The Time Cost of Information in Financial Markets

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Abstract

I model a financial market in which traders acquire private information through time-consuming research. A time cost of information arises due to competition - through the expected adverse price movements due to others' trades - causing traders to rush to trade on weak information. This cost monotonically increases with asset value uncertainty, so that, exactly opposite to the result under the standard modeling assumption of a monetary cost of information, traders acquire the least information when this uncertainty is largest. The model makes several novel testable predictions regarding volume and order imbalances, some of which have existing empirical support.

1 Introduction

Incentives to acquire information about financial assets are crucial to the informational efficiency of market prices, which is in turn important for the efficient allocation of resources, specifically capital.[1] As such, a large literature studies information acquisition in financial market settings (examples include Grossman and Stiglitz (1980), Verrecchia (1982), Admati and Pfleiderer (1988), Kyle (1989), Barlevy and Veronesi (2000,2007), Veldkamp (2006a,b),

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[1]The idea that prices serve an important role in allocating resources goes back to at least Hayek (1945). In particular, firm prices are socially valuable because they allow capital to be allocated efficiently across firms and serve as a signal to managers that internal resources are being used appropriately. See Bond, Edmans, and Goldstein (2012) for a recent review of the literature on the real effects of secondary financial markets.

Chamley (2007), Lew (2013), and Nikandrova (2014)). This literature typically models the cost of information in monetary (or utility) units - as some weakly increasing function of signal quality. This monetary cost may be interpreted literally, or, given that it takes time to acquire and process information into a trading decision, as the reduced-form of a time cost. In this paper, I model the time cost explicitly and show that it takes a very different form than is typically assumed.

As in the existing literature, traders can produce a private, informative signal about an asset's value. Here, however, the quality of the signal is an increasing function of *time*, rather than of money (or utility).[2] In deciding how much research to do, the trader faces a trade-off: better information increases her expected profits through improved trading decisions. But, if another trader front runs her, the other's trade moves prices closer to the asset's true value in expectation, eating into her potential profit.[3] This endogenous cost of research is therefore a function of more than just the signal quality a trader obtains. In fact, it varies in such a way that the conditions under which traders obtain better quality information are completely opposite to that in a model with a purely monetary cost.

The model builds upon the classic trading model of Glosten and Milgrom (1985) in which risk-neutral agents trade an asset with a market maker.[4] I introduce an information acquisition decision in which a trader can obtain a relatively weak, private signal immediately (*rushing*) or a stronger, private signal by investing an additional time period (*waiting*). I first consider a single trader in order to illustrate the main force in the model in the simplest possible setting.[5] To introduce a cost of waiting in this set up, I allow arbitrary public information to arrive between the two periods, which, as with others' trades, moves prices closer on average to the true asset value, reducing profits.[6] I later endogenize this cost by

[2]More generally, one would expect signal quality to be a function of both time and money. In my analysis, I consider the two types of costs independently in order to emphasize the differences in equilibrium information acquisition strategies.

[3]In the model, prices reflect all available public information, so that public information imposes a cost. This contrasts with models of common investment opportunities, such as Chamley and Gale (1994), Gul and Lundholm (1995), and Chari and Kehoe (2004), in which information revealed by others' decisions may be beneficial.

[4]Related papers include those that allow for trade timing, such as Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Ostrovsky (2012), Dugast and Foucault (2016), and Malinova and Park (2014). I discuss the most relevant of these papers in the following, and when contrasting empirical predictions in Section 5.

[5]It is a relatively simple exercise to extend the model to allow sequential arrival as in the original Glosten and Milgrom (1985) model, provided that the information acquisition decisions of each trader do not interact (i.e. the subsequent trader arrives after the last possible time the previous trader trades). Because public beliefs about the asset value are a sufficient statistic for the past history, subsequent traders can be analyzed in a manner identical to that of a single trader. Extending the model in this manner, however, leads to relatively little new insight. Instead, I add a second trader whose information acquisition decision interacts with that of the first.

[6]The high-frequency trading model of Foucault, Hombert, and Roşu (2016) also features fast traders

introducing a second trader.

The key insight of the model is that others' trades have the largest impact on prices and profits, and therefore lead to the greatest time cost, precisely when information is privately most valuable - when uncertainty is high. Standard intuition suggests that when uncertainty is high, traders are most willing to pay for private information because it creates a large difference between private and public beliefs and hence a large expected profit. After confirming this result in a model with monetary costs, I show that, because public information generates large price movements when uncertainty is high, they then impose the largest cost in terms of forgone profits. Therefore, competition causes the greatest reduction in incentives to do research exactly when traders are most willing to pay a monetary cost for it. In fact, in equilibrium, traders actually do the *least* research at this time. I also show that at a time of high uncertainty, information contributes most to the long-term informational efficiency of the market, making it all the more relevant that traders acquire little information at this time.

The key assumption for this result is that traders choose a signal quality prior to trading.[7] This assumption is, for example, a natural consequence of a trader choosing where to look for information: reading the financial statements of a firm is faster than doing a complete industry analysis, but also likely to provide less private information about the value of the firm. Both activities consume time such that, if one reads the financial statements, one cannot simultaneously be performing an industry analysis. Under this interpretation, the prediction of the model is that fast, weak sources of information are more likely to be exploited when uncertainty is high. However, the model does *not* imply that the better quality source of information is never exploited - as uncertainty is resolved, traders more frequently access it.

As in Glosten and Milgrom (1985), all trades take place with a risk-neutral market maker who faces perfect competition, earning zero expected profits. Traders can be either informed traders that choose the quality of their information, or uninformed traders that facilitate trade by trading for reasons exogenous to the model. The market maker accounts for the possibility that he faces an informed trader and posts separate bid and ask prices that are conditional on the type of order. The difference between the bid and ask prices, the bid-ask spread, imposes a trading cost that the informed trader can attempt to avoid by masquerading as an uninformed trader. For example, if the informed trader were to always rush, the market maker would increase the first period spread accordingly. Then, however, the informed trader would prefer to wait, pretending to be an uninformed trader in the

trading in front of public information, but with information of exogenous, fixed quality.

[7]Kendall (2016) considers a variation of the model in which traders always receive an initial signal, but can obtain an additional signal by waiting. Under the assumption that traders trade only once, the trade-off they face is very similar to that here and produces similar results.

second period.

This strategic interaction naturally arises when a trader with market power decides when to trade, but is orthogonal to the main question of interest and not necessary for the results.[8] Furthermore, in most major financial markets, the strategic interaction with the market maker is likely to play less of a role than the threat of being front run by other traders given the small bid-ask spreads observed in these markets.[9] For these reasons, I focus on the case of a small probability of informed trading such that the bid-ask spreads are small enough relative to the time cost of acquiring better information that the latter determines equilibrium behavior. I demonstrate through numerical simulations that the upper limit on the probability of informed trading required for the main results to hold is large - larger than even the most conservative estimates from empirical research.

The model makes several testable predictions about the effects of uncertainty, competition, and information quality on trade timing and order imbalances. Indirect evidence exists for some of these predictions, including that the order imbalance decreases with competition. Predictions with respect to uncertainty and information quality, are more novel and yet to be tested. Kendall (2016) tests the predictions of a closely related model in a controlled laboratory setting and finds evidence supporting the theoretical mechanisms of this paper.

In addition to the literature on information acquisition, this paper is related to the literature on rational panics which shares the intuition that waiting to trade is costly (see Romer (1993), Bulow and Klemperer (1994), Smith (1997), Lee (1998), Barlevy and Veronesi (2003), Brunnermeier and Pedersen (2005), and Pedersen (2009)). Information is exogenous in these models and has emphasized the role that rushing to trade plays in producing price crashes, whereas I show that it also affects the informational content of trades.

A contemporaneous paper by Dugast and Foucault (2016) also studies a model in which traders benefit from time-consuming research. In their model, traders choose between raw information, which may be completely uninformative, and perfect information. A time cost of acquiring perfect information arises when raw information is sufficiently precise. However, Dugast and Foucault (2016) fix the prior asset value uncertainty so do not study how incentives to acquire better information vary with this uncertainty, instead focusing on the informational efficiency of prices. As such, the contributions of the papers are complementary in nature.[10]

[8]One can introduce a continuum of types based on a private discount factor or private cost of delay as in Harsanyi (1973), to convert the mixing strategies to threshold strategies based on type. The main results are unchanged in this variation.

[9]On the other hand, Malinova and Park (2014), in a two period set up similar to mine, but with exogenous information of varying precision, show that this strategic interaction with the market maker can produce intraday trading patterns consistent with empirical data.

[10]The model of Dugast and Foucault (2016) does not always produce a cost of waiting. If raw information

The paper is organized as follows. I begin with the model of a single trader in Section 2. After describing the model, I establish a benchmark for the value of information by considering a model with a monetary cost of information (Section 2.3). I then study the time cost of information by replacing the monetary cost with a public information event (Section 2.4). Section 3 provides the main results of the paper by introducing competition that endogenizes the time cost. Section 4 shows numerically that the upper limit on the probability of informed trading necessary for the results is not restrictive. Section 5 provides testable empirical predictions. Section 6 discusses the robustness of the model to various modeling assumptions, and Section 7 concludes.

2 Single Trader Model

2.1 Description

There are two trading periods, $t \in \{0, 1\}$. A single asset of unknown value, $V \in \{0, 1\}$, with common prior, $p_0 = Pr(V = 1) \in (0, 1)$ is traded in each period. Its value is realized prior to the trading periods, but becomes public knowledge only at $t = 2$.[11] Trade takes place between a risk-neutral trader and risk-neutral market maker. As in Glosten and Milgrom (1985), all trades are for a fixed size, normalized to a single unit: either a purchase or a short sale.[12]

With probability $1 - \mu$, $\mu \in (0, 1)$, the trader is an uninformed trader who buys or sells with equal probability in either trading period. Uninformed traders represent traders that trade for exogenous reasons, such as their own liquidity needs, portfolio re-balancing, etc. Their presence prevents the adverse selection problem faced by the market maker from precluding all trade.

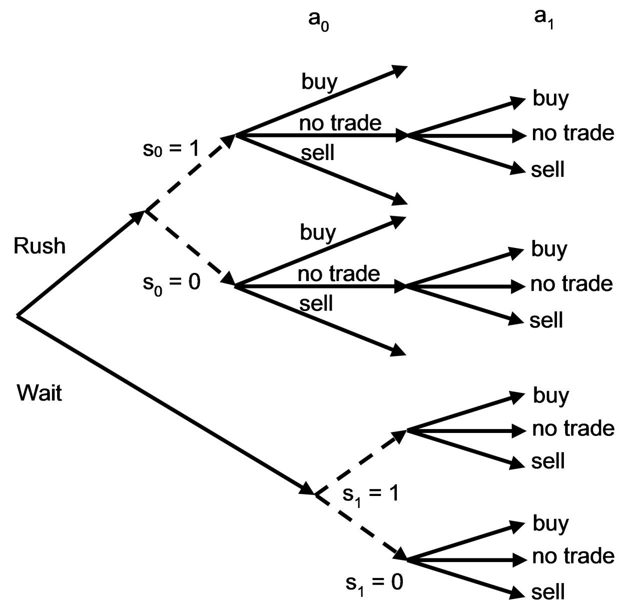
With probability μ, the trader is informed and can generate a private, binary signal, the quality of which is increasing in the amount of time spent doing research (and is otherwise costless). Due to the discrete nature of the model, potential signal qualities are also discrete. In particular, if the trader invests little time doing research, she receives a private, binary signal at $t = 0$, $s_0 \in \{0, 1\}$, which correctly identifies V with probability $q_0 = Pr(s_0 =$

is noisy enough, the fact that first period traders may be trading on a completely uninformative signal can produce a mispricing that benefits those that process information. Instead, here, first period signals are always partially informative so always produce a time cost.

[11]This assumption is standard in models of informed trading and can be motivated by the value of the asset becoming public through an earnings announcement, for example. See Glosten and Milgrom (1985), Kyle (1985), Admati and Pfleiderer (1995), and Foster and Viswanathan (1995), among others.

[12]This assumption is also standard in models of informed trading with risk-neutral traders. In addition to Glosten and Milgrom (1985), see Cipriani and Guarino (2014), Dugast and Foucault (2016), and Malinova and Park (2010,2014).

Figure 1: Decision Tree for the Informed Trader



$1|V = 1) = Pr(s_0 = 0|V = 0) \in (\frac{1}{2}, 1)$. In this case, I say she *rushes*. If, instead, the trader invests an additional time period doing research, she receives a private, binary signal at $t = 1$, $s_1 \in \{0, 1\}$, which identifies V with the larger probability $q_1 = Pr(s_1 = 1|V = 1) = Pr(s_1 = 0|V = 0) \in (q_0, 1]$.[13] In this case, I say she *waits*.

I denote the trader's action in each period, $a_t \in \{buy, sell, no\ trade\}$, and I make two restrictions to simplify the exposition and analysis. The first is that a trader may not place a trade in the first period if she waits. This assumption is innocuous because trading prior to receiving private information earns zero expected profits at best.[14] The second restriction is that if she acquires information in the first period, she may only trade once (in either period). Formally, $a_1|a_0 \in \{buy, sell\} = no\ trade$. I argue in Section 6 that the main insights of the model are unchanged if this restriction is relaxed. Figure 1 provides a simple representation of the decision tree faced by the informed trader.

Between the two trading periods, information about the asset's value may become public. In the simplest case, this public information could be in the form of public news about the asset. But, it could also be information revealed by others' trades, a possibility I consider explicitly in Section 3. To allow for both possibilities, let e denote a generic public

[13] I impose the restriction, $q_1 > q_0$, because it is natural to assume that information improves with time. The nature of the time cost is the same when $q_1 \leq q_0$, but in this case, when the probability of informed trading becomes small, no trade-off exists and the informed trader rushes with probability one.

[14] The trader would be willing to trade only if the bid-ask spread in the first period is zero. Allowing a trade in this case does not change the equilibrium otherwise because, given that the trader has no private information, the bid-ask spread remains zero even with her trade.

event. I assume that the event: (i) has at least two possible realizations from the set, $E = \{e^1, e^2, \ldots, e^n\}$, (ii) is informative, $Pr(e = e^i|V = 1) \neq Pr(e = e^i|V = 0)$, for at least one realization, and (iii) is symmetric, $Pr(e = e^i|V = 1) = Pr(e = e^j|V = 0)$ for some j, for all realizations, e^i. Information contained in the public event and the private signals are assumed to be independent conditional on the asset value.

As is standard (Glosten and Milgrom (1985)), the market maker is assumed to face (unmodeled) perfect competition, earning zero expected profits. He accounts for the private information contained in the current order and posts separate bid and ask prices, B_t and A_t, at which he is willing to buy and sell, respectively. Let I_t denote the information set of publicly available information at time t, including trades in previous periods and, at $t = 1$, information revealed by the public event. If the informed trader rushes and then trades at time $t \in \{0, 1\}$, her expected profit is given by $E[V|I_t, s_0] - A_t$ if she buys, and $B_t - E[V|I_t, s_0]$ if she sells. If she waits, the corresponding expressions are $E[V|I_1, s_1] - A_1$ and $B_1 - E[V|I_1, s_1]$.

2.2 Preliminaries

The solution concept is Perfect Bayesian Equilibrium. It is convenient to denote the public expectation of the asset's value, $p_t = E[V|I_t] = Pr[V = 1|I_t]$. Due to the assumption that the market maker earns zero expected profits, it is easily shown that the bid and ask prices in each period are given by the expected value of the asset conditional on public information and the information contained in the current order, $B_t = E[V|I_t, a_t = sell]$ and $A_t = E[V|I_t, a_t = buy]$. These posted prices depend upon the market maker's belief that a trade contains information, which, due to the presence of uninformed traders, is pinned down by Bayes' rule at all possible histories. As usual, equilibrium requires that beliefs are correct.

Given the posted prices, the informed trader maximizes her expected profit. Lemma 1 first characterizes the optimal trading strategy given any possible information acquisition strategy. All proofs are contained in the Appendix.

Lemma 1: *In any equilibrium, an informed trader buys with a positive signal ($s_t = 1$) and sells with a negative signal ($s_t = 0$).*

The result of Lemma 1 is standard and intuitive: an informed trader with a positive signal buys because her private belief exceeds the ask price, and conversely. The presence of uninformed traders ensures a gap between the posted prices and the informed trader's private belief, allowing her to make a positive expected profit from trading. The market maker breaks even, profiting from an uninformed trader and losing to an informed trader.

Knowing her optimal trading strategy, the informed trader chooses the quality of research to undertake to maximize her expected profit. I denote the probability that the she rushes to acquire information at $t = 0$, $\hat{\beta}$. The probability that she waits is then $1 - \hat{\beta}$. I primarily study the equilibrium in which the informed trader trades in the period in which she acquires information. Provided that the probability of informed trading, μ, is not too large, I show in Section 2.4 that this is in fact the unique equilibrium. Furthermore, in Section 4, I show numerically that, for all but the most extreme parameterizations, the upper limit on μ is not binding - any $\mu \in (0, 1)$ has only this equilibrium. In the remaining minority of cases, this equilibrium may fail to exist and the informed trader may instead acquire information at $t = 0$ but delay trading until $t = 1$. See Section 6.1 for further discussion of this case.

An equilibrium in which the informed trader trades immediately upon acquiring information is fully characterized by $\hat{\beta}$, along with the trading strategy of Lemma 1. In such an equilibrium, her expected profit, π_0, from rushing is given by

$$\begin{aligned} \pi_0(\beta, p_0, q_0) &= Pr(s_0 = 1)\left(Pr(V = 1|s_0 = 1) - A_0\right) \\ &+ Pr(s_0 = 0)\left(B_0 - Pr(V = 1|s_0 = 0)\right) \end{aligned} \tag{1}$$

which depends upon the market maker's belief, β, about her information acquisition strategy, $\hat{\beta}$, through the bid and ask prices. The first term corresponds to the profit from buying if she receives a positive signal, and the second from selling with a negative signal. In the Appendix, I show that we can use Bayes' rule and the symmetry of the problem (the probability of a positive signal - and therefore a buy order - when the asset value is good is the same as the probability of a negative signal - and therefore a sell order - when the asset value is bad) to obtain

$$\pi_0(\beta, p_0, q_0) = \omega_0 m(2q_0 - 1)\left(\frac{1}{Pr(a_0 = buy)} + \frac{1}{Pr(a_0 = sell)}\right) \tag{2}$$

where $\omega_0 = Var(V) = p_0(1 - p_0)$ is the prior variance in the asset's value and $m = \frac{1-\mu}{4}$, is the probability of observing a buy (or sell) order from an uninformed trader in either period.

If no public information arrives, the expected profit from waiting, denoted $\pi_1^{NP}(\beta, p_0, q_1)$, is very similar:[15]

[15] In deriving (3), note that observing no trade in the first period generally provides information about whether or not the trader is informed, but this information becomes redundant upon observing an order in the second period because, by assumption, a trade can only occur in the second period if no trade occurred in the first (i.e. $Pr(V = 1|a_0 = no\ trade, a_1 = buy) = Pr(V = 1|a_1 = buy)$, and similarly for a sell order).

$$\pi_1^{NP}(\beta, p_0, q_1) = \omega_0 m(2q_1 - 1)\left(\frac{1}{Pr(a_1 = buy)} + \frac{1}{Pr(a_1 = sell)}\right) \tag{3}$$

If public information arrives on the other hand, one must sum account for the possible realizations of the public event and their impact on prices. In this case, the expected profit from waiting is given by

$$\begin{aligned}\pi_1(\beta, p_0, q_1) &= \omega_0 m(2q_1 - 1)\sum_{e^i \in E}\left(\frac{Pr(e = e^i|V = 0)Pr(e = e^i|V = 1)}{Pr(a_1 = buy\&e = e^i)}\right.\\ &+ \left.\frac{Pr(e = e^i|V = 0)Pr(e = e^i|V = 1)}{Pr(a_1 = sell\&e = e^i)}\right) \end{aligned} \tag{4}$$

Armed with expressions for the expected profits, I turn first to the question of the value of additional information.

2.3 The Value of Information

In this section, I establish the benchmark result with a monetary cost of information. Throughout the analysis, I focus on how the informed trader behaves as a function of the prior , p_0, which maps one-to-one to the prior uncertainty in the asset value, $\omega_0 = p_0(1-p_0)$. Intuition suggests that when uncertainty is high, private information of fixed quality is more valuable because it results in a larger difference between a trader's belief and the public belief, allowing her to earn a larger expected profit. Lemma 2 shows that this intuition is correct: given either period's information quality, the informed trader's expected profit peaks at the highest value of uncertainty which occurs when $p_0 = \frac{1}{2}$. It also establishes a symmetry property due to the fact that the profit from obtaining a particular signal realization at a belief of p_0 is the same as that from obtaining the opposite realization at a belief of $1 - p_0$.

Lemma 2: *The expected profits at $t = 0$ and $t = 1$ are both symmetric and concave in the prior: they peak at maximum prior uncertainty, $p_0 = \frac{1}{2}$, and decrease to zero at $p_0 = \{0, 1\}$.*

Lemma 2 suggests that the informed trader would be willing to pay the most for information when $p_0 = \frac{1}{2}$. In fact, this result has been previously established in models with monetary costs. In a sequential trading model with a fixed cost of information, Nikandrova (2014) shows that traders are only willing to pay for information when p_0 is near $\frac{1}{2}$. Lew (2013) instead allows traders to pay to increase the probability that they learn the asset value for certain. With a cost that is quadratic in this probability, he also shows that traders acquire the most information when $p_0 = \frac{1}{2}$.

Here, traders face a choice between signals of different quality, unlike in the papers by Nikandrova (2014) and Lew (2013). The question is then whether or not the *marginal* value of the better signal peaks at $p_0 = \frac{1}{2}$. Perhaps surprisingly, the answer is - not always. The reason is that there can be a value (or cost) to waiting that has little to do with information quality, but instead has to do with the difference between trading costs due to differences in the bid-ask spreads across periods. My approach to understand the value of information is two-fold. First, I take a revealed preference approach: if the informed trader is more willing to pay a fixed cost to trade in the second period with better information, then this option must be more valuable to her. Second, I consider the limit as the probability of informed trading becomes small such that the trading costs due to the bid-ask spread become negligible.

Consider a setup identical to that described in Section 2.1 except that, instead of public information arriving while a trader waits, the trader faces a cost of delay, $c \in (0, 1)$, which must be paid if she trades in the second period. This cost may reflect an opportunity cost, for example. I could instead assume the cost is only paid when a trader obtains a high quality signal, but to be consistent with the public information model, I assume the cost is paid even if one acquires the low quality signal and then delays trading to the second period. In this way, the models are identical except that the cost of forgone profits due to a public event is replaced with a fixed cost. If the informed trader is willing to pay this fixed cost, then the value of trading in the second period must be higher.

When better information takes time to generate, the profits $\pi_0(\beta, p_0, q_0)$ and $\pi_1^{NP}(\beta, p_0, q_1)$ depend upon the market maker's belief, β, about the information acquisition strategy of the informed trader. In equilibrium, this belief must coincide with the trader's actual information acquisition strategy, $\beta = \hat{\beta}$. Denoting the equilibrium probability of rushing for the fixed cost case, β^{C*}, an equilibrium must satisfy

$$
\begin{array}{rclcrcl}
\beta^{C*} & = & 1 & if & \pi_1^{NP}(1, p_0, q_1) - c - \pi_0(1, p_0, q_0) & \leq & 0 \\
\beta^{C*} & \in & (0,1) & if & \pi_1^{NP}(\beta^{NP*}, p_0, q_1) - c - \pi_0(\beta^{NP*}, p_0, q_0) & = & 0 \\
\beta^{C*} & = & 0 & if & \pi_1^{NP}(0, p_0, q_1) - c - \pi_0(0, p_0, q_0) & \geq & 0
\end{array}
$$

For many parameterizations, β^{C*} is interior: intuitively, the higher the probability that the market maker places on informed trade in a period, the larger the spread between the bid and ask prices must be. An increase in the spread in turn reduces the profit from trading, causing the trader to shift trade to the other period. This argument suggests a unique equilibrium, which I confirm in Proposition 2.

Proposition 1: *With a monetary cost of information, if the probability of an informed trader is not too large (i.e.* $\exists \hat{\mu} > 0$ *such that* $\forall\ 0 < \mu \leq \hat{\mu}$*), a unique equilibrium exists in which the informed trader rushes with probability,* $\beta^{C*} \in [0, 1]$*, trades in the period she acquires information, and buys or sells according to Lemma 1.*

Proposition 1 establishes a unique equilibrium in which the informed trader trades in the period she acquires information. As I discuss in the introduction, this result, and most of the subsequent results, require that the probability of informed trading not be too large so that trading costs due to the bid-ask spread do not dominate the value of information. In Section 4, I briefly discuss the intuition in more detail and, more importantly, show numerically that the upper limits on μ required for the results to hold are large relative to the most conservative empirical estimates.

Turning to the question of how the marginal value of information varies as a function of the prior uncertainty, Proposition 2 establishes that traders are willing to pay more for better information (wait more) when uncertainty is higher: the marginal value of the better quality signal is higher at this time.[16]

Proposition 2: *With a monetary cost of information, if the probability of an informed trader is not too large, the informed trader acquires the better signal most often when the prior uncertainty is largest:* β^{C*}*, is a minimum when uncertainty is highest,* $p_0 = \frac{1}{2}$*, and weakly increases as* $p_0 \longrightarrow \{0, 1\}$*.*

Proposition 2 shows that, with monetary costs, informed traders acquire the better signal most often when $p_0 = \frac{1}{2}$. In the following section, we see that the nature of the time cost of public information arrival is such that this result is abruptly overturned: traders in fact acquire the *least* information in precisely this situation.

2.4 The Time Cost of Information

This section studies how public information arrival between trading periods creates a time cost of performing research using the model described in Section 2.1. I first establish that public information does in fact impose a cost. Lemma 3 shows that the expected profit from acquiring better information is smaller when public information arrives than when it does not.

[16]Specifically, the informed trader pays a monetary cost of $c(1 - \beta^{C*})$ which decreases in β^{C*}, peaking at maximum uncertainty.

Lemma 3: *The expected profit from waiting decreases with the arrival of public information,* $\pi_1(\beta, p_0, q_1) < \pi_1^{NP}(\beta, p_0, q_1)$.

Lemma 3 stems from the fact that the value of additional information is proportional to prior uncertainty about the asset value, and that public information reduces this uncertainty. This same effect is present in the model of Holden and Subrahmanyam (1992) who show that information released by others' trades causes traders to act more quickly on their own information. However, in their model, traders know the asset value perfectly so that acting more quickly improves short-term informational efficiency, while leaving long-term efficiency unchanged. Here, traders face a trade-off in terms of the quality of information, so we can ask when the effect of the information release on incentives to do time-consuming research, and therefore the effect on long-term informational efficiency, is greatest. I answer this question in Proposition 4, after establishing the existence of a unique equilibrium in Proposition 3.

Proposition 3: *If the probability of an informed trader is not too large, a unique equilibrium exists in which the informed trader rushes with probability,* $\beta^* \in [0, 1]$, *trades in the period she acquires information, and buys or sells according to Lemma 1.*

Proposition 4: *If the probability of an informed trader is not too large, the equilibrium probability that she rushes,* β^*, *is largest when uncertainty is highest,* $p_0 = \frac{1}{2}$, *and weakly decreases as* $p_0 \longrightarrow \{0, 1\}$. *The decrease is strict whenever* $\beta^* \in (0, 1)$.

Proposition 4 characterizes the equilibrium probability of rushing as a function of prior uncertainty, showing that, opposite to the case of a monetary cost of information, the informed trader acquires better information *least* often when $p_0 = \frac{1}{2}$. The result stems from the fact that the reduction in uncertainty due to the public information release is more consequential when the prior uncertainty is higher. Consider the case of very low uncertainty, $p_0 \to \{0, 1\}$. In this case, public information reveals almost nothing new and therefore changes prices and the expected profit from waiting only negligibly. On the other hand, when p_0 is close to $\frac{1}{2}$, the change in expected profit due to a public event is large.

Alternatively, we can think of the impact in terms of price movements. Due to unconditional correlation between the trader's information and the information that becomes public, in expectation these price movements move in such a way as to reduce the trader's profit. When p_0 is near one half, the public event causes market prices to move substantially, whereas prices move very little when uncertainty is low.

To put the time cost of public information in another light, we can calculate the monetary cost equivalent of the public information event: the cost that induces the same probability of rushing. Provided the equilibrium probability of rushing, β^*, is interior, this cost is defined by $c^P(p_0, q_0, q_1) \equiv \pi_1^{NP}(\beta^*, p_0, q_1) - \pi_0(\beta^*, p_0, q_1)$.[17] Proposition 5 shows that c^P monotonically increases with uncertainty as one would expect given the previous results.

> **Proposition 5:** *If the probability of an informed trader is not too large and the equilibrium probability of rushing that public information arrival induces is interior, $\beta^* \in (0, 1)$, then the monetary cost, $c^P(p_0, q_0, q_1)$, required to induce the same equilibrium probability of rushing is largest when uncertainty is highest, $p_0 = \frac{1}{2}$, and strictly decreases to zero as $p_0 \longrightarrow \{0, 1\}$.*

Comparing Propositions 2 and 4, we see a stark difference between a model with monetary costs and one with time costs due to public information arrival. Although one is willing to pay more in monetary terms when uncertainty is large, due to the nature of the time cost, less information is acquired under this same condition. Thus, although it is possible to interpret the monetary cost in a standard model as a time cost, when we model it explicitly, we see that it must take a non-standard form. Rather than simply being a weakly increasing, convex function of signal quality (see, for example, Grossman and Stiglitz (1980) and Verrecchia (1982)), the cost must also monotonically increase with the prior uncertainty in the asset value (Proposition 5).[18]

A further implication of traders rushing more often when uncertainty is highest is that they are forgoing information precisely when it contributes most to the long-term informational efficiency of the market. To measure efficiency, we can consider the standard pricing error given by $\mathbb{E}_2 \equiv E\left[(V - p_2)^2\right]$, where p_2 is the public belief after both trading periods. A larger pricing error corresponds to less informative prices. As I show in part B of the online Appendix, for a given information acquisition strategy, the improvement in the long-term informational efficiency is greatest in relative and absolute terms when $p_0 = \frac{1}{2}$. To the extent that it is the long-term informational efficiency of market prices that matters for real decisions, the time cost of information is significant, preventing information acquisition when it most improves informational efficiency.

[17] When β^* is a corner solution, the monetary cost is bounded, but not uniquely defined. For ease of exposition, I focus on the interior case.

[18] In Veldkamp (2006a,b), competitive markets for information generate prices which depend upon the number of investors buying the information, which in turn depend upon asset variance. Her model therefore also generates a dependence of the cost of information on asset variance, but through a very different mechanism.

3 Competitive Model

This section provides the main results of the paper, introducing a second trader to demonstrate that competition can induce traders to rush to trade on weak information, and does so more when uncertainty is higher.

I replace the public event with the possibility that a second trader, who is informed with probability, $\mu_2 \in (0,1)$, arrives to the market during the time the first trader does research.[19] The second trader arrives with probability $\alpha \in (0,1]$, which serves as a measure of the level of competition. The structure of the asset and the signals available to the first trader are identical to the single trader model. I also allow the second trader to choose either a weak signal in the first period or a strong signal in the second period to be consistent with the choice faced by the first. Signals are independent across traders. I use superscripts to identify the trader associated with an action (e.g. a_0^1 is the trade of trader 1 in period 0) and with a probability of rushing (e.g. β^1 is the probability trader 1 rushes).

The potential arrival of the second trader results in the overlapping structure shown in Figure 2.[20] Rather than describe the model in terms of four periods, to preserve notation, I continue to label the periods 0 and 1, but divide each period into two subperiods, with trader 1 having the first opportunity to trade in each. In each period, after a trade by the first trader, the market maker updates his beliefs and posts new bid and ask prices for the second trader.[21]

My primary focus is on the behavior of trader 1 who must be concerned that trader 2 may arrive and preempt her trade. However, note that trader 1 also impacts trader 2 when both wait. I assume trader 2 observes whether or not trader 1 trades and therefore knows when she can freely do research without any possible intervening trade.[22] I study the Perfect Bayesian Equilibrium in which each trader accounts for the actions of the other in choosing her information acquisition and trading strategies.

For each trader, the potential informed trade by the other trader is a form of a public event so that many of the results of the previous sections apply directly to the model with competition. In particular, Lemma 1 applies immediately so that both traders trade according to their private signals, and, assuming a trader trades immediately after acquiring information, each has an expected profit from rushing given by equation (2).

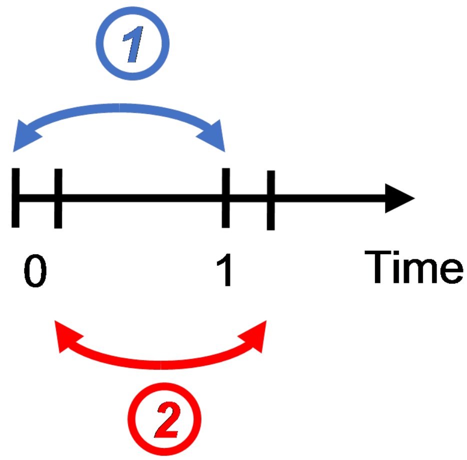
[19] I relabel the probability trader 1 is informed, μ_1, and define $m_1 \equiv \frac{1-\mu_1}{4}$ and $m_2 \equiv \frac{1-\mu_2}{4}$.

[20] The overlapping structure reflects the reality of most markets that trades occur sequentially, but it also frees me from making assumptions about how to handle simultaneous orders. See Malinova and Park(2014) for an example of a model with simultaneous orders.

[21] Note that none of the bid and ask prices depend upon α because, upon observing a second trade within a period, the market maker directly learns of the second trader's presence.

[22] If she can't observe the trade directly, she can infer it from the resulting change in prices.

Figure 2: Overlapping Timing Structure



Note: Trader identifies are shown in circles. Time periods are identified below the time line.

Trader 1's expected profit if she waits is given by $\Pi_1^1(\beta^1, \beta^2, p_0, q_1) = \alpha\pi_1^1(\beta^1, \beta^2, p_0, q_1) + (1-\alpha)\pi_1^{NP}(\beta^1, p_0, q_1)$, where the second trader arrives with probability α. $\pi_1^1(\beta^1, \beta^2, p_0, q_1)$ is given by (4) and depends on β^2 through the probabilities of three possible public events corresponding to the possible trades by trader 2 : buy, sell, or no trade.[23]

In the case of trader 2, if she observes trader 1 trade, she rushes with probability β^{NP*}, defined to be the probability a trader rushes when no public information event occurs.[24] On the other hand, if she observes trader 1 not trade, then her expected profit from waiting must be modified to condition on this event:

$$\begin{aligned}\pi_1^2(\beta^1, \beta^2, p_0, q_1) &= \frac{\omega_0 m_2(2q_1-1)}{\mu(1-\beta^1)+2m_1}\sum_{e^i\in E}\left(\frac{Pr(e^i|V=0)Pr(e^i|V=1)}{Pr(a_1^2=buy\&e=e^i)}\right. \\ &+ \left.\frac{Pr(e^i|V=0)Pr(e^i|V=1)}{Pr(a_1^2=sell\&e=e^i)}\right) \qquad (5)\end{aligned}$$

β^1 enters $\pi_1^2(\beta^1, \beta^2, p_0, q_1)$ through the probabilities of two public events, a purchase or sale by trader 1. Proposition 6 establishes that, provided μ_1 and μ_2 are not too large, a unique equilibrium exists in which each trader trades in the period they acquire information.

[23]This set of public events is consistent with the assumptions made about public events in Section 2.1. The buy and sell events are informative and symmetric ($Pr(a_0^2 = buy|V=1) = Pr(a_0^2 = sell|V=0)$ and vice versa). The no trade event is uninformative and symmetric ($Pr(a_0^2 = no\,trade|V=1) = Pr(a_0^2 = no\,trade|V=0)$)=(1-$\alpha$) + $\alpha(\mu\beta^{2*} + m_2)$.

[24]β^{NP*} depends upon the difference between $\pi_1^{NP}(\beta, p_0, q_1)$ and $\pi_0(\beta, p_0, q_0)$ and its uniqueness can be shown in an identical manner to the proof of Proposition 1. It is also possible to show $\beta^{NP*} \in [0, \frac{1}{2})$. Intuitively, with equal signal strengths, the symmetry of the problem requires $\beta^{NP*} = \frac{1}{2}$, and in the absence of any cost, a stronger second period signal induces the informed trader to rush with lower probability (details available upon request).

Proposition 6: *If the probability that each trader is informed is not too large (i.e. $\exists \hat{\mu}_1, \hat{\mu}_2$ such that $\forall\ 0 < \mu_1 \le \hat{\mu}_1, 0 < \mu_2 \le \hat{\mu}_2$), a unique equilibrium exists in which each trader trades in the period she acquires information, and buys or sells according to Lemma 1. Trader 1 rushes with probability, $\beta^{1*} \in [0,1]$. Trader 2 rushes with probability, $\beta^{NP*} \in [0, \frac{1}{2})$, when trader 1 rushes, and $\beta^{2*} \in [0,1]$ when trader 1 waits.*

The uniqueness of the rushing probabilities in Proposition 2 is not immediate because there is strategic complementarity in the timing decisions from the point of view of trader 1. As trader 2 waits more often, it reduces the impact on trader 1, causing her to best respond by waiting more often. However, the rushing probabilities are unique because of strategic substitutability from the point of view of trader 2: as trader 1 waits more often, it increases the impact on trader 2 causing her to best respond by rushing more often.[25]

Proposition 7 establishes several properties of the equilibrium which form the basis for the empirical predictions in Section 5.

Proposition 7: *If the probability that each trader is informed is not too large then, in equilibrium:*

a) Trader 1 rushes more often due to the potential competition of trader 2, $\forall \alpha > 0$, $\beta^{1*} \ge \beta^{NP*}$, with strict inequality whenever $\beta^{NP*} > 0$.

b) As the probability that trader 2 arrives increases, trader 1 rushes weakly more often, $\frac{d\beta^{1*}}{d\alpha} \ge 0$, with strict inequality whenever $\beta^{NP*} > 0$ and $\beta^{1*} \in (0,1)$.

c) *The probability that trader 1 rushes, β^{1*}, is largest when uncertainty is highest, $p_0 = \frac{1}{2}$, and weakly decreases as $p_0 \longrightarrow \{0,1\}$. The decrease is strict whenever $\beta^{1*} \in (0,1)$.*

d) *The probability that trader 1 rushes is strictly increasing in the first period signal strength, q_0, whenever it is interior: $\frac{d\beta^{1*}}{dq_0} > 0\ \forall \beta^{1*} \in (0,1)$.*

Part a) of Proposition 7 shows that the potential arrival of the second trader causes trader 1 to rush more often than when trader 2 doesn't exist, and part b) shows that trader 1 rushes more often as trader 2's probability of arrival increases. Intuitively, the increased chance of a price impact makes it less profitable to wait. Given that trader 2 impacts trader 1, we expect from Proposition 4 that i) trader 1 rushes most often when uncertainty is highest. However, this result does not follow immediately as a corollary because, unlike in the case of an exogenous public event, one must account for the fact that β^{2*} also changes when the

[25] The nature of strategic substitutability and complementarity are very different here from in static models such as Grossman and Stiglitz (1980). In a static model, the strategic interaction in information acquisition comes about through simultaneous learning through prices due to rational expectations. Here, instead, the strategic interaction is through the dynamic price impacts (or lack of) of others' trades.

prior uncertainty changes. Nevertheless, part c) of Proposition 7 establishes that the result holds. Finally, part d) provides an intuitive comparative static for the initial signal quality. As q_0 increases, trader 1 rushes more often both because she receives a larger profit in the first period and because trader 2's price impact becomes larger if trader 1 waits.[26]

4 Limits on the Probability of Informed Trading

In this section, I briefly discuss why many of the previous results rely on a probability of informed trading that is not too large, and then show numerically that the upper limits required are rarely binding.

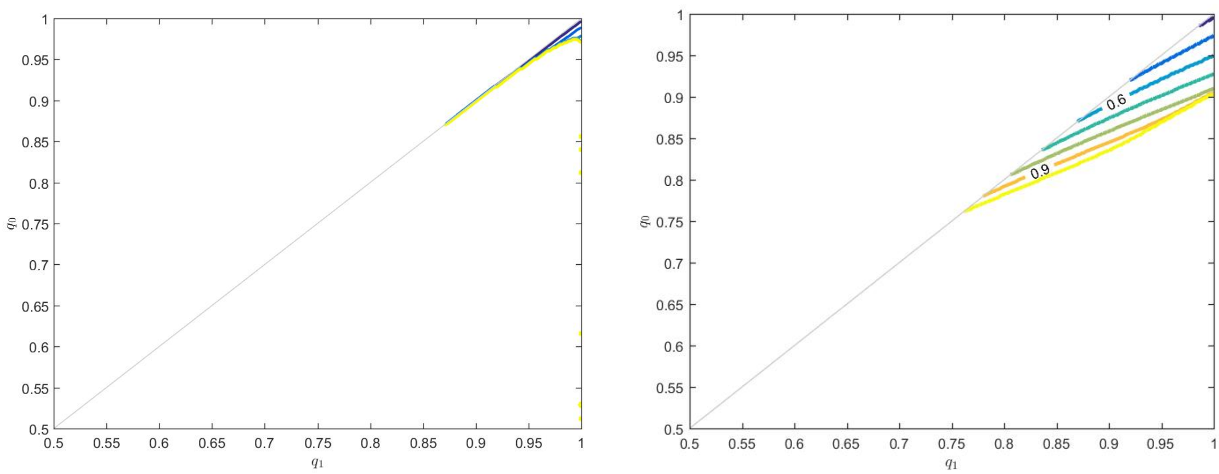
Propositions 1, 3, and 6 establish a unique equilibrium in which the informed trader, after rushing, trades immediately. When the probability of informed trading becomes large, the informed trader may no longer find this strategy optimal, instead preferring to delay her trade to $t = 1$ (or to mix between the two periods). Intuitively, when the cost of waiting is large so that informed trade is concentrated at $t = 0$, then the spread at this time is large. In this case, after acquiring a signal at $t = 0$, it can be optimal to delay trading until $t = 1$ where the spread is smaller. When μ is not too large, spreads are also small relative to the cost of waiting so that they have little impact.

The results that characterize the equilibrium information acquisition strategy of the informed trader (Propositions 2, 4, and 7) also require the probability of informed trading not be too large, but for a slightly different reason. When the probability of informed trade is small, because (by Lemma 2) the expected profit in each period is concave and approaches zero as public beliefs become extreme, as we move towards $p_0 = \frac{1}{2}$, the expected profit from waiting increases more than that from rushing, causing the trader to wait more often (in the absence of any additional cost beyond the trading cost). In this case, we have the standard intuition that the marginal value of better information is highest at high uncertainty. However, when the probability of informed trading is large, trades in the second period can be very informative if trade is (endogenously) concentrated there. In this case, as we move towards $p_0 = \frac{1}{2}$, is it possible that the expected profit from waiting actually increases *less* than that of the expected profit from rushing so that a trader may actually rush more often. The marginal value of information is distorted by the large trading costs in this case.

I now show through numerical simulations when these distortions become significant,

[26]It is possible to establish this comparative static in the case of a single trader as well, but I omit it for brevity. The comparative static with respect to the second period signal quality is more cumbersome in the competitive case, but it is easy to show the trader waits more as q_1 increases in the single trader case. Numerical simulations suggest that the comparative statics with respect to the probabilities of informed trading, μ_1 and μ_2, are non-monotonic.

Figure 3: Threshold Probability of Informed Trading



Note: Contour plots of the upper limit on the probability of informed trading, $\hat{\mu}$, necessary to guarantee the unique equilibrium is one in which informed traders trade in the period they obtain information (left graph), and to guarantee the probability that the first trader rushes peaks at $p_0 = \frac{1}{2}$ (right graph). For the left graph, I plot the minimum value of $\hat{\mu}$ over the whole range of the prior, $p_0 \in (0, 1)$. The contour lines in both graphs are in increments of 0.1 with the lowest corresponding to $\hat{\mu} = 1$.

focusing on the main case of interest - the model with competition. I numerically calculate the upper limit, $\hat{\mu}$, such that the unique equilibrium is one in which each trader trades in the period she acquires information (Proposition 6), and also the upper limit that ensures the first trader acquires the least information at highest uncertainty (Proposition 7, part c)). To do so, I set $\mu = \mu_1 = \mu_2$ and calculate the minimum limit for each pair of signal qualities, q_0 and q_1. For the limit guaranteeing equilibrium uniqueness, I take the minimum limit over all $p_0 \in (0, 1)$.[27] Figure 3 plots contour maps of the calculated limits.

The left graph of Figure 3 shows that the limit required to ensure equilibrium uniqueness only binds in the extreme case in which both signal qualities approach one. From the right graph, we see that when both signal qualities exceed about 0.75, the $\hat{\mu}$ necessary to guarantee the time cost is largest at maximum uncertainty begins to bind: the combination of a high probability of informed trade and strong signal qualities makes the effect of the bid-ask spread more pronounced. To put this limit on the probability of informed trade into perspective, the most conservative estimate of it which I'm aware is that obtained by Cipriani and Guarino (2014). They estimate $\mu = 0.42$ where their model defines μ to be conditional on information being available, and allows for informed traders to have imperfect

[27] I calculate $\hat{\mu}$ on a grid of 0.002 for each pair of signal qualities. Due to symmetry, it is only necessary to simulate for $p \in [0.5, 1)$, but one must check that neither trader 1 nor trader 2 prefers to delay when receiving either a positive or negative signal.

information.[28] Comparing their estimate with $\hat{\mu}$ in Figure 3, we see that the main results of the paper continue to hold except in the very extreme case of signal qualities that approach one. One could argue instead that, if information is only available on news days (as in the models of Cipriani and Guarino (2014) and Easley et al. (1996)), and that the marker maker can't condition on news being available or not, then the unconditional probability of informed trading is the correct measure for comparison because it determines the spread. Cipriani and Guarino (2014) estimate this probability to be 0.19. At this value, the main results hold for all parameterizations of the model.[29]

5 Empirical Implications

The primitives of the competitive model are the prior, the two signal qualities, and the degree of competition (arrival rate of subsequent trader), α. It makes predictions about the information that trades contain and also the time at which trades occur. Because information is not directly observable, in this section, I develop testable predictions in terms of observables. I continue to focus on the first trader in the two trader model. For the length of a period, I have in mind times on the order of seconds, minutes, or perhaps hours. For information that takes longer (days) to produce, factors in addition to price movements are likely to play an important role in information acquisition decisions, such as opportunity costs.

For many common observables, competition and uncertainty have a countervailing effect to the direct effect of information quality, so that no straightforward prediction exists. For example, volatility directly increases with α and uncertainty, but decreases when information is weaker because each trade reveals less information. Similarly, the bid-ask spread and the price impact of a trade ($p_{t+1} - p_t$) decrease when information is weaker, but increase with uncertainty. The net effects on these variables are therefore ambiguous.

An observable for which unambiguous predictions exists is the order flow imbalance: the difference in buy vs. sell-initiated orders. Intuitively, a larger imbalance suggests traders have stronger private information. This intuition forms the basis for the identification of the popular probability of informed trading, or PIN, measure of Easley et al. (1996). When considering the first trader only, the expected difference between the number of buys and sells at the end of the second period is simply, $Pr(a_0 = buy) + Pr(a_1 = buy) - Pr(a_0 = sell) - Pr(a_1 = sell)$. Because positive and negative differences are equally informative, I consider the absolute value of this measure, $|E[IB]| \equiv |Pr(a_0 = buy) + Pr(a_1 = buy) -$

[28]The traditional probability of informed trading measure, PIN, of Easley et al. (1996), assumes instead that informed trades are perfectly informed, resulting in lower probability of informed trading estimates.

[29]The minimum value of $\hat{\mu}$ is 0.38.

$Pr(a_0 = sell) - Pr(a_1 = sell)|$.[30] Accounting for the probability that the first trader rushes, it is easy to show that

$$|E[IB]| = \mu_1 |2p_0 - 1| \left((2q_0 - 1)\beta^{1*} + (2q_1 - 1)(1 - \beta^{1*})\right)$$

I also consider the expected change in volume across periods, given by[31]

$$E[Vol_1 - Vol_0] \equiv Pr(a_1 = buy) + Pr(a_1 = sell) - Pr(a_0 = buy) - Pr(a_0 = sell) = \mu_1(1 - 2\beta^{1*})$$

The expected change in volume is a measure of the frequency of rushing.

Consider first an increase in the probability that another trader arrives , α. Because an increase in α induces more rushing (Proposition 7, part b)), we have[32]

> **Prediction 1:** *As the probability that another trader arrives increases, the order imbalance decreases and volume shifts earlier in time.*

Prediction 1 is perhaps most easily tested in the time immediately following an earnings announcement. If one assumes that public news releases are differentially interpreted (as in Holthausen and Verrecchia (1990), Indjejikian (1990), and Kandel and Pearson (1995)), then news generates private information, which takes time to process into a trading decision. Prediction 1 states that when it is more likely other traders are processing the same information (which could perhaps be proxied with total volume over some time period, such as a day) then volume concentrates around the announcement and the order flow is more balanced.[33] Although not a direct test of the model, Li (2015) provides evidence that market prices do not immediately incorporate the public information in earnings announcements, and that it can be profitable to rush based on weak information. He shows that following a simple rule of buying a stock as soon as possible if both the revenue and earnings per share targets are met, and selling if neither is met, beats the market by 11.5% per year after costs.

Models with fixed information quality that allow traders to time their trading decisions, such as Holden and Subrahmanyam (1992) and Malinova and Park (2014), make the same prediction with respect to volume, but, to my knowledge, the order imbalance prediction is novel. It is driven by the fact that as traders rush, the overall quality of information

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[30] If one instead takes the expectation of the absolute value, as in Malinova and Park (2010), the prediction is trivially always one. For this reason, I use a slightly different definition.

[31] I don't consider overall volume because, when considering a change in the probability of trader arrival, α, overall volume clearly increases exogenously, and for other comparative statics, the model predicts total volume is constant.

[32] The proofs of the predictions are straightforward calculations and are available upon request.

[33] Alternatively, as suggested by Malinova and Park (2010) changes in α could be a result of index inclusion, deregulation, cross-listing, or international market openings.

decreases.[34,35]

Easley et al. (1996) provide indirect evidence of the order imbalance prediction of Prediction 1 through a structural model. They use the order flow imbalance to simultaneously identify days with news and the rates of arrival of informed and uninformed traders. They find that higher volume is associated with a lower PIN and hence more balanced order flows.[36]

Next, consider the comparative static with respect to prior uncertainty about the asset value, ω_0. Because higher uncertainty is associated with a higher probability of rushing (Proposition 7, part c)), we have[37]

> **Prediction 2:** *When the prior uncertainty about the asset value is higher, the order imbalance decreases and volume shifts earlier in time.*

The prediction about order imbalance in Prediction 2 is driven by two reinforcing effects. First, when uncertainty is higher, order flows are naturally more balanced ($p_0 = \frac{1}{2}$ results in an order imbalance of zero). Second, when traders rush more, they have weaker information. Due to the first effect, the order flow imbalance prediction occurs even with exogenous information, and is therefore not unique to the model. The volume prediction is, however, unique to my knowledge, being driven by the fact that the time cost of waiting increases with

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[34]Setting $q_0 = q_1$ in the model, we can see that an increase in α has no effect on the order imbalance when information is of constant quality. In Malinova and Park (2014), the trade imbalance is higher in the first period. They do not perform the comparative static exercise on the trade imbalance as competition increases, but because information is of constant quality, it is not likely to decrease. The models of Admati and Pfleiderer (1988) and Foster and Viswanathan do not directly consider order imbalances or changes in the level of competition, but their models do make predictions about how volume and price informativeness covary over time. In Admati and Pfleiderer (1988), volume and price informativeness are positively correlated because traders are more willing to pay a fixed, monetary cost to obtain information when liquidity trading volume is higher. My model's prediction is the opposite due to the endogenous time cost of information. In Foster and Viswanathan (1990) informed trading volume shifts towards the first period due to the release of public information, as in my model, but either relationship between price informativeness and volume is possible.

[35]Models of salience, such as that in Barber and Odean (2008), predict that individual investors are attracted to salient events, such as news releases. If these individual investors are more likely to buy, as they find, then higher volume is associated with higher order imbalances, contrary to Prediction 1. However, Prediction 1 comes about from informed traders trading on weaker information, whereas their finding is driven by individual, presumably uninformed, investors. If uninformed traders do not trade in a balanced fashion, as assumed in the model, it creates a confound that needs to be controlled for.

[36]The other traditional method of estimating the informational content of trades is to measure the persistence in prices using a VAR approach (Hasbrouck (1991)). However, the model does not make an unambiguous prediction about the price impact as I note at the start of this section.

[37]In addition to predicting smaller order flow imbalances, the model also predicts more frequent deviations from fundamental values (mispricing) during times of high uncertainty ($Pr(buy|V=0) = Pr(sell|V=1) = \mu_1 \left(\beta^{1*}(1-q_0) + (1-\beta^{1*})(1-q_1)\right)$ increases with uncertainty). Models with exogenous information predict no relationship. Limits to arbitrage (see Shleifer and Vishny (1997) and Mitchell et al. (2002)) can also cause mispricing when volatility is high due to increased arbitrage risk. To the extent volatility is related to uncertainty, these two explanations provide similar predictions. However, as discussed at the start of this section, volatility and uncertainty are not necessarily positively correlated.

uncertainty.[38] Considering news releases, we'd expect to see announcements that generate more uncertainty having more balanced order flows and also higher concentrations of volume immediately after the announcement, relative to those that resolve uncertainty.[39]

Because estimates of the PIN are identified off of order flow imbalances, a corollary of Prediction 2 is that traditional estimates of the PIN should be smaller when uncertainty is higher. This relationship has been explored in cross-sectional studies (Kumar (2009) and Aslan et al. (2011)) with mixed results. The main difficulty is in finding a proxy for uncertainty about the fundamental value of the asset that is uncorrelated with the information traders possess through any other channel.[40] Because of this difficulty, a time-series study in which one can control for firm-specific effects may be better able to test Prediction 2. Potential proxies for uncertainty include the implied volatility index of the market (VIX), or dispersion in analyst forecasts.[41]

Finally, the model makes a prediction with respect to the initial signal quality. Proposition 7, part d), states that we expect more rushing when q_0 increases which leads to Prediction 3.[42]

Prediction 3: *When the first period signal quality increases, volume shifts earlier in time.*

Signal qualities may be affected by changes in regulations regarding disclosure or transparency, or improvements in technology. In particular, the internet and improvements in computing power have almost certainly improved the quality of information available immediately, while potentially leaving longer-term information quality unchanged. In this case, we'd expect volume to currently be more concentrated around announcements than in the

[38] This result does not require $q_1 > q_0$, but with fixed information quality, $q_0 = q_1$, the probability of informed trading must be large enough to induce waiting to avoid a large spread. If not, traders always rush so that uncertainty has no predicted impact on when trades occur.

[39] For example, the announcement of the initiation of a merger or litigation may create uncertainty which is resolved at a later date.

[40] Both Kumar (2009) and Aslan et al. (2011)) use proxies including firm age, firm size, monthly volume turnover, industry, and idiosyncratic volatility, and volatility in earnings. Each of these proxies may plausibly affect information through other channels or, as with volatility, is an output of the model that has no straightforward relationship to uncertainty.

[41] For other cross-sectional evidence that is suggestive of less informed trades when uncertainty is higher, the underreaction in stock prices is stronger in stocks with higher uncertainty (Zhang (2006) and Jiang et al. (2005)). One interpretation of underreaction, as summarized by Zhang (2006), is that underreaction is "more likely to reflect slow absorption of ambiguous information into stock prices than to reflect missing risk factors". Under this interpretation, the fact the prices of stocks with higher uncertainty more slowly absorb information is consistent with the model. However, strictly speaking, the model does not capture underreaction (prices follow a martingale). Extending the model to capture underreaction is an interesting avenue for future research.

[42] An increase in q_0 leads to an ambiguous change in the order imbalance. The direct effect is to increase the imbalance, but this is countered by an increase in rushing.

past.

Rather than running reduced-form regressions to test the model, one could impose structure to provide identification, in the spirit of Easley at al. (1996). They assume informed traders are perfectly informed so that on a day with positive news, informed traders only buy, and conversely on days with negative news. The predictions of the model, however, are with respect to the quality of information informed traders possess, so one must allow for the possibility that they are misinformed. A recent paper by Cipriani and Guarino (2014) takes a significant step in this direction, using the order flow imbalance to jointly identify the arrival rates of informed and uninformed traders, but also the quality of information. They find that informed traders in fact have misinformation 40% of the time, and are able to reject the null of perfect information. Cipriani and Guarino (2014) analyze only a single stock using the entire available time series, but their methodology is well suited for testing the main mechanisms of the model. For example, one could estimate their model for a single stock at different points in time. The model predicts that low estimates of information quality coincide with high estimates for the rates of trader arrival. Or, one could divide time into periods of high and low uncertainty, perhaps based on the VIX, to test Prediction 2.

The novel empirical predictions of the model are driven both by the endogenous time cost of acquiring information and the fact that poor quality information can result in misinformed traders. An important consequence is that the way in which one tries to assess the informational content of trades depends upon how it is measured (PIN, bid/ask spread, order flow imbalance, volatility, etc.): not all measures need necessarily deliver the same results. Future empirical work will hopefully account for this variability in assessing the quality of traders' information.

6 Robustness

The key assumption that drives results is that better information arrives over time and traders must choose between either a weak or a strong signal. In this section, I argue that the other assumptions are less critical.

6.1 Other Equilibria

In Section 4, I show that, for almost all parameterizations of the model, the unique equilibrium is one in which traders trade immediately upon receiving their signals. For the small remaining region of the parameter space, this equilibrium may fail to exist. Instead, an equilibrium involves a careful construction of interdependent mixing probabilities. The probability with which the informed trader delay's her trade to $t = 1$ after rushing (which may depend on the signal one receives) generally involves mixing. An equilibrium then requires mixing probabilities that ensure indifference ex ante (prior to receiving any signal) and ex post (after receiving a signal at $t = 0$). While not conceptually difficult to construct such an equilibrium, it is unlikely to lead to any new insight, so I don't pursue it in detail.

6.2 Multiple Trades

To simplify the exposition I restricted the informed trader to a single trade if she rushes. Here, I argue that the main insight of the model is unlikely to change if she could acquire (or sell short) an additional unit in the second period, or instead unwind her position. In the latter case, the equilibrium is unchanged because, given that she doesn't receive any additional private information and that her information is not fully revealed by her trade, she still has a private belief more extreme than the market maker, so prefers to hold her position.

On the other hand, because the trader's belief is more extreme than that of the market maker even after trading, she would like to acquire (or sell) an additional unit in the second period if she could, earning a small additional profit and revealing more of her private signal to the market.[43] Therefore, if I were to allow a second trade, it would make rushing (weakly) more attractive which I conjecture can only lead to more rushing in equilibrium.[44] Certainly, by changing the profit to rushing, this variation changes the equilibrium probability of acquiring better information, but it doesn't change the nature of the impact of competition on second period trading profits, and therefore is unlikely to change the main message of the paper.

6.3 Multiple Signals

A natural question raised by the model is, why not obtain the weak signal, trade, and then obtain the strong signal? Conceptually, one can think about extending the model to more than two periods. Under the assumption that time spent obtaining the weak signal cannot be spent obtaining the stronger signal, one cannot obtain a stronger signal at $t = 1$ after obtaining a weak signal at $t = 0$. Instead, one has to wait until $t = 2$ for a stronger signal, but then one again has the option of obtaining a weaker signal at $t = 1$. For example, if weak signals come from processing news and stronger signals can only be obtained by in-depth

[43] In fact, this feature of the model is shared by the original Glosten and Milgrom (1985) model and the subsequent literature.

[44] The profit from rushing only weakly increases because, if the bid-ask spread in the second period is large enough, the second trade is not profitable.

financial modeling, one typically has the option to continue processing different aspects of the news. Given this, after obtaining a weak signal at $t = 0$, one faces a similar trade-off to that in the two period model.

Extending the model to multiple periods is non-trivial, however, because additional forces come into play. First, traders may possess residual private information not revealed by past trades, and their decision to obtain a weak or strong signal can be conditioned on this information, enlarging the strategy space. Second, position limits may alter the value of additional information (i.e. the value of information may be different if a trader can only unwind her initial position). Third, with many periods to trade over, a trader will want to spread out her trades to hide her information (as in Kyle (1985)). Finally, strategic incentives to manipulate prices in order to profit in the future may exist (Chakraborty and Yilmaz, 2004). For these reasons, although the basic intuition about the nature of the time cost extends beyond two periods, I leave the analysis to future work.

6.4 Non-binary Asset Value

The result in Proposition 7 that traders acquire less information when the prior uncertainty is highest is a result of two features of the time cost in the model. First, expected profits increase with uncertainty. Second, others' trades reduce uncertainty in the asset value when one waits, and more so when uncertainty is higher. Together these features imply the largest drop in the expected profit from waiting when uncertainty is high, which in turn encourages more rushing. In part C of the online Appendix, I show that these two features are also present in a model à la Kyle (1985) in which the asset value is normally distributed and the informed trader learns the asset value for certain in the second period, suggesting that the results are not driven by the binary nature of the asset's value.

6.5 The Role of the Bid-Ask Spread

In the model, the market maker can post different prices in each period, conditional on the quality of information available in each period. In some real life situations, such as after an earnings announcement, this knowledge of a 'time zero' is natural, but in other situations, it may be more natural to assume that the market maker instead only knows the average quality of information. With this assumption, I suspect the main results of the paper continue to hold for two reasons. The first is that I obtain results for very small values of the probability of informed trading where the bid-ask spread is close to zero in each period, so that it plays little role. Second, it is possible to write down a model in which the market maker posts only a single price equal to the expected value of the asset. In this case, I obtain a result very similar to Proposition 4 - better information is forgone at $p_0 = \frac{1}{2}$ when a trader would be most willing to pay a monetary cost for it. Therefore, the precise way the bid-ask spread is determined is not important for the results. And, neither is the fact that the trader has market power, because in this version she doesn't affect the price until after she trades.

7 Conclusion

This paper considers a model in which information can only be acquired through time-consuming research. Adverse price movements due to others' trades impose a time cost on acquiring better information. This time cost varies in a manner that causes traders to acquire the least amount of information at high prior uncertainty, the same condition under which they acquire the most information if information is acquired at a monetary cost. The model therefore suggests that it is not without loss of generality to model information costs as monotonically increasing functions of signal quality when studying dynamic settings.

The fact that information is acquired least when it contributes most to the long-term informational efficiency of the market suggests that, if long-term information efficiency is socially desirable, we may want to reduce or eliminate the time cost, if possible. This issue relates to the debate on market design.[45] In markets that are continuously open, the fear of being front run by other traders is present which, as shown, can cause traders to rush to trade on weak information. On the other hand, in markets that are cleared through call auctions, as long as the auctions are not too frequent relative to the time it takes to process information, traders can process information without fear. Thus, the model suggests that markets that are cleared through call auctions may be less prone to traders trading on weak information. Similarly, the model provides a potential justification for the practice of regulatory trading halts on stock exchanges when news is to be released. The temporary trading halt ensures traders have time to process information, which again prevents rushing to trade on weak information.[46] Future work could extend the main idea of the model to more explicitly address the market design question using a suitable welfare criterion.

Future theoretical work could also extend the model to non-binary asset values and more than two periods. Kendall (2016) instead extends the model to allow multiple traders to trade simultaneously over many periods, but simplifies along another dimension, removing the bid-ask spread. There, rushing is predicted to cause not only informational losses but also spikes in volume as a panic ensues: traders all trade in the first period, each attempting

[45] For a recent paper in this area that summarizes the previous theoretical and empirical work, see Kuo and Li (2011).

[46] Allowing after-hours trading, however, undoes this benefit.

to preempt the others. These predictions are confirmed in laboratory data. Future empirical work can use the ideas of the model to better understand volume and the informational content of trades, but, to do so, must allow for the possibility that informed traders may not have perfect information.

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Appendix

Proofs

Preliminaries:
Throughout, I use the abbreviated notation:

$$Pr(a_t = buy|V = x) \equiv buy_{t|Vx}, x \in \{0,1\}, t \in \{0,1\}$$

$$Pr(a_t \;=\; sell|V=x) \equiv sell_{t|Vx}, x \in \{0,1\}, t \in \{0,1\}$$

$$Pr(e \;=\; e^i|V=x) \equiv e^i_{|Vx}, x \in \{0,1\}$$

To derive (2), apply Bayes' rule to (1). We have

$$\begin{aligned}
\pi_0(\beta, p_0, q_0) &= (p_0q_0 + (1-p_0)(1-q_0)) \left(\tfrac{p_0q_0}{p_0q_0+(1-p_0)(1-q_0)} - \tfrac{p_0buy_{0|V=1})}{Pr(a_0=buy)}\right) \\
&+ (p_0(1-q_0) + (1-p_0)q_0) \left(\tfrac{p_0sell_{0|V=1})}{Pr(a_0=sell)} - \tfrac{p_0(1-q_0)}{p_0(1-q_0)+(1-p_0)q_0}\right) \\
&= \omega_0 \left(\tfrac{q_0buy_{0|V=0}-(1-q_0)buy_{0|V=1})}{Pr(a_0=buy)}\right) \\
&+ \omega_0 \left(\tfrac{q_0sell_{0|V=1}-(1-q_0)sell_{0|V=0}}{Pr(a_0=sell)}\right)
\end{aligned}$$

Symmetry implies $buy_{0|V=0} = sell_{0|V=1}$ and $buy_{0|V=1} = sell_{0|V=0}$ so that

$$\begin{aligned}
\pi_0(\beta, p_0, q_0) &= \omega_0 \left(q_0buy_{0|V=0} - (1-q_0)buy_{1|V=1}\right) \\
&\times \left(\frac{1}{Pr(a_0 = buy)} + \frac{1}{Pr(a_0 = sell)}\right)
\end{aligned}$$

We have $buy_{0|V=0} = \mu(1-q_0)\beta + m$, where the first term is the probability of observing a purchase order from an informed trader (who is believed to rush with probability β) and the second term, $m = \frac{1-\mu}{4}$, is the probability of observing a purchase order at $t = 0$ from an uninformed trader. Similarly, $buy_{1|V=1} = \mu q_0\beta + m$. Substituting these expressions then gives (2).

The following expressions are used repeatedly and follow from straightforward algebra:

$$\begin{aligned}
&Pr(a_t = buy\&e = e^i)Pr(a_t = sell\&e = e^i) \\
&= Pr(e^i)^2 buy_{t|V1}buy_{t|V0} + \omega_0 e^i_{|V1}e^i_{|V0}\left(buy_{t|V1} - buy_{t|V0}\right)^2
\end{aligned} \tag{6}$$

$$\begin{aligned}
&Pr(a_t = buy)Pr(a_t = sell) \\
&= buy_{t|V1}buy_{t|V0} + \omega_0\left(buy_{t|V1} - buy_{t|V0}\right)^2
\end{aligned} \tag{7}$$

$$\begin{aligned}
&Pr(a_t = buy)Pr(a_t = sell) \\
&= (2p_0-1)^2 buy_{t|V1}buy_{t|V0} + \omega_0\left(buy_{t|V1} + buy_{t|V0}\right)^2
\end{aligned} \tag{8}$$

The mathematical claim of Lemma A1 is used to prove several propositions.

Lemma A1: *The following inequality holds for any $x, y \in \mathbb{R}^+$, $n \geq 1$, and any $c_i, d_i \in [0,1]\ \forall i = 1\ldots n$ satisfying $\sum_{i=1}^n c_i = \sum_{i=1}^n d_i = 1$ and at least one of c_i or d_i greater than zero $\forall i = 1\ldots n$. Furthermore, it holds with equality if and only if $c_i = d_i \neq 0\ \forall i = 1\ldots n$.*

$$\sum_{i=1}^{n} \frac{c_i d_i}{c_i x + d_i y} \quad \leq \quad \frac{1}{x+y}$$

Proof of Lemma A1: See the online Appendix.□

Proof of Lemma 1:

The key to the proof is that the bid and ask prices are bounded by the possible beliefs the market maker may hold. Consider first a trader that has waited and received a positive signal, $s_1 = 1$. I claim that her private belief exceeds the maximum possible bid or ask price so that buying is optimal. The informed trader's belief is given by $Pr(V = 1|s_1 = 1) = \frac{p_0 q_1}{p_0 q_1 + (1-p_0)(1-q_1)}$ by Bayes' rule. The market maker's belief conditional on a buy is $Pr(V = 1|a_0 = no\, trade, a_1 = buy)$, where no trade in the first period may reveal information if the informed trader rushes with positive probability and conditions her decision to delay her trade based upon the signal she receives. Regardless of the information acquisition and trading strategies of the informed trader, the highest possible belief the market maker can have, conditional on observing a buy decision, is

$$\max A_1 \quad = \quad \frac{p_0(\mu q_1 + m)}{p_0(\mu q_1 + m) + (1-p_0)(\mu(1-q_1)+m)} \tag{9}$$

This belief arises when only a trader with a positive, strong signal buys and is the highest possible because of two facts. First, the posterior is clearly higher if a buy order reveals a positive signal as opposed to a negative signal, or some combination of both (if, for example, those with strong, positive signals and those with weak, negative signals both buy). Second, conditional on revealing a positive signal, the belief is increasing in the quality of the signal.

Simple algebra shows that (9) is strictly smaller than the informed trader's belief for $\mu < 1$. The maximum possible bid price is the same as the maximum possible ask price, so is also smaller than the trader's belief. For a trader that has waited and received a negative signal, $s_1 = 0$, the argument reverses: her private belief must be below the minimum possible bid or ask price, so she must sell.

A similar argument establishes that a trader that has rushed and trades at $t = 0$ must buy with a positive signal and sell with a negative signal. In this case, her private belief with a positive signal is $Pr(V = 1|s_0 = 1) = \frac{p_0 q_0}{p_0 q_0 + (1-p_0)(1-q_0)}$ and the maximum ask price is the strictly smaller value given by

$$\max A_0 \quad = \quad \frac{p_0(\mu q_0 + m)}{p_0(\mu q_0 + m) + (1-p_0)(\mu(1-q_0)+m)}$$

Finally, consider the trading strategy of a trader who rushes but delays her trade until $t = 1$. In this case, the above argument no longer applies because the private belief of a trader with a positive signal, $Pr(V = 1|s_0 = 1) = \frac{p_0 q_0}{p_0 q_0 + (1-p_0)(1-q_0)}$, is not necessarily larger than the maximum ask price in this period, given by (9). It is, however, larger than the maximum bid price, so selling is not optimal: given that a trader who waits to receive a strong signal sells when it is negative, the maximum possible bid price occurs when no trader waits and a trader who rushes but delays her trade buys with a negative signal:

$$\max B_1 = \frac{p_0(\mu q_0 + m)}{p_0(\mu q_0 + m) + (1 - p_0)(\mu(1 - q_0) + m)}$$

which is strictly less than the trader's private belief. Similarly, a trader who rushes and receives a negative signal can not buy at $t = 1$. A trader who rushes but delays her trading decision must then either not trade or trade in the direction of her private information. But, we can rule out delaying and not trading in equilibrium, because the trader can always instead trade in the direction of her signal at $t = 0$ and make a strictly positive expected profit.□

Proof of Lemma 2:

I first show that both $\pi_1(\beta, p_0, q_1)$ and $\pi_0(\beta, p_0, q_0)$ are symmetric with respect to $p_0 = \frac{1}{2}$, $\pi_i(\beta, p_0, q_1) = \pi_i(\beta, 1 - p_0, q_1)$, $i \in \{0, 1\}$. I provide the proof for $\pi_1(\beta, p_0, q_1)$. That for $\pi_0(\beta, p_0, q_0)$ is similar, but simpler. From (4), we can write

$$\pi_1(\beta, p_0, q_1) = \omega_0 m(2q_1 - 1) \sum_{e^i \in E} \left(\frac{Pr(e = e^i|V = 0)Pr(e = e^i|V = 1)Pr(e^i)}{Pr(a_1 = buy\&e = e^i)Pr(a_1 = sell\&e = e^i)} \right)$$

Applying (6), we have

$$\pi_1(\beta, p_0, q_1) = \omega_0 m(2q_1 - 1) \sum_{e^i \in E} \left(\frac{Pr(e = e^i|V = 0)Pr(e = e^i|V = 1)Pr(e^i)}{Pr(e^i)^2 buy_{1|V1} buy_{1|V0} + \omega_0 e^i_{|V1} e^i_{|V0} \left(buy_{1|V1} - buy_{1|V0}\right)^2} \right)$$

Note that the prior only enters through $Pr(e^i)$ and ω_0. ω_0 is symmetric in p_0. $Pr(e^i)$ is not, but by the assumed symmetry of the set of possible public events, $Pr(e^i) = p_0 e^i_{|V1} + (1 - p_0)e^i_{|V0} = p_0 e^j_{|V0} + (1 - p_0)e^j_{|V1}$ for some event realization, e^j, and $p_0 e^j_{|V0} + (1 - p_0)e^j_{|V1}$is the probability of event realization e^j in the expression for $\pi_1(\beta, 1 - p_0, q_1)$. So, when one sums over all possible event realizations, $\pi_1(\beta, p_0, q_1) = \pi_1(\beta, 1 - p_0, q_1)$.

To establish that the expected profits are concave and peak at $p_0 = \frac{1}{2}$, consider first $\pi_0(\beta, p_0, q_0)$. Take the derivative with respect to p_o:

$$\begin{aligned}\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial p_0} &= (1 - 2p_0)m(2q_0 - 1) \left(\frac{1}{Pr(a_0 = buy)} + \frac{1}{Pr(a_0 = sell)} \right) \\ &- \omega_0 m(2q_0 - 1) \left(\frac{\frac{\partial Pr(a_1 = buy)}{\partial p_0}}{Pr(a_0 = buy)^2} + \frac{\frac{\partial Pr(a_1 = sell)}{\partial p_0}}{Pr(a_0 = sell)^2} \right)\end{aligned}$$

Now, $\frac{\partial Pr(a_0 = buy)}{\partial p_0} = buy_{0|V1} - buy_{0|V0}$ and $\frac{\partial Pr(a_0 = sell)}{\partial p_0} = sell_{0|V1} - sell_{0|V0} = buy_{0|V0} - buy_{0|V1}$

by symmetry so that simple algebra gives

$$\begin{aligned} & \tfrac{\partial \pi_0(\beta,p_0,q_0)}{\partial p_0} \\ = \ & (1-2p_0)m(2q_0-1)\tfrac{Pr(a_0=sell)+Pr(a_0=buy)}{Pr(a_0=buy)Pr(a_0=sell)}\left(1-\omega_0\tfrac{\left(buy_{0|V1}-buy_{0|V0}\right)^2}{Pr(a_0=buy)Pr(a_0=sell)}\right) \\ = \ & (1-2p_0)m(2q_0-1)\tfrac{buy_{0|V1}buy_{0|V0}(Pr(a_0=sell)+Pr(a_0=buy))}{Pr(a_0=buy)^2Pr(a_0=sell)^2} \end{aligned} \tag{10}$$

where the last step uses (7). This derivative is zero if and only if $p_0 = \frac{1}{2}$.

For the second derivative:

$$\begin{aligned} & \tfrac{\partial^2 \pi_0(\beta,p_0,q_0)}{\partial p_0^2} \\ = \ & -2m(2q_0-1)\left(Pr(a_0=sell)+Pr(a_0=buy)\right)\tfrac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)^2Pr(a_0=sell)^2} \\ - \ & (1-2p_0)m(2q_0-1)\left(Pr(a_0=sell)+Pr(a_0=buy)\right) \\ & \times buy_{0|V1}buy_{0|V0}\tfrac{2\left(Pr(a_0=buy)\frac{\partial Pr(a_0=sell)}{\partial p_0}+Pr(a_0=sell)\frac{\partial Pr(a_0=buy)}{\partial p_0}\right)}{Pr(a_0=buy)^3Pr(a_0=sell)^3} \end{aligned}$$

Canceling common terms, its sign is given by

$$\begin{aligned} & -1-(1-2p_0)\tfrac{\left(Pr(a_0=buy)\frac{\partial Pr(a_0=sell)}{\partial p_0}+Pr(a_0=sell)\frac{\partial Pr(a_0=buy)}{\partial p_0}\right)}{Pr(a_0=buy)Pr(a_0=sell)} \\ = \ & -1-(1-2p_0)(buy_{V0}-buy_{V1})\tfrac{(Pr(a_0=buy)-Pr(a_0=sell))}{Pr(a_0=buy)Pr(a_0=sell)} \\ = \ & -1-\tfrac{(1-2p_0)^2(buy_{0|V0}-buy_{0|V1})^2}{Pr(a_0=buy)Pr(a_0=sell)} \end{aligned}$$

which is always negative, so that the expected profit is everywhere concave.

For the concavity of $\pi_1(\beta, p_0, q_1)$, note first that we can write the profit from waiting as the following expectation

$$\pi_1(\beta,p_0,q_1) = \sum_{e^i \in E} Pr(e_i)\pi_1^{NP}(\beta,p_{e_i},q_1) \tag{11}$$

where p_{e^i} is the public belief after public event e^i has realized.[47] Taking the first and second derivatives of (11) with respect to the initial public belief, we obtain

$$\begin{aligned} \frac{\partial \pi_1(\beta,p_0,q_1)}{\partial p_0} &= \sum_{e^i\in E}\left\{(e^i_{|V1}-e^i_{|V0})\pi_1^{NP}(\beta,p_{e^i},q_1)+Pr(e=e^i)\frac{\partial \pi_1^{NP}(\beta,p_{e^i},q_1)}{\partial p_{e^i}}\frac{dp_{e^i}}{dp_0}\right\} \\ \frac{\partial^2 \pi_1(\beta,p_0,q_1)}{\partial p_0^2} &= \sum_{e^i\in E}\left\{2(e^i_{|V1}-e^i_{|V0})\frac{\partial \pi_1^{NP}(\beta,p_{e^i},q_1)}{\partial p_{e^i}}\frac{dp_{e^i}}{dp_0}\right. \\ &+ \left. Pr(e=e^i)\left(\frac{\partial^2 \pi_1^{NP}(\beta,p_{e^i},q_1)}{\partial p_{e^i}^2}\left(\frac{dp_{e^i}}{dp_0}\right)^2+\frac{\partial \pi_1^{NP}(\beta,p_{e^i},q_1)}{\partial p_{e^i}}\frac{d^2p_{e^i}}{dp_0^2}\right)\right. \end{aligned}$$

We also have $p_{e^i} = Pr(V=1|e^i) = \frac{p_0 e^i_{|V1}}{Pr(e^i)}$ by Bayes' rule. Its first and second derivatives are $\frac{dp_{e^i}}{dp_0} = \frac{e^i_{|V1}e^i_{|V0}}{Pr(e^i)^2}$ and $\frac{d^2p_{e^i}}{dp_0^2} = -2\frac{e^i_{|V1}e^i_{|V0}}{Pr(e^i)^3}(e^i_{|V1}-e^i_{|V0})$. Substituting these expressions and

[47] Mathematical details available upon request.

canceling common terms gives

$$\frac{\partial^2 \pi_1(\beta, p_0, q_1)}{\partial p_0^2} \quad = \quad \sum_{e^i \in E} Pr(e = e^i) \frac{\partial^2 \pi_1^{NP}(\beta, p_{e^i}, q_1)}{\partial p_{e^i}^2} \left(\frac{e^i_{|V1} e^i_{|V0}}{Pr(e^i)^2} \right)^2$$

In a manner identical to the proof for $\pi_0(\beta, p_0, q_0)$, one can show that $\pi_1^{NP}(\beta, p_0, q_1)$ is concave. Then, because the sum of concave functions is concave, $\pi_1(\beta, p_0, q_1)$ must be concave. Finally, it is trivial to show that both $\pi_1(\beta, p_0, q_1)$ and $\pi_0(\beta, p_0, q_0)$ are zero at $p_0 = \{0, 1\}$.$\square$

Proof of Proposition 1:

Consider first the decision of an informed trader who has rushed. Denote the probability with which she trades immediately as $\hat{\beta}_i$ (with corresponding belief of the market maker, β_i) where $i \in (0, 1)$ is the signal she receives. A trader who has a positive signal ($s_0 = 1$) prefers to trade at $t = 0$ if

$$Pr(V \quad = \quad 1|s_0 = 1) - A_0 \geq Pr(V = 1|s_0 = 1) - A_1 - c$$

From Lemma 1, only a trader with a positive signal buys at $t = 1$ so that, after observing a buy order at this time, observing no trade at $t = 0$ reveals no further information. Applying Bayes' rule, the above expression becomes

$$\begin{aligned} A_1 - A_0 &\geq -c \\ \iff \frac{p_0 buy_{1|V1}}{p_0 buy_{1|V1} + (1-p_0) buy_{1|V0}} - \frac{p_0 buy_{0|V1}}{p_0 buy_{0|V1} + (1-p_0) buy_{0|V0}} &\geq -c \end{aligned} \tag{12}$$

A trader with a negative signal prefers to trade at $t = 0$ when

$$\begin{aligned} B_0 - B_1 &\geq -c \\ \iff \frac{p_0 sell_{0|V1}}{p_0 sell_{0|V1} + (1-p_0) sell_{0|V0}} - \frac{p_0 sell_{1|V1}}{p_0 sell_{1|V1} + (1-p_0) sell_{1|V0}} &\geq -c \end{aligned} \tag{13}$$

With extreme parameters, it is possible to construct examples in which (12) and/or (13) do not hold at the ex ante equilibrium probability of rushing calculated assuming they hold, so that an equilibrium in which the informed trader always trades in the period she acquires information does not exist. But, in the limit as the probability of informed trading approaches zero, all trades are by uninformed traders so that the probabilities of observing a buy or a sell are independent of the time period and state and equal to $\frac{1}{4}$. In this case, the left-hand side of both (12) and (13) become zero so that, for a strictly positive cost, both hold with strict inequality. By continuity of the expressions, both must then also hold at a strictly positive value of μ. Therefore, for μ sufficiently small, we must have $\beta_0^{C*} = \beta_1^{C*} = 1$: a trader that rushes must trade immediately.

With $\beta_0^{C*} = \beta_1^{C*} = 1$, the informed trader's expected profits from trading in periods 0 and 1 are given by (2) and (3), respectively. To establish uniqueness of the probability she rushes, β^{C*}, I show that $\pi_1^{NP}(\beta, p_0, q_1) - c - \pi_0(\beta, p_0, q_0)$ monotonically increases in β, by showing that $\pi_1^{NP}(\beta, p_0, q_1)$ monotonically increases in β, and that $\pi_0(\beta, p_0, q_0)$ monotonically decreases. For the former:

$$\frac{\partial \pi_1^{NP}(\beta, p_0, q_1)}{\partial \beta} = -\omega_0 m(2q_1 - 1)\left(\frac{\frac{\partial Pr(a_1=buy)}{\partial \beta}}{Pr(a_1 = buy)^2} + \frac{\frac{\partial Pr(a_1=sell)}{\partial \beta}}{Pr(a_1 = sell)^2}\right)$$

Evaluating the derivatives of the probabilities, we see that they are both negative. For brevity, I show only the derivative of the first probability:

$$\begin{aligned}\frac{\partial Pr(a_1=buy)}{\partial \beta} &= \frac{\partial (p_0(\mu q_1(1-\beta)+m)+(1-p_0)(\mu(1-q_1)(1-\beta)+m))}{\partial \beta} \\ &= -p_0\mu q_1 - (1-p_0)\mu(1-q_1)\end{aligned}$$

which is strictly less than zero.

$\frac{\partial \pi_0(\beta,p_0,q_0)}{\partial \beta}$ is evaluated similarly, but because the probabilities of buy and sell at $t = 0$ depend on β rather than $1 - \beta$, the sign of the derivative is reversed so that π_0 decreases in β.□

Proof of Proposition 2:

Consider first the case in which the equilibrium probability of waiting, β^{C*}, is interior over some range of p_0 so that the implicit function theorem applies:

$$\frac{\partial \pi_1^{NP}(\beta^{C*}, p_0, q_1)}{\partial p_0} + \frac{\partial \pi_1^{NP}(\beta^{C*}, p_0, q_1)}{\partial \beta^{C*}}\frac{d\beta^{C*}}{dp_0} = \frac{\partial \pi_0(\beta^{C*}, p_0, q_0)}{\partial p_0} + \frac{\partial \pi_0(\beta^{C*}, p_0, q_0)}{\partial \beta^{C*}}\frac{d\beta^{C*}}{dp_0}$$

or

$$\frac{d\beta^{C*}}{dp_0} = \frac{\frac{\partial \pi_0(\beta^{C*},p_0,q_0)}{\partial p_0} - \frac{\partial \pi_1^{NP}(\beta^{C*},p_0,q_1)}{\partial p_0}}{\frac{\partial \pi_1^{NP}(\beta^{C*},p_0,q_1)}{\partial \beta^{C*}} - \frac{\partial \pi_0(\beta^{C*},p_0,q_0)}{\partial \beta^{C*}}} \tag{14}$$

From the proof of Proposition 1, the denominator of (14) is strictly positive. Using the expression for the first derivative with respect to p_0 from the proof of Lemma 2, (10), the numerator of (14) becomes

$$\begin{aligned}&\frac{\partial \pi_0(\beta^{C*},p_0,q_0)}{\partial p_0} - \frac{\partial \pi_1^{NP}(\beta^{C*},p_0,q_1)}{\partial p_0} \\ =\ & (1-2p_0)m\Big[(2q_0-1)\left(Pr(a_0 = sell) + Pr(a_0 = buy)\right)\frac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)^2 Pr(a_0=sell)^2} \\ & -(2q_1-1)\left(Pr(a_1 = sell) + Pr(a_1 = buy)\right)\frac{buy_{1|V1}buy_{1|V0}}{Pr(a_1=buy)^2 Pr(a_1=sell)^2}\Big] \\ =\ & \frac{(1-2p_0)}{\omega_0}\Big[\pi_0(\beta^{C*}, p_0, q_0)\frac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)Pr(a_0=sell)} \\ & -\pi_1^{NP}(\beta^{C*}, p_0, q_1)\frac{buy_{1|V1}buy_{1|V0}}{Pr(a_1=buy)Pr(a_1=sell)}\Big] \\ =\ & \frac{(1-2p_0)}{\omega_0}\Big[\pi_0(\beta^{C*}, p_0, q_0)\left(\frac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)Pr(a_0=sell)} - \frac{buy_{1|V1}buy_{1|V0}}{Pr(a_1=buy)Pr(a_1=sell)}\right) \\ & -c\frac{buy_{1|V1}buy_{1|V0}}{Pr(a_1=buy)Pr(a_1=sell)}\Big]\end{aligned} \tag{15}$$

where the second equality uses the expressions for the expected profits, (2) and (3), and the last inequality uses the fact that we must have $\pi_1^{NP}(\beta^{C*}, p_0, q_1) - c = \pi_0(\beta^{C*}, p_0, q_0)$ in an interior equilibrium.

In the limit as the probability of informed trading goes to zero, all trades are uninformed so that $\lim_{\mu\to 0} \frac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)Pr(a_0=sell)} = \lim_{\mu\to 0} \frac{buy_{1|V1}buy_{1|V0}}{Pr(a_1=buy)Pr(a_1=sell)} = 1$. Then,

$$\begin{aligned} & \lim_{\mu\to 0} \frac{\partial \pi_0(\beta^{C*},p_0,q_0)}{\partial p_0} - \frac{\partial \pi_1^{NP}(\beta^{C*},p_0,q_1)}{\partial p_0} \\ = & -c\frac{(1-2p_0)}{\omega_0} \end{aligned}$$

Therefore, in the limit, the sign of the numerator of (14) depends upon the sign of $(2p_0-1)$ so that $\frac{d\beta^{C*}}{dp_0} < 0$ for $p_0 < \frac{1}{2}$, $\frac{d\beta^{C*}}{dp_0} = 0$ for $p_0 = \frac{1}{2}$, and $\frac{d\beta^{C*}}{dp_0} > 0$ for $p_0 > \frac{1}{2}$, whenever it is interior. Because $\frac{d\beta^{C*}}{dp_0}$ is strictly negative or positive in the limit (other than at $p_0 = \frac{1}{2}$), by continuity of the numerator of (14) in μ, there must exist a strictly positive value of μ such that the properties of $\frac{d\beta^{C*}}{dp_0}$ are the same as those in the limit.

As p_0 increases from one half, if β^* reaches one at some $\hat{p}_0 \geq \frac{1}{2}$, then $\beta^* = 1$ is the unique (by Proposition 3) equilibrium for all $p_0 \geq \hat{p}_0$. To see this fact, consider the difference in the expected profits for $p_0 \geq \hat{p}_0$ as $\mu \to 0$:

$$\begin{aligned} & \lim_{\mu\to 0} \pi_1(\beta, p_0, q_1) - c - \pi_0(\beta, p_0, q_0) \\ = & \quad 8\omega_0 \left[(2q_1 - 1) - (2q_0 - 1)\right] - c \end{aligned} \tag{16}$$

using the fact that all traders become uninformed. As p_0 increases from $\hat{p}_0 \geq \frac{1}{2}$, (16) strictly decreases. Therefore, in the limit, if $\pi_1(0, \hat{p}_0, q_1) - c - \pi_0(0, \hat{p}_0, q_0) = 0$ so that $\beta^* = 1$ at $\hat{p}_0$, then the difference in profits is strictly less than zero for all $p_0 > \hat{p}_0$ so that $\beta^* = 1$ remains an equilibrium. Because the difference in profits is strictly negative in the limit, for μ positive, but sufficiently small, continuity ensures this fact remains true. By the same reasoning, if β^* reaches zero at $\hat{p}_0 > \frac{1}{2}$ as we decrease p_0 from one, then $\beta^* = 0$ is the unique equilibrium over $\frac{1}{2} \leq p_0 \leq \hat{p}_0$. The corresponding statements for $p_0 < \frac{1}{2}$ follow by symmetry around $p_0 = \frac{1}{2}$.□

Proof of Lemma 3:

$\pi_1(\beta, p_0, q_1) < \pi_1^{NP}(\beta, p_0, q_1)$ is equivalent to

$$\sum_{e^i\in E}\left(\frac{e^i_{|V0}e^i_{|V1}}{Pr(a_1 = buy\&e = e^i)} + \frac{e^i_{|V0}e^i_{|V1}}{Pr(a_1 = sell\&e = e^i)}\right) \quad < \quad \left(\frac{1}{Pr(a_1 = buy)} + \frac{1}{Pr(a_1 = sell)}\right)$$

where the factor, $\omega_0 m(2q_1 - 1)$, cancels on either side of the inequality for $p_0 \in (0, 1)$. To see that this inequality holds, apply the mathematical claim of Lemma A1 to the buy and sell terms separately. For the buy term, $Pr(a_1 = buy\&e = e^i) = p_0 buy_{1|V1}e^i_{|V1} + (1-p_0)buy_{1|V0}e^i_{|V0}$ so set $c_i = e^i_{|V1}$, $d_i = e^i_{|V0}$, $x = p_0 buy_{1|V1}$, and $y = (1 - p_0)buy_{1|V0}$ to apply the claim, and similarly for the sell term. We also know from the mathematical claim that as long as the event is informative, the inequality is strict.□

Proof of Proposition 3:

Consider first the informed trader's decision conditional on rushing and receiving a positive signal ($s_0 = 1$). She prefers to trade at $t = 0$ if

$$\begin{array}{rrcl} & Pr(V=1|s_0=1) - A_0 & \geq & \sum_{e^i \in E} Pr(e=e^i)\left(Pr(V=1|s_0=1,e^i)\right. \\ & & & \left. -Pr(V=1|a_1=buy, a_0=no\,trade, e^i)\right) \\ \Longleftrightarrow & \frac{p_0 q_0}{Pr(s_0=1)} - \frac{p_0 buy_{0|V1}}{Pr(a_0=buy)} & \geq & \sum_{e^i \in E}\left[Pr(e=e^i)\right. \\ & & & \left. \times\left(\frac{p_0 q_0 e^i_{|V1}}{Pr(s_0=1\&e=e^i)} - \frac{p_0 buy_{1|V1} e^i_{|V1}}{Pr(a_1=buy\&e=e^i)}\right)\right] \\ \Longleftrightarrow & \frac{q_0 buy_{0|V0} - (1-q_0) buy_{0|V1})}{Pr(s_0=1)Pr(a_0=buy)} & \geq & \sum_{e^i \in E}\left[e^i_{|V1} e^i_{|V0} Pr(e=e^i)\right. \\ & & & \left. \times\frac{\left(q_0 buy_{1|V0} - (1-q_0) buy_{1|V1}\right)}{Pr(s_0=1)Pr(e=e^i)Pr(a_1=buy\&e=e^i)}\right] \end{array} \tag{17}$$

where the second equivalence uses the independence of s_0 and the public event. As $\mu \to 0$ so that $Pr(a_0 = buy) = \frac{1}{4}$ and $Pr(a_1 = buy\&e = e^i) = \frac{1}{4}Pr(e = e^i)$ (all trades are uninformative), the inequality becomes

$$(2q_0 - 1) \;\; \geq \;\; \sum_{e^i \in E} \frac{e^i_{|V1} e^i_{|V0}(2q_0 - 1)}{Pr(e = e^i)}$$

The mathematical claim of Lemma A1 shows that, as long as one of the realizations of the public event is informative, $\sum_{e^i \in E} \frac{e^i_{|V1} e^i_{|V0}}{Pr(e=e^i)} < 1$, so that the informed trader has a strict incentive to trade at $t = 0$ (set $c_i = e^i_{|V1}$, $d_i = e^i_{|V0}$, $x = p_0$, and $y = 1 - p_0$ to apply the claim). Because both sides of (17) are continuous in μ and the inequality is strict in the limit, there exists a $\hat{\mu} > 0$ such that for all $\mu \leq \hat{\mu}$, the informed trader also prefers to trade at $t = 0$. Thus, for μ sufficiently small, $\beta^*_1 = 1$ in any equilibrium. A similar calculation shows $\beta^*_0 = 1$ as well.

It remains to show uniqueness of β^*. In the proof of Proposition 1, I show that $\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial \beta} < 0$ and $\frac{\partial \pi_1^{NP}(\beta, p_0, q_1)}{\partial \beta} > 0$. Writing $\pi_1(\beta, p_0, q_1) = \sum_{e^i \in E} Pr(e = e^i)\pi_1^{NP}(\beta, p_{e^i}, q_1)$, it is clear that $\frac{\partial \pi_1(\beta, p_0, q_1)}{\partial \beta} > 0$, given that $\frac{\partial \pi_1^{NP}(\beta, p_0, q_1)}{\partial \beta} > 0$. Therefore, as in the proof of Proposition 1, the equilibrium probability of rushing, β^*, is unique. $\square$

Proof of Proposition 4:

Assume μ is small enough that an equilibrium in which each trader trades in the period she acquires information exists (by Proposition 3). Consider first the case in which the equilibrium probability of waiting, β^*, is interior over some range of p_0. Applying the implicit function theorem as in Proposition 2, we obtain:

$$\frac{d\beta^*}{dp_0} \;\; = \;\; \frac{\frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial p_0} - \frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial p_0}}{\frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial \beta^*} - \frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial \beta^*}} \tag{18}$$

From the proof of Proposition 4, the denominator of (18) is strictly positive. For the numerator, from the proof of Proposition 2, we have

$$\frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial p_0} = \frac{(1-2p_0)}{\omega_0}\pi_0(\beta^*, p_0, q_0)\frac{buy_{0|V1}buy_{0|V0}}{Pr(a_0 = buy)Pr(a_0 = sell)}$$

Taking the derivative of (4) with respect to p_0 after combining the buy and sell terms in the summation, we get

$$\frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial p_0} = \frac{(1-2p_0)}{\omega_0}\pi_1(\beta^*, p_0, q_1) + \Sigma$$

where

$$\begin{aligned}\Sigma \equiv\ & \omega_0 m(2q_1 - 1)(buy_{1|V0} + buy_{1|V1}) \\ & \times \sum_{e^i \in E} e^i_{|V=0} e^i_{|V=1} \left[\frac{(e^i_{|V=1} - e^i_{|V=0})Pr(a_1 = buy\&e = e^i)Pr(a_1 = sell\&e = e^i)}{Pr(a_1 = buy\&e = e^i)^2 Pr(a_1 = sell\&e = e^i)^2} \right. \\ & \left. - \frac{Pr(e = e^i)\frac{\partial}{\partial p_0}\left(Pr(a_1 = buy\&e = e^i)Pr(a_1 = sell\&e = e^i)\right)}{Pr(a_1 = buy\&e = e^i)^2 Pr(a_1 = sell\&e = e^i)^2} \right]\end{aligned}$$

and

$$\begin{aligned} & \tfrac{\partial}{\partial p_0}\left(Pr(a_1 = buy\&e = e^i)Pr(a_1 = sell\&e = e^i)\right) \\ = & \tfrac{\partial}{\partial p_0}\left(Pr(e^i)^2 buy_{1|V1}buy_{1|V0} + \omega_0 e^i_{|V1}e^i_{|V0}\left(buy_{1|V1} - buy_{1|V0}\right)^2\right) \\ = & 2Pr(e^i)(e^i_{|V=1} - e^i_{|V=0})buy_{1|V1}buy_{1|V0} + (1-2p_0)e^i_{|V1}e^i_{|V0}\left(buy_{1|V1} - buy_{1|V0}\right)^2 \end{aligned}$$

where the first equivalence uses (6). The numerator of (18) is then

$$\begin{aligned} & \tfrac{\partial \pi_0(\beta^*,p_0,q_0)}{\partial p_0} - \tfrac{\partial \pi_1(\beta^*,p_0,q_1)}{\partial p_0} \\ = & \tfrac{(1-2p_0)}{\omega_0}\pi_0(\beta^*, p_0, q_0)\tfrac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)Pr(a_0=sell)} - \tfrac{(1-2p_0)}{\omega_0}\pi_1(\beta^*, p_0, q_1) - \Sigma \end{aligned}$$

When β^* is interior, $\pi_0(\beta^*, p_0, q_0) = \pi_1(\beta^*, p_0, q_1)$, so that this expression becomes

$$\begin{aligned} & \tfrac{\partial \pi_0(\beta^*,p_0,q_0)}{\partial p_0} - \tfrac{\partial \pi_1(\beta^*,p_0,q_1)}{\partial p_0} \\ = & \tfrac{(1-2p_0)}{\omega_0}\pi_0(\beta^*, p_0, q_0)\left[\tfrac{buy_{0|V1}buy_{0|V0}}{Pr(a_0=buy)Pr(a_0=sell)} - 1\right] - \Sigma \end{aligned} \tag{19}$$

In the limit as $\mu \to 0$, the first term approaches zero as in Proposition 2. The sign of $\frac{d\beta^*}{dp_0}$ is then determined by

$$\begin{array}{rl} & -\lim_{\mu\to 0}\Sigma \\ = & -\omega_0(2q_1-1)\frac{1}{8}\sum_{e^i\in E} e^i_{|V=0}e^i_{|V=1}\left[\frac{16(e^i_{|V=1}-e^i_{|V=0})}{Pr(e=e^i)^2}-\frac{32(e^i_{|V=1}-e^i_{|V=0})}{Pr(e=e^i)^2}\right] \\ = & \omega_0 2(2q_1-1)\sum_{e^i\in E} e^i_{|V=0}e^i_{|V=1}\frac{(e^i_{|V=1}-e^i_{|V=0})}{Pr(e=e^i)^2} \end{array}$$

From Lemma A3 of the online Appendix, $\sum_{e^i\in E} e^i_{|V=0}e^i_{|V=1}\frac{(e^i_{|V=1}-e^i_{|V=0})}{Pr(e=e^i)^2}$ has the same sign as $(1-2p_0)$, so that, in the limit, $\frac{d\beta^*}{dp_0}$ is strictly increasing for $p_0<\frac{1}{2}$, zero at $p_0=\frac{1}{2}$, and strictly decreasing for $p_0>\frac{1}{2}$. By continuity, because these properties of $\frac{d\beta^*}{dp_0}$ are strict, there exists a $\hat{\mu}>0$ such that for all $\mu\leq\hat{\mu}$, $\frac{d\beta^*}{dp_0}$ peaks at $p_0=\frac{1}{2}$.

As p_0 increases from one half, if β^* reaches zero at some $\hat{p}_0\geq\frac{1}{2}$, then $\beta^*=0$ is the unique (by Proposition 3) equilibrium for all $p_0\geq\hat{p}_0$.To see this fact, consider the relationship between the expected profits for $p_0\geq\hat{p}_0$ as $\mu\to 0$:

$$\begin{array}{rl} & \lim_{\mu\to 0}\pi_1(\beta,p_0,q_1)-\pi_0(\beta,p_0,q_0) \\ = & 8\omega_0\left[(2q_1-1)\sum_{e^i\in E}\frac{e^i_{|V=0}e^i_{|V=1}}{Pr(e=e^i)}-(2q_0-1)\right] \end{array} \tag{20}$$

using the fact that all traders become uninformed. The sign of (20) is determined by the term in brackets which is strictly increasing for $p_0>\frac{1}{2}$ because

$$\begin{array}{rl} & \frac{\partial}{\partial p_0}\sum_{e^i\in E}\left[(2q_1-1)\sum_{e^i\in E}\frac{e^i_{|V=0}e^i_{|V=1}}{Pr(e=e^i)}-(2q_0-1)\right] \\ = & -(2q_1-1)\sum_{e^i\in E} e^i_{|V=0}e^i_{|V=1}\frac{(e^i_{|V=1}-e^i_{|V=0})}{Pr(e=e^i)^2} \end{array}$$

which, by Lemma A3, is is strictly greater than zero over this range. Therefore, in the limit, if $\pi_1(0,\hat{p}_0,q_1)-\pi_0(0,\hat{p}_0,q_0)=0$ so that $\beta^*=0$ at $\hat{p}_0$, then the difference in profits is strictly greater than zero for all $p_0>\hat{p}_0$ so that $\beta^*=0$ remains an equilibrium. Because the difference in profits is strictly positive in the limit, for μ positive, but sufficiently small, continuity ensures this fact remains true. By the same reasoning, if β^* reaches one at $\hat{p}_0>\frac{1}{2}$ as we decrease p_0 from one, then $\beta^*=1$ is the unique equilibrium over $\frac{1}{2}\leq p_0\leq\hat{p}_0$. The corresponding statements for $p_0<\frac{1}{2}$ follow by symmetry around $p_0=\frac{1}{2}$.□

Proof of Proposition 5:

Assume μ is small enough that an equilibrium in the model with public information arrival in which each trader trades in the period she acquires information exists (by Proposition 3). With $\beta^*\in(0,1)$ so that the monetary cost, $c^P(p_0,q_0,q_1)$, is uniquely defined, by the implicit function theorem, we have

$$\frac{\partial c^P(p_0,q_0,q_1)}{\partial\beta^*}=\left(\frac{\partial\pi_1^{NP}(\beta^*,p_0,q_1)}{\beta^*}-\frac{\partial\pi_0(\beta^*,p_0,q_0)}{\beta^*}\right)\frac{\partial\beta^*}{\partial p_0}$$

From the proof of Proposition 1, the term in parentheses is strictly positive so that $\frac{\partial c^P(p_0,q_0,q_1)}{\partial\beta^*}$ has the same sign as $\frac{\partial\beta^*}{\partial p_0}$. But, from Proposition 4, we know that, provided it is interior, β^* strictly increases when $p_0<\frac{1}{2}$, reaches a maximum at $p_0=\frac{1}{2}$, and then strictly decreases for $p_0>\frac{1}{2}$. Therefore, $c^P(p_0,q_0,q_1)$ does as well.□

Proof of Proposition 6:

If either trader faces an intervening trade by the other with any positive probability, then provided μ_1 and μ_2 are sufficiently small, Proposition 3 shows that the trader must trade immediately if she rushes. On the other hand, if the trader faces no possibility of intervening trade, the proof is straightforward.[48] First, because there is no cost, it is easy to show that traders with both positive and negative signals face exactly the same benefit and cost of trading immediately or delaying and trading at $t = 1$. But, given this fact, we cannot have an equilibrium in which both rush and trade immediately - they would be strictly better off by instead waiting to acquire the better signal since they would face the same bid-ask spread but have better quality information.

It remains to show that β^{1*} and β^{2*} (when trader 1 has not traded) are unique. As discussed in the main text, when trader 1 rushes, trader 2 knows there is no possibility of trade and therefore rushes with probability β^{NP*}, which is easily shown to be unique as in Proposition 1.

From the proofs of Proposition 3, each $\pi_1^i(\beta^{1*}, \beta^{2*}, p_0, q_1) - \pi_0(\beta^{i*}, p_0, q_0)$ is strictly monotonic in β^{i*} which ensures $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) - \pi_0(\beta^{1*}, p_0, q_0)$ is also strictly monotonic in β^{1*}.[49] Therefore, each trader has a unique best response to the timing strategy of the other trader. I now show that β^{1*} is weakly increasing in β^{2*}, and β^{2*} is weakly decreasing in β^{1*} which, together with the uniqueness of best responses, ensures a unique fixed point in best responses.

Consider trader 1. I show that her expected profit from waiting, $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) = \alpha\pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) + (1-\alpha)\pi_1^{NP}(\beta^{1*}, p_0, q_1)$, is decreasing in β^{2*}. The second term is independent of trader 2's strategy. The first term depends upon trader 2's strategy through the summation over the set of public events corresponding to the possibility of her trade: $e^1 = Pr(buy)$, $e^2 = Pr(sell)$, and $e^3 = Pr(no\ trade)$. I first show that the summation over events when trader 1 buys is decreasing in β^{2*}. Expanding this term:

$$
\begin{aligned}
& \textstyle\sum_{e^i \in E} \frac{Pr(e^i|V=0)Pr(e^i|V=1)}{Pr(a_1^1=buy \& e=e^i)} \\
= \;& \frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^1=buy \& e=e^1)} + \frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^1=buy \& e=e^2)} + \frac{e^3_{|V0}}{Pr(a_1^1=buy)} \\
= \;& Pr(a_1^1 = buy)\left(\frac{e^1_{|V0}e^1_{|V1}\left(\mu_2\beta^{2*}+2m_2\right)}{Pr(a_1^1=buy \& e=e^1)Pr(a_1^1=buy \& e=e^2)} + \frac{e^3_{|V0}}{Pr(a_1^1=buy)^2}\right)
\end{aligned}
$$

where the first equality uses symmetry: $e^1_{|V0} = e^2_{|V1} = \mu_2\beta^{2*}(1-q_0) + m_2$, $e^1_{|V1} = e^2_{|V0} = \mu_2\beta^{2*}q_0 + m_2$, and $e^3_{|V0} = e^3_{|V1} = \mu_2(1-\beta^{2*}) + 2m_2$. Dividing by $Pr(a_1^1 = buy)$, which is independent of β^{2*},, and taking the derivative with respect to β^{2*}:

$$
\begin{aligned}
& \frac{1}{Pr(a_1^1=buy)}\frac{\partial}{\partial\beta^{2*}}\textstyle\sum_{e^i \in E} \frac{Pr(e^i|V=0)Pr(e^i|V=1)}{Pr(a_1^1=buy \& e=e^i)} \\
= \;& \mu_2\left(\frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^1=buy \& e=e^1)Pr(a_1^1=buy \& e=e^2)} - \frac{1}{Pr(a_1^1=buy)^2}\right) \\
& + \left(\mu_2\beta^{2*} + 2m_2\right)\frac{\partial}{\partial\beta^{2*}}\frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^1=buy \& e=e^1)Pr(a_1^1=buy \& e=e^2)}
\end{aligned} \tag{21}
$$

[48] Mathematical details available upon request.

[49] The additional term in the denominator of $\pi_1^2(\beta^{1*}, \beta^{2*}, p_0, q_1)$ is independent of β^{2*} so does not affect this monotonicity result.

Each of the two terms in (21) is separately negative. Expanding the product of probabilities gives $Pr(a_1^1 = buy\&e = e^1)Pr(a_1^1 = buy\&e = e^2) = e^1_{|V0}e^1_{|V1}Pr(a_1^1 = buy)^2 + buy_{|V0}buy_{|V1}\omega_0(e^1_{|V1} - e^1_{|V0})^2$. Cross-multiplying the first term and substituting this expression, its sign becomes that of $Pr(a_1^1 = buy)^2 e^1_{|V0}e^1_{|V1} - e^1_{|V0}e^1_{|V1}Pr(a_1^1 = buy)^2 - buy_{|V0}buy_{|V1}\omega_0(e^1_{|V1} - e^1_{|V0})^2 = -buy_{|V0}buy_{|V1}\omega_0(e^1_{|V1} - e^1_{|V0})^2$, which is weakly negative.

For the second term, to evaluate the derivative it is convenient to first multiply by the term $Pr(a_1^1 = buy)^2$, which is independent of β^{2*} . Doing so, we see that the denominator of the derivative is positive and the sign of the numerator is determined by

$$\begin{aligned}
& Pr(a_1^1 = buy)^2 \tfrac{\partial}{\partial \beta^{2*}} \left(e^1_{|V0}e^1_{|V1}\right) \left(e^1_{|V0}e^1_{|V1}Pr(a_1^1 = buy)^2 + buy_{|V0}buy_{|V1}\omega_0(e^1_{|V1} - e^1_{|V0})^2\right) \\
& -Pr(a_1^1 = buy)^2 e^1_{|V0}e^1_{|V1} \left(\tfrac{\partial}{\partial \beta^{2*}} \left(e^1_{|V0}e^1_{|V1}\right) Pr(a_1^1 = buy)^2 + buy_{|V0}buy_{|V1}\omega_0 \tfrac{\partial}{\partial \beta^{2*}}(e^1_{|V1} - e^1_{|V0})^2\right) \\
= \; & Pr(a_1^1 = buy)^2 buy_{|V0}buy_{|V1}\omega_0 \left(\tfrac{\partial}{\partial \beta^{2*}} \left(e^1_{|V0}e^1_{|V1}\right)(e^1_{|V1} - e^1_{|V0})^2\right. \\
& \left. -2e^1_{|V0}e^1_{|V1}(e^1_{|V1} - e^1_{|V0}) \tfrac{\partial}{\partial \beta^{2*}}(e^1_{|V1} - e^1_{|V0})\right)
\end{aligned}$$

which has a sign determined by

$$\begin{aligned}
& (e^1_{|V1} - e^1_{|V0}) \tfrac{\partial}{\partial \beta^{2*}} \left(e^1_{|V0}e^1_{|V1}\right)(e^1_{|V1} - e^1_{|V0}) - 2e^1_{|V0}e^1_{|V1} \tfrac{\partial}{\partial \beta^{2*}}(e^1_{|V1} - e^1_{|V0}) \\
= \; & (e^1_{|V1} - e^1_{|V0}) \left((\mu_2(1-q_0)e^1_{|V1} + \mu_2 q_0 e^1_{|V0})\mu_2\beta^{2*}(2q_0 - 1) - 2\mu_2 e^1_{|V0}e^1_{|V1}(2q_0 - 1)\right) \\
= \; & (e^1_{|V1} - e^1_{|V0})(2q_0 - 1)\mu_2 \left(\mu_2\beta^{2*}((1-q_0)e^1_{|V1} + q_0 e^1_{|V0}) - 2e^1_{|V0}e^1_{|V1}\right) \\
= \; & (e^1_{|V1} - e^1_{|V0})(2q_0 - 1)\mu_2 (\mu_2\beta^{2*}((1-q_0)(\mu_2\beta^{2*}q_0 + m_2) \\
& +q_0(\mu_2\beta^{2*}(1-q_0) + m_2) - 2(\mu_2\beta^{2*}q_0 + m_2)(\mu_2\beta^{2*}(1-q_0) + m_2) \\
= \; & (e^1_{|V1} - e^1_{|V0})(2q_0 - 1)\mu_2 (-\mu_2 m_2 \beta^{2*} - 2m_2^2) \\
< \; & 0
\end{aligned}$$

Similarly, the second term (corresponding to trader 1 selling) also strictly decreases in β^{2*}, so that her overall expected profit does as well. The best response of trader 1, $\beta^{1*}(\beta^{2*})$ is governed by $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) - \pi_0(\beta^{1*}, p_0, q_0) = 0$ when β^{1*} is interior. From the monotonicity results in the proof of Propositions 2 and 4, if $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1)$ strictly decreases in β^{2*}, then we must have β^{1*} strictly increase in β^{2*} when interior. If, instead, $\beta^{1*} \in \{0, 1\}$ then it is unaffected by β^{2*}.

Now consider trader 2. Opposite to the case of trader 1 above, we can show that $\pi_1^2(\beta^{1*}, \beta^{2*}, p_0, q_1)$ is increasing in β^{1*}. When trader 1 has not already traded, the set of public events corresponding to the trade of trader 1 is $e^1 = Pr(buy)$ and $e^2 = Pr(sell)$. The conditional probabilities are $e^1_{|V0} = e^2_{|V1} = \mu_1(1-\beta^{1*})(1-q_1) + m$ and $e^1_{|V1} = e^2_{|V0} = \mu_1(1-\beta^{1*})q_1 + m$. The term in $\pi_1^2(\beta^{1*}, \beta^{2*}, p_0, q_1)$ corresponding to trader 2 buying is given by

$$\begin{array}{rl} & \frac{1}{\mu_1(1-\beta^{1*})+2m_1}\sum_{e^i\in E}\frac{Pr(e^i|V=0)Pr(e^i|V=1)}{Pr(a_1^2=buy\&e=e^i)} \\ = & \frac{1}{\mu_1(1-\beta^{1*})+2m_1}\left(\frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^2=buy\&e=e^1)}+\frac{e^1_{|V0}e^1_{|V1}}{Pr(a_1^2=buy\&e=e^2)}\right) \\ = & \frac{e^1_{|V0}e^1_{|V1}Pr(a_1^2=buy)}{Pr(a_1^2=buy\&e=e^1)Pr(a_1^2=buy\&e=e^2)} \end{array} \tag{22}$$

because $Pr(a_1^2 = buy\&E = e^1) + Pr(a_1^2 = buy\&E = e^2) = Pr(a_1^2 = buy)(Pr(e = e^1) + Pr(e = e^2)) = Pr(a_1^2 = buy)(\mu_1(1-\beta^{1*}) + 2m_1)$. If we divide (22) by $Pr(a_1^2 = buy)$, which is independent of β^{1*}, we can see that its derivative with respect to β^{1*} is identical to the second term in (21) except that the public events depend upon $1-\beta^{1*}$ instead of β^{2*} so it is strictly positive instead of strictly negative. Similarly, the term in $\pi_1^2(\beta^{1*}, \beta^{2*}, p_0, q_1)$ corresponding to trader 2 selling, and therefore her expected profit from waiting overall, is strictly increasing in β^{1*}. If $\pi_1^2(\beta^{1*}, \beta^{2*}, p_0, q_1)$ strictly increases in β^{1*} then from the monotonicity properties of Propositions 2 and 4, β^{2*} must strictly decrease in β^{1*} if interior. If $\beta^{1*} \in \{0, 1\}$, it is unaffected by β^{2*}. Because the two best response functions are weakly monotonic (strict when interior), and of opposite sign, the fixed point of best responses is guaranteed to be unique.□

Proof of Proposition 7:

Assume μ_1 and μ_2 are small enough that the unique equilibrium is one in which each trader trades in the period she acquires information (by Proposition 6).

Part a). To see that we must have trader 1 rush more often than if there is no competition, suppose not: $\beta^{1*} \leq \beta^{NP*}$. Because $\beta^{NP*} < \frac{1}{2}$, trader 1 waits with positive probability and her trade impacts trader 2 whenever she does. By Proposition 6, we must then have $\beta^{2*} \geq \beta^{NP*}$ (with strict inequality if $\beta^{NP*} > 0$). This fact in turn implies trader 2 impacts trader 1's expected profit from waiting so that $\beta^{1*} \geq \beta^{NP*}$ (with strict inequality if $\beta^{NP*} > 0$). We then have a contradiction if $\beta^{NP*} > 0$. If $\beta^{NP*} = 0$, $\beta^{1*} = \beta^{NP*} = 0$ is possible.

Part b). Consider an increase in α. If $\beta^{2*} = 0$, trader 2 has no impact on trader 1 so that clearly any change in α has no effect on trader 1's probability of rushing. If $\beta^{2*} > 0$, on the other hand, the direct effect of an increase in α on trader 1's expected profit from waiting is given by $\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \alpha} = \pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) - \pi_1^{NP}(\beta^{1*}, p_0, q_1)$ which is strictly negative due to the impact of trader 2 (see the proof of Proposition 6).

When $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1)$ decreases, both β^{1*} and β^{2*} may change provided $\beta^{1*} \in (0, 1)$ (if not, β^{1*} may remain unchanged). After the change, I claim that β^{1*} must be strictly larger so that $\frac{d\beta^{1*}}{d\alpha} > 0$. Suppose not. If β^{1*} remains unchanged, then β^{2*} remains unchanged given that trader 2's best response is unique (Proposition 6), so we have $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) < \pi_0(\beta^{1*}, p_0, q_0)$ contradicting $\beta^{1*} \in (0, 1)$ which requires the two expected profits to be equal. If β^{1*} were to decrease, then β^{2*} weakly increases as seen in the proof of Proposition 6. But, as also shown there, this increase in the probability that trader 2 rushes further decreases $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1)$ so that again $\Pi_1^1(\beta^{1*}, \beta^{2*}, p_0, q_1) < \pi_0(\beta^{1*}, p_0, q_0)$, contradicting $\beta^{1*} \in (0, 1)$. Therefore β^{1*} doesn't change if either $\beta^{2*} = 0$ or $\beta^{1*} \in \{0, 1\}$, but must otherwise strictly increase when α increases.

Part c). Consider first the case of β^{1*} interior so that we can apply the implicit function theorem

$$\frac{d\beta^{1*}}{dp_0} = \frac{\frac{\partial \pi_0^1(\beta^{1*},p_0,q_0)}{\partial p_0} - \frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0} - \frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta^{2*}}\frac{d\beta^{2*}}{dp_0}}{\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_1^*} - \frac{\partial \pi_0^1(\beta^{*},p_0,q_0)}{\partial \beta_1^*}}$$

Comparing to the exogenous public event case, (18), we see that the additional equilibrium effect of a change in β^{2*} enters the numerator. If β^{1*} is interior, then either β^{2*} is interior or $\beta^{2*} = 1$.[50]

If $\beta^{2*} = 1$, $\frac{d\beta^{2*}}{dp_0} = 0$. From the proof of Proposition 6, $\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_1^*} - \frac{\partial \pi_0^1(\beta^{*},p_0,q_0)}{\partial \beta_1^*} > 0$ so that the sign of $\frac{d\beta^{1*}}{dp_0}$ depends upon the sign of $\frac{\partial \pi_0^1(\beta^{1*},p_0,q_0)}{\partial p_0} - \frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0}$. Exactly as in the proof of Proposition 4, we can relate the derivatives to their corresponding expected profits and then use the equilibrium relationship between expected profits to substitute out one of them out. Doing so, in the limit as $\mu_1 \to 0$, we have

$$\begin{aligned} & \lim_{\mu_1\to 0} \frac{\partial \pi_0^1(\beta^{1*},p_0,q_0)}{\partial p_0} - \frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0} \\ = \; & -\lim_{\mu_1\to 0} \alpha\Sigma \end{aligned}$$

As in the proof of Proposition 4, $-\lim_{\mu_1\to 0}\Sigma$ has the same sign as $(1-2p_0)$ provided at least one of the event realizations is informative, which is the case if $\beta^{2*} = 1$. Therefore, in this case, $\frac{d\beta^{1*}}{dp_0}$ has a strict maximum at $p_0 = \frac{1}{2}$, which, by continuity, ensures it also has a strict maximum for μ sufficiently small and positive.

If β^{2*} is interior, we can substitute out $\frac{d\beta^{2*}}{dp_0}$ by applying the implicit function theorem to the equilibrium relationship for β^{2*}. Doing so, and solving for $\frac{d\beta^{1*}}{dp_0}$, results in

$$\frac{d\beta^{1*}}{dp_0} = \frac{\frac{\partial \pi_0^1(\beta^{1*},p_0,q_0)}{\partial p_0} - \frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0} - A\left(\frac{\partial \pi_0^2(\beta^{2*},p_0,q_0)}{\partial p_0} - \frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0}\right)}{\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_1^*} - \frac{\partial \pi_0^1(\beta^{*},p_0,q_0)}{\partial \beta_1^*} - A\frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta^{1*}}} \tag{23}$$

where

$$A \equiv \frac{\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_2^*}}{\frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_2^*} - \frac{\partial \pi_0^2(\beta^{2*},p_0,q_0)}{\partial \beta_2^*}}$$

From the proof of Proposition 6, $\frac{\partial \Pi_1^1(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_2^*} < 0$ and $\frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta_2^*} - \frac{\partial \pi_0^2(\beta^{2*},p_0,q_0)}{\partial \beta_2^*} > 0$ so that $A < 0$. In addition, $\frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial \beta^{1*}} > 0$, so that the denominator of (23) is strictly positive.

For the numerator, consider the limit as $\mu_1 \to 0$. As in the $\beta^{2*} = 1$ case, the difference of the first two terms peaks at $p_0 = \frac{1}{2}$. For the last term, in the limit as $\mu_1 \to 0$, trader 1 has no impact on trader 2 so that trader 2 rushes with probability $\beta^{2*} = \beta^{NP*}$. In this case, it

[50] If $\beta^{2*} = 0$, trader 1 rushes with probability β^{NP*}. But then because we are considering the case of β^{1*} interior, β^{NP*} must be interior. This fact in turn implies trader 1 impacts trader 2 because trader 1 waits with positive probability, so we must have $\beta^{2*} \geq \beta^{NP*}$ which contradicts $\beta^{2*} = 0$ when β^{NP*} is interior.

is easy to show that $\frac{\partial \pi_0^2(\beta^{2*},p_0,q_0)}{\partial p_0} - \frac{\partial \pi_1^2(\beta^{1*},\beta^{2*},p_0,q_1)}{\partial p_0}$ vanishes (see (15) with $c=0$). Therefore, for μ sufficiently small, $\frac{d\beta^{1*}}{dp_0}$ has a strict maximum at $p_0 = \frac{1}{2}$, provided β^{1*} is interior.

Using the same argument as in the proof of Proposition 4, we can show that, as p_0 increases from one half, if β^{1*} reaches zero at some $\hat{p}_0 \geq \frac{1}{2}$, then $\beta^{1*} = 0$ is the unique (by Proposition 6) equilibrium for all $p_0 \geq \hat{p}_0$. And, if β^{1*} reaches one at $\hat{p}_0 > \frac{1}{2}$ as we decrease p_0 from one, then $\beta^{1*} = 1$ is the unique equilibrium over $\frac{1}{2} \leq p_0 \leq \hat{p}_0$. The only difference from the public event case is that the relationship between the expected profits for $p_0 \geq \hat{p}_0$ as $\mu_1 \to 0$ is given by

$$\begin{aligned} &\lim_{\mu_1 \to 0} \alpha\pi_1^1(\beta^1,\beta^2,p_0,q_1) + (1-\alpha)\pi_1^{NP}(\beta^1,p_0,q_1) - \pi_0(\beta^1,p_0,q_0) \\ = \quad & 8\omega_0 \left[(2q_1-1)\left(\alpha \sum_{e^i \in E} \frac{e^i_{|V=0} e^i_{|V=1}}{Pr(e=e^i)} + 1 - \alpha\right) - (2q_0-1)\right] \end{aligned} \tag{24}$$

Using the fact that trader 2 impacts trader 1 by rushing with probability β^{NP*} in the limit, if $\beta^{NP*} > 0$, the summation term in (24) contains an informative event. Therefore, given $\alpha > 0$, the term in brackets is strictly increasing for $p_0 > \frac{1}{2}$ as is the case in the proof of Proposition 4. The rest of the proof follows similarly. If $\beta^{NP*} = 0$, $\beta^{2*} = 0$ is possible such that trader 2 has no impact on trader 1. But, in this case we must also have $\beta^{1*} = \beta^{NP*} = 0$. If, as p_0 increases from one half, β^{1*} reaches zero at some $\hat{p}_0 \geq \frac{1}{2}$, then $\pi_1^{NP}(0,\hat{p}_0,q_1) - \pi_0(0,\hat{p}_0,q_0) = 0$. For $p_0 \geq \hat{p}_0$, we have[51]

$$\begin{aligned} &\pi_1^{NP}(0,p_0,q_1) - \pi_0(0,p_0,q_0) \\ = \quad & \omega_0 \left[(2q_1-1)\left(\frac{1}{Pr(a_0=buy)} + \frac{1}{Pr(a_0=sell)}\right) - 8(2q_0-1)\right] \end{aligned} \tag{25}$$

The sign of (25) is determined by the term in brackets which is easily shown to be increasing in p_0 for $p_0 > \frac{1}{2}$. Therefore, if it is zero at some $\hat{p}_0 \geq \frac{1}{2}$, it is strictly positive for all $p_0 \geq \hat{p}_0$ so that $\beta^{1*} = 0$ remains an equilibrium.

Part d). To see that trader 1 rushes more often as q_0 increases, suppose not: $\beta^{1*} \leq \hat{\beta}^{1*}$ where $\hat{\beta}^{1*}$ is the initial equilibrium probability of rushing prior to the increase. By assumption, $\hat{\beta}^{1*} \in (0,1)$ so that trader 1 waits with positive probability and her trade impacts trader 2 whenever she does. From the proof of Proposition 6, if β^{1*} decreases, β^{2*} must weakly increase. In addition, π_0^2 increases in q_0 and π_1^2 decreases due to the increased price impact of trader 1's trades (see Lemma A2 of the online Appendix), which both also cause β^{2*} to increase. Therefore, we must have $\beta^{2*} \geq \hat{\beta}^{2*}$, where $\hat{\beta}^{2*}$ is trader 2's initial equilibrium probability of rushing prior to the increase. But, the increase in β^{2*}, combined with the fact that π_0^1 strictly increases in q_0, then implies $\beta^{1*} > \hat{\beta}^{1*}$, a contradiction.□

[51] The limiting case of $\mu_1 \to 0$ is not helpful here because, in the limit, trader 1 always has a strict incentive to wait so no such $\hat{p}_0$ exists for any parameterization.