component of the microprocessor product with the monthly production of 200,000 units. During the first month, the company actually produced 190, 000 units. The accounting department provided the following unit chip cost information in table 1**:**

Department	*Standard Cost*	*Actual Cost*	*Variance*	*(%) Variance*	*(%)variation*
Wafer cost	$ 40.00	$ 42.00	$ 2.00	2/32=6.25%	5%
Fabrication Cost	150.00	162.00	12.00	12/32=37.5%	8%
Probe Cost	80.00	93.00	13.00	13/32=40.625%	16.25%
Assembly Cost	10.00	13.00	3.00	3/32 = 9.375%	30%
Test Cost	30.00	31.00	1.00	1/32=3.125%	3.33%
Package Cost	40.00	41.00	1.00	1/32=3.125%	2.5%
Total Cost	$ 350.00	$ 382.00	$ 32.00	100%	100%

Table 1 – Cost Information

Since the variance in the total cost is about 10% (32/350), management feels that some of the process variations need to be investigated. A three-person team is formed in order to proceed with the investigation. The team consists of a cost accountant, a supervisor, and a process engineer from each of the three subdivisions. They need to decide which processes should be investigated further and make proposal to the Vice President of Operations.

The team members agree on that they should focus on three processes: fabricating, probing, and assembling. The costs in these processes account for the majority of the total variation. For those three departments, the standard costs are $230, and the variance is $28. As a result, those three departments account for 87.5% of the total variance and the variance of those 3 is over 12%, compared to 3.6% for the rest of the processes.

However, the opinions of three members vary on which process is most likely out of control. Since no single opinion is dominant, they turn to a decision aid based on the belief functions to aggregate their assessments.

We assume that we are able to gather the m values for each of the actors (supervisor, process engineer, and cost accountant) directly or we can assign m values based on statistical evidence. We assume that there are three m values, where the m values are defined as

m(ic) – belief the process is in control,

m(oc) – belief the process is out of control, and
m(ic,oc) – uncertainty as to whether the process is in control or out of control.

Assume that the nine different sets of m-values, for each of the three actors are given in Table 2.

Department	*Supervisor*	*Process Engineer*	*Cost Accountant*
Fabrication process	m(ic)=0.9, m(oc)=0, m(ic,oc)=0.1	m(ic)=0.8,m(oc)=0 m(ic,oc)=0.2	m(ic)=0.7, m(oc)=0 m(ic,oc)=0.3
Probe Process	m(ic)=0.8, m(oc)=0, m(ic,oc)=0.2	m(ic)=0.1, m(oc)=0.7, m(ic,oc)=0.2	m(ic)=0, m(oc)=0.6, m(ic,oc)=0.4
Assembly Process	m(ic)=0.8, m(oc)=0, m(ic,oc)=0.2	m(ic)=0.5,m(oc)=0.4, m(ic,oc)=0.1	m(ic)=0.3,m(oc)=0.2, m(ic,oc)=0.5

Table 2 – m values

We can see that for the Fabrication process, the belief that the process is out of control is 0 for all three, however, we can also see that that there is some uncertainty for each of them that the process is either in control or out of control. We will let m(ic) + m(oc) + m(ic,oc) =1 in this case.

To illustrate, we make the calculations simple and adopt two-step approach in aggregating the assessments of three persons, a pair at a time. The first assessment is combined with the second assessment using the following formula.

$m'(ic) = [m_1(ic)*m_2(ic) + m_1(ic)*m_2(ic,oc) + m_2(ic)*m_1(ic,oc)] / K$
$m'(ic) = [m_1(oc)*m_2(oc) + m_1(oc)*m_2(ic,oc) + m_2(oc)*m_1(ic,oc)] / K$
$m'(ic,oc) = [m_1(ic,oc) * m_2(ic,oc)]$

The combined assessment is then aggregated with the third assessment to obtain the total belief. The constant K is one when there is no conflict evidence in each assessment, and less than one when there is conflict evidence in each assessment. We see in fabrication process all three persons assign zero value to m(oc) though the belief in in-control is different. There is no obvious evidence showing that the process is out of control. However, in Probe and Assembly processes, the assessments among three persons are significantly different.

For the Fabrication process, using Dempster's rule, we first combine the supervisor's and the process engineer's evidence to obtain m'(.).

m'(ic) = .9 *.8 + .9 * .2 + .8 * .1 = .98
m'(oc) = 0 * 0 + 0 * .2 + 0 * .1 = 0
m'(ic,oc) = .1 * .2 = .02

Then using m'(.), and Dempster's rule, we combine the resulting combined supervisor and process engineer evidence with the cost accountant's beliefs to obtained the overall aggregated beliefs.

m"(ic) = .98 * .7 + .98 * .3 + .7 * .02 = .994
m"(oc) = 0 * 0 + 0 * .3 + 0 * .2 = 0
m"(ic,oc) = .02 * .3 = .006

As a result, after combining the evidence across each of the supervisor, process engineer and cost accountant, the aggregation results are as follows:

For Fabrication process:	m"(ic) = 0.994,	m"(oc) = 0,	m"(ic,oc)=0.006
Probe process:	m"(ic) = 0.365,	m"(oc) = 0.579,	m"(ic,oc)=0.056
Assembly process:	m"(ic) = 0.878	m"(oc) =0.105,	m"(ic,oc)=0.017

Assume that we investigate the process if the probability that the process is in control does not exceed 0.9. In that case, the recommendation is to investigate both Probe and Assembly process, but not the Fabrication process since for the Fabrication process, m"(ic) $\geq$ .9.

7. Summary and Conclusions

Belief functions are a powerful technique to model uncertain beliefs faced by managers and management accountants. Using belief functions we can aggregate beliefs as to whether a process is in control or out of control across multiple decision makers. Belief functions can be gathered directly from the participants or we could generate the values using other processes, such as statistical analysis.

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