REAL YIELD VARIABILITY; A SIMPLE EXPLANATION FOR THE UIRP AND RELATED "PUZZLES" IN INTERNATIONAL FINANCE

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ABSTRACT

I derive a dynamic version of the Dornbusch "overshooting" model in which real yields and inflation vary stochastically, and the exchange rate (FX) delivers UIRP in expectations.

Tests using the model provide support for the UIRP proposition. Simulations show that the "disconnect" of FX rates from fundamentals as well as their very high volatility is a necessary consequence of UIRP when real yields are autocorrelated. I also show that FX rates display some predictability as required by the model. Finally, the model shows that the statistical patterns on which "carry trade" is based are consistent with equilibrium.

Applicable JEL Categories: E44, E47, F31, F41.

Keywords: Exchange rates, puzzles, UIRP, real yields, overshooting, predictability, carry trade.

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In this paper I derive a dynamic version of the Dornbusch (1976) "overshooting" model where uncovered interest parity (UIRP) holds in expectations. I show that when real yields are not perfectly correlated, the model offers a simple explanation to several FX "puzzles." The model is informed by the empirical evidence which shows that real yields are variable, highly persistent, and only moderately correlated across currencies.[1] In this model the FX rate behavior cannot be described by the traditional law of motion labeled UIRP. In the new formulation, the real FX rate path is conditional on the real yield processes, and long-run PPP and UIRP are embedded in a single law of motion for the FX rate.[2]

Stylized facts of international finance that have resisted explanation by current models have been labeled "puzzles." These puzzles are discussed extensively by Bachus et al. (2010), Boudoukh et al. (2011), Engel and West (2004, 2005, 2006, 2010), Engel, Mark, and West (2007), Evans (2012), Lewis (1995), Mark (2009), McCallum (1994), Obstfeld and Rogoff (2000, 2002), and Taylor (1995), among many others. The failure to convincingly explain the stylized facts of exchange rate behavior has given rise to the notion that exchange rates are "disconnected" from their fundamentals.

The foremost puzzle is the spectacular empirical failure of UIRP –Uncovered Interest Rate Parity; UIRP says that in the presence of sufficient speculation the current foreign yield together with the expected appreciation of the foreign currency will equal the current home yield, up to a risk premium.[3] There have been several recent attempts to build models in which time varying risk premia help explain, at least in principle, the failure of UIRP (see Bachus et. al. (2010), Bansal and Shaliastovich (2010), Evans (2012), Moore and Rosh (2010), Sierra (2009),

[1] In the developed country sample used in this paper AR1 coefficients are almost always in excess of 0.90, and often much higher, and the cross-country correlations average 40%.
[2] *If and only if* real yields are perfectly correlated across currencies does this expression simplify to the traditional UIRP law of motion, in which case PPP holds in the short-run as well.

and others). The idea is that general equilibrium risk premia that arise from particular forms of consumer risk aversion or monetary rules result in the empirical failure of UIRP.[4]

The new law of motion derived here provides strong empirical support for UIRP. In addition, simulations with data-based parameters show that the model simultaneously explains several yet-unexplained stylized facts (puzzles) of FX rate behavior: (i) the volatility of FX rates relative to their "fundamentals" is similar to that in the data, (ii) the FX rate behavior exhibits a remarkably similar "disconnect" from the fundamentals, (iii) short run PPP doesn't hold, and (iv) the stylized facts that underlie the "carry trade" are shown to be equilibrium phenomena. An implication of the model is that the FX rate ought to have a predictable component, and not be random walks. This has been noted by several authors (see any of Engel & West) but is in contradiction to empirical findings. In this paper I provide novel empirical evidence that FX rates are not simple random walks.[5]

The model developed here is partial equilibrium, because the real yield processes are assumed rather than derived. However, the resulting law of motion is consistent with any well-specified general equilibrium model, because it assumes rational expectations, no arbitrage in the FX market, and it obeys the usual transversality conditions.

The model also abstracts from risk premia because there is no basis for introducing risk premia in this model; in any case it is useful to understand how much real yield variation alone

[3] The lack of empirical support of the UIRP is also known as the "forward premium puzzle."

[4] See Bacchetta and Von Wincoop (2006, 2010) for models that can explain qualitatively the failure of UIRP based on limited information or limited attention. For other types of mechanisms that can explain UIRP failure: Alvarez, Atkinson and Kehoe (2005) –varying degree of market participation; Bacchetta and Van Wincoop (2005) –rational expectations errors; Gourinchas and Tornell (2004) –distorted beliefs; Frankel & Froot (1987) – liquidity premia and deviations of market expectations from their statistical counterpart; additional citations are in Engel and West (2010). Fama (1984) elaborates on the statistical characteristics risk premia must possess to "explain" the empirical failure of UIRP.

[5] Important puzzles not considered here are: the home-goods consumption bias puzzle, the home assets preference bias, and why consumptions are not more correlated internationally or why are current accounts so-small compared to saving and investment.

can explain. It does not deny the existence of such premia, and they can be easily incorporated into the model; in that case UIRP would hold ex ante up to the risk premia. Finally, the model takes no position as to why real yields behave the way they do. However, it is quite clear that lack of complete arbitrage is fundamentally responsible for variation in real yields across currencies. With perfect goods and capital markets arbitrage, measured real yields would be very highly correlated; they would differ only by country risk premia. In the sample of developed countries with open financial markets studied here, such risk premia are likely to be quite small, and it is highly unlikely that they would account for the low correlations and the empirical differences in sample averages of the estimated real yields.

The model is a dynamic version of the "overshooting" model initially formulated by Dornbusch (1976). It also has a lot in common with the present value models of the FX rate, developed by Engel and West (2004, 2005, 2006, 2010), and Engel, Mark, and West (2007), who develop a present value relation similar to this model which they apply to the "fundamentals."

The remaining paper is organized as follows. Empirical evidence that established the FX puzzles is presented in Section I; the new law of motion for the FX rate is derived in Section II. Section III contains description of the data and of the procedures used to estimate expected inflation and real yields. Simulations of the model and the conditions under which it produces the familiar patterns of FX, real FX, and PPP-FX rates are shown in Section IV. Initial tests of the model are presented in Section V, and concluding remarks are in Section VI. The Appendix contains detailed proofs of important results.

I. The Empirical Evidence for the FX Puzzles

The FX rate *S* is expressed in foreign currency per unit of home currency –taken to be the US$ (fc/\$)– and the forward rate is *F*. Also, $x \equiv \ln X$ and $\underset{t-1}{E} x_t \equiv x_t^{e,t-1}$. The general notation, ${}_{t-k}x_{h(f),t+j}$, denotes the value of *x* formed at *t-k* and relevant for the period *t-k* to *t+j*; *h* and *f* stand for home and foreign values, respectively. When clarity is not impaired the pre-subscripts are dropped.

The statement of UIRP is $s_t^{e,t-1} - s_{t-1} = {}_{t-1}i_{f,t} - {}_{t-1}i_{h,t}$; the difference between currently quoted home and foreign yields is made up by the expected appreciation of the foreign currency.[6] UIRP can be derived by analogy to covered interest rate parity (CIRP) or formally from models in which PPP holds.[7]

It is important to distinguish between the UIRP *proposition*, a statement about the effect of sufficient arbitrage (or speculation) in the FX markets, and the resulting law of motion, which traditionally has been stated as, $s_t - s_{t-1} = {}_{t-1}i_{f,t} - {}_{t-1}i_{h,t} + rp_t + \varepsilon_t$, where *rp* is a possibly time-varying risk premium and ε is the error term. I refer to this formulation of UIRP as the "traditional" UIRP formulation and reserve the term UIRP to refer to the arbitrage condition itself. Crucially, in the derivation of the traditional UIRP law of motion there is no provision for

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[6] This is the continuous-time (or log) approximation of the traditional UIRP discrete-time expression: $\frac{S_t^{e,t-1}}{S_{t-1}} = \frac{1 + {}_{t-1}i_{f,t}}{1 + {}_{t-1}i_{h,t}}$.

[7] Covered Interest Rate Parity, (CIRP) is ${}_{t-1}f_t - s_{t-1} = {}_{t-1}i_{f,t} - {}_{t-1}i_{h,t}$; a higher foreign yield implies an appreciated forward rate over the spot, so that the home investor gets the identical return from the home and foreign yields, covered. CIRP holds extremely well in the data once bid-ask spreads are taken into account. While CIRP can be arbitraged near-risklessly, this is not so for UIRP.

independent movements of home and foreign real yields, even thought there is general agreement that real yield and inflation premia can have different effects on the FX rate.[8]

Tests of the traditional UIRP formulation have been conducted by estimating the law of motion, $s_t - s_{t-1} = \beta_0 + \beta_1\left({}_{t-1}i_{f,t} - {}_{t-1}i_{h,t}\right) + \varepsilon_t$; UIRP requires that β_0 = 0.0 and β_1 = 1.0. In test after test this formulation fails, and spectacularly so; β_1 has been found to be negative, close to zero or positive but generally insignificant; only occasional significance is reported in the literature. Chinn (2006) shows that the regression coefficient can be as high as 0.70 for UIRP tests done with longer-horizon maturity yields, and Chinn & Quayyum (2012) find the result holds in a now-longer sample but the coefficients values are somewhat attenuated.[9]

Table 1 replicates the traditional UIRP regression results for 14 currencies, using monthly data.[10] All but one of the coefficients (β_1) are negative, and none are statistically different from 0.0, at the 5% confidence level. The miniscule R-squared statistics show that the yield differential contains little information about FX rates. This conclusion is emphasized by Martin (2012).

A second puzzle is illustrated in Figure 1. It shows the extent to which the volatility of the FX rate's growth rate (GFX) is at least an order of magnitude higher than the volatility of the associated nominal yield differential, for four major currencies relative to the $US. The pattern illustrated in Figure 1 is typical of all the free-floating currencies. The average ratio of the

[8] A corollary of deriving UIRP from PPP is that the real yields are equal across country (Real IRP).

[9] According to the model developed below this finding is to not surprising because as the horizon lengthens the effects of inflation become increasingly more important for FX rates compared to the effects of real yields, the higher the inflation rates and the more inflation differs across countries.

[10] In the estimation results presented here and later, I use data for Australia, Canada, Denmark, France, Germany/Eurozone, Italy, Japan, Netherlands, New Zealand, Norway, Singapore, Sweden, Switzerland, and the U.K.; the U.S. is the "home" country. See the data section for details.

standard deviations of GFX to the yield differentials for the 14 currencies in the sample is 17.1, with a range of 24.3-12.5.

A third puzzle is that the real FX rate, the FX rate adjusted for CPI differences –RFX– and the FX rate itself are highly correlated and they are virtually disconnected from the PPP-FX rate (the FX rate at which PPP would hold). Figure 2 shows these data patterns for the four currencies in Figure 1; again, these patterns are typical of all free-floating currencies. The only connection between the FX and PPP-FX rates appears to be that the FX rates loosely follow the overall trend of PPP-FX.[11] If PPP held in the short run, the FX and the PPP-FX rates would coincide and they would respond only to inflation differentials; there would be no correlation between *FX* and *RFX*. Regressions that test short-run PPP using inflation differentials on FX growth rates are about as successful as UIRP regressions. The positive evidence on long-run PPP comes mainly from cross-sectional regressions or from cointegration-type studies (see Engel (2000)). Longer-term evidence suggests that the RFX is most likely stationary, i.e., that long-run PPP holds but this is not a settled question in the literature.[12]

Finally, the reported success of the "carry trade" is based on two related stylized facts. One is that returns from holding foreign currencies and converting the proceeds back into $US (the $-equivalent return) are correlated with the currencies' local yield. The other stylized fact is that a speculative portfolio long in the highest yield local currency and short in the lowest local yield currency (a Hi-Lo portfolio) has economically significant positive returns. These stylized

[11] The *RFX* and *PPP-FX* in Figure 2 are benchmarked to the over- or under-valuation of each currency reported by the March 1999 Big Mac Index. This is the last date for which Big Mac prices for the individual Eurozone countries are reported.

[12] For an extensive analysis of issues relating to long run PPP see Engel (2000), Engel, Mark and West (2007), and references therein. The ocular evidence on long run PPP is largely related to a stylized fact in the literature, that substantial inflation differences are detectable in the trends of FX rates. In Figure 2, Germany/Eurozone, Japan, and Switzerland have had lower inflation rates than the U.S., and the secular appreciation of their currencies is

facts directly contradict the traditional formulation of UIRP and remain unexplained; these correlations ought to be zero and no excess returns ought to accrue to such Hi-Lo portfolios according to the traditional UIRP.

Table 2 shows the evidence for the 14 currencies studied here. The table results are broken down into three periods because the observations do not substantially overlap for all the currencies.[13] The ordinary correlation coefficients between the local yields and the $-equivalent returns are very high and statistically significant (average is 97%); however, the Spearman rank correlation between them is low and generally statistically insignificant. The Hi-Lo returns are substantial (average is 4.5%) but they are not statistically different from zero because of the very high volatility of the $-equivalent returns.

II. A New Model of the FX Rate

Dornbusch (1976) shows that if the home real yield declines (due to an unexpected but temporarily looser monetary policy), the home currency has to *depreciate*, so that *after* the surprise it can appreciate and continue to deliver UIRP in expectations, i.e., the now-lower domestic yield is equalized with the higher foreign yield, expressed in either currency. This is known as the "overshooting model." Short-run PPP has to be violated so that the FX rate conforms to the UIRP until real yields are equalized.

The model developed here assumes that UIRP holds in expectations, and derives the law of motion for the FX rate when real yields are stochastic and not perfectly correlated across currencies; inflation rates are also stochastic.

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detectable, in that the spot rates loosely follow *PPP-FX*. Similarly, the U.K. has had higher inflation rates than the U.S. and the secular depreciation of its spot rate loosely follows the depreciating *PPP-FX*.

[13] Data for France, Italy, and the Netherlands end in 1999 with the implementation of the euro, while data for Australia, New Zealand, Norway, and Sweden are available from 1997. The remaining countries have data for the whole period, 1986-2013 (Singapore data start in 1988).

A. The Relation of the Real Exchange Rate to the Real Yields

The real FX rate is, $RFX_t \equiv \frac{S_t P_{h,t}}{P_{f,t}}$, where the *P*s are the respective national price levels; *S* is fc/hc. In logs, this becomes $rfx_t \equiv s_t + p_{h,t} - p_{f,t}$, or $s_t = rfx_t + p_{f,t} - p_{h,t}$. In what follows, the behavior of RFX is derived first and the effects of inflation are added later to derive a complete expression for the FX rate.

Rational expectations, arbitrage, long-run PPP, and boundary conditions require that RFX converges to its possibly time varying equilibrium value, RFX_{eq}, and that the expected path of *S* conforms to UIRP in expectations. At any date, the market has to rationally forecast the future real yield paths and the *RFX* has to take a value that will deliver UIRP in expectations along a path that converges to RFX_{eq} when the *r*s return to their equilibrium values.

To derive the behavior of the RFX, let the equilibrium value of *rfx* be rfx_{eq} and examine how the *rfx* is set when real yields, *r*, are not equal to their equilibrium values, $r_{j,0} \neq r_{eq}$, for $j = f,h$.[14] The strategy is to solve for rfx_0. Start with periods 0 and 1. From UIRP,

$$rfx_1^{e,0} - rfx_0 = {}_0r_{f,1} - {}_0r_{h,1}. \tag{1a}$$

For UIRP to hold, if the foreign yield is higher the home currency has to appreciate. A home-country agent investing in the foreign asset will receive the higher yield, r_f, but will lose the advantage over r_h. Pursuing the process forward,

$$rfx_2^{e,0} - rfx_1^{e,0} = {}_1r_{f,2}^{e,0} - {}_1r_{h,2}^{e,0}, \text{ and } rfx_2^{e,0} = rfx_0 + {}_0r_{f,1} + {}_1r_{f,2}^{e,0} - {}_0r_{h,1} - {}_1r_{h,2}^{e,0}, \text{ etc.} \tag{1b}$$

This can be written compactly as,

[14] Implicit in the derivations that follow is that there exists a real short term bond, so ${}_tr_{t+1}$ is known at time *t*. This eliminates the need to burden the notation further by keeping track of expected real rates but it is not a critical assumption.

$$rfx_0 = rfx_n^{e,0} + \left({}_0 r_{h,1} + \sum_{j=1}^{n} {}_j r_{h,j+1}^{e,0} \right) - \left({}_0 r_{f,1} + \sum_{j=1}^{n} {}_j r_{f,j+1}^{e,0} \right), \tag{1c}$$

and because as $n \to \infty$, $rfx_n^e \to rfx_{eq}$,

$$\begin{aligned} rfx_0 &= rfx_{eq,0}^{e,0} + \left({}_0 r_{h,1} + \sum_{j=1}^{n} {}_j r_{h,j+1}^{e,0} \right) - \left({}_0 r_{f,1} + \sum_{j=1}^{n} {}_j r_{f,j+1}^{e,0} \right), \text{ or} \\ rfx_0 &= rfx_{eq,0}^{e,0} + \left({}_0 r_{h,1} - {}_0 r_{f,1} \right) + \sum_{j=1}^{n} \left({}_j r_{h,j+1}^{e,0} - {}_j r_{f,j+1}^{e,0} \right) \end{aligned} \tag{2}$$

The process produces a finite rfx_0 if and only if $r_{f,n}^{e,0} \to r_{h,n}^{e,0} \to r_{eq,0}^{e,0}$ as $n \to \infty$, where $r_{eq,0}^{e,0}$ is the expected equilibrium yield at $t = 0$.[15] For rfx_t to be I(0) and long run PPP to hold, the $r_{eq}^{e,0}$ and $rfx_{eq}^{e,0}$ have to be I(0) as well.[16] rfx_0 deviates from $rfx_{eq,0}^{e,0}$, the expected equilibrium RFX at $t = 0$, by the undiscounted sum of the expected real yield deviations from r_{eq}; of course equation (2) holds for any rfx_t.

Equation (2) is the key to understanding the model and why this law of motion is fundamentally different from that of the traditional UIRP formulation. The *RFX* deviates from its long run value, RFX_{eq}, to the extent that home and foreign real yields differ from one another; this deviation depends on how large the differences are and how long they will persist. The longer the real yields are expected to differ, the bigger the deviation of RFX from its equilibrium value; differences that are expected to persist for a long time will cause the RFX to wander far from its equilibrium value. This result is general and it is fully consistent with any rational general equilibrium model.

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[15] More precisely, in a stochastic environment, $\operatorname{plim}\left(\sum_{n=0}^{\infty} \left(r_{h,n} - r_{f,n} \right) \right) = 0$.

[16] I(0) refers to a stationary series while I(1) refers to an integrated series of order 1. For the rest of the paper, I assume that r_{eq} and rfx_{eq} are I(0).

Below I assume exogenous AR1 processes for the real yields to derive a closed-form solution to equation (2); inflations are also taken to be exogenous AR1 processes for the simulations that follow.[17]

B. The Yield Processes

Let the real yields be described by AR1 processes: ${}_t r_{t+1} = r_c + \rho \; {}_{t-1}r_t + \varepsilon_t$, where r_c is a constant, ρ is a parameter, and ε is an error term. The possibly time varying equilibrium real yield has to be the same for all currencies, so that $r_{eq} = \dfrac{r_c}{1-\rho}$; thus $r_c = r_{eq}(1-\rho)$ and $\rho < 1$ are restrictions on the processes; ε is i.i.d., and E(ε) = 0.

The real yield forecast from an initial yield ${}_0 r_1$ is:

$$ {}_1 r_2^{e,0} = r_c + \rho \; {}_0 r_1 , \tag{3a} $$

${}_2 r_3^{e,0} = r_c + \rho {}_1 r_2^{e,0}$ leads to, ${}_2 r_3^{e,0} = r_c + \rho(r_c + \rho \; {}_0 r_1)$ and to ${}_2 r_3^{e,0} = r_c(1+\rho) + \rho^2 \; {}_0 r_1$.

The general term is ${}_n r_{n+1}^{e,0} = r_c\left(1 + \rho + \rho^2 + \rho^3 + ... + \rho^{n-1}\right) + \rho^n \; {}_0 r_1$; substitute in r_{eq} to get:

$$ {}_n r_{n+1}^{e,0} = r_{eq,0}^{e,0}\left(1 - \rho^n\right) + \rho^n \; {}_0 r_1 ; \quad {}_n r_{n+1}^{e,0} \to r_{eq,0}^{e,0} \text{ as } n \to \infty. \tag{3b} $$

C. The Laws of Motion for RFX and FX

To derive the expression for *rfx* substitute the successive time = *t* real yield forecasts, ${}_{t+j} r_{h,t+j+1}^{e,t}$, ${}_{t+j} r_{f,t+j+1}^{e,t}$, for the home and foreign currencies and collect terms.

$$ \begin{aligned} rfx_t =& \left(\frac{1-\rho_h^n}{1-\rho_h}\right) {}_t r_{h,t+1} - \left(\frac{1-\rho_f^n}{1-\rho_f}\right) {}_t r_{f,t+1} \\ &+ \left\{ \left(\rho_f^{n+1} - \rho_h^{n+1}\right) + \frac{\rho_f\left(1-\rho_f^n\right)}{1-\rho_f} - \frac{\rho_h\left(1-\rho_n^n\right)}{1-\rho_h} + n\left(\rho_h^{n+1} - \rho_f^{n+1}\right) \right\} r_{eq,t}^{e,t} + rfx_{t+n}^{e,t} \end{aligned} \tag{5a} $$

[17] In Dornbusch (1976), McCallum (1994), and others, the real yields deviate due to monetary policy and because prices are not completely flexible.

Now let $n \to \infty$, and write equation (2) compactly as (the complete derivation is in the appendix):

$$rfx_t = \left(\frac{1}{1-\rho_h}\right)_t r_{h,t+1} - \left(\frac{1}{1-\rho_f}\right)_t r_{f,t+1} + \left\{\frac{\rho_f}{(1-\rho_f)} - \frac{\rho_h}{(1-\rho_h)}\right\} r^{e,t}_{eq,t} + rfx^{e,t}_{eq,t}. \tag{5b}$$

The FX rate is the RFX augmented by the price levels, from the definition of the RFX; the new law of motion of the FX rate is:

$$s_t = \left(\frac{1}{1-\rho_h}\right)_t r_{h,t+1} - \left(\frac{1}{1-\rho_f}\right)_t r_{f,t+1} + \left\{\frac{\rho_f}{(1-\rho_f)} - \frac{\rho_h}{(1-\rho_h)}\right\} r^{e,t}_{eq,t} + rfx^{e,t}_{eq,t} + p_{f,t} - p_{h,t} \tag{6}$$

Absent disturbances, s_t will deliver UIRP along its path as it converges to rfx_{eq} as the real yields converge to each other and to r_{eq}. Furthermore, s_t obeys “relative” PPP in the limited sense that it adjusts to nominal price changes in the two economies; PPP holds only in the long run. In growth rates the model is,

$$\Delta s_t = \Phi_h \Delta\ {}_t r_{h,t+1} - \Phi_f \Delta\ {}_t r_{f,t+1} + \Theta \Delta r^{e,t}_{eq,t} + \Delta rfx^{e,t}_{eq,t} + {}_{t-1}\pi_{f,t} - {}_{t-1}\pi_{h,t}\ , \tag{7}$$

where Δ stands for first differences, $\Phi_j \equiv \dfrac{1}{1-\rho_j} > 1$, $\Theta = \left\{\dfrac{\rho}{1-\rho_f} - \dfrac{\rho}{1-\rho_h}\right\}$, and the πs are the respective inflation rates. From this point forward, I refer to the model in equations (6) or (7) as the new model or the new UIRP formulation.

D. The Economics of the Model

Equation (6) shows that there is little apparent connection between this model and the traditional formulation of UIRP. The conclusion is that the traditional law of motion based on UIRP cannot describe the behavior of the FX rate when real yields move independently. The equations also show the intimate interconnectedness of the explanations to the international finance puzzles discussed above.

The distinguishing features of the new model are:

1. Real yields and inflation rates have opposite effects. Furthermore, the real yields need not have symmetric impacts on the FX rate.
2. In equation (6), real yields and price levels (not inflation rates as in the traditional formulation) enter together. This means that the expression cannot be written in terms of nominal yields alone.
3. Equation (6) shows that it is the *current* rather than the past values of the real yields (as in the traditional formulation) that drive the FX rate.
4. The explanation for the high volatility of the FX rate is that the coefficients of the real yields can be very large, amplifying the yields' volatility. The yield coefficients summarize the persistence of the real yields (ρ_j); the higher the persistence, the larger the coefficients, and the longer it takes for the FX rate to return to equilibrium while delivering UIRP along the path. This amplification will be tempered by the degree of correlation of the real yields.
5. The term in the curly brackets in equation (6) adjusts for the difference between the convergence rates of the two real yields; if $\rho_h = \rho_f$, the term is zero; it increases with the difference in the convergence rates (see the Appendix for details).
6. Equation (6) shows that there ought to be some ability to forecast the RFX and therefore the FX rates because real yields, as I(0) processes, regress to the mean.[18]

Equation (6) combines ex ante UIRP and long run PPP into a single law of motion. If PPP is to "hold" in the short run, enough goods and real capital arbitrage as well as full price flexibility are required, for the real yields will obey $r_h = r_f = r_{eq}$. Only then will short-run PPP hold, regardless of variation in r_{eq}. In such a case, the new law of motion is observationally

[18] Additionally, forecasting ability can arise from forecasting inflation, r_{eq}, rfx_{eq}, and risk premia (excluded here).

indistinguishable from the traditional one because it contains the identical information as the PPP equation, $\Delta s_t = {}_{t-1}\pi_{f,t} - {}_{t-1}\pi_{h,t}$. But in the absence of such perfect arbitrage, the FX rate can deviate a great deal from its PPP prediction because of real yield variation.

Finally, equation (6) shows that the short run dynamics of the FX rate are likely to be dominated by the real yield, and that the short term effects of shocks in macro variables other than inflation will be transmitted to the FX through their effects on the real yields. This may make it difficult to link most macro variables to FX rates, particularly if not all the relevant variables are considered simultaneously.[19]

Macro variables affect the FX rate through the equilibrium values, $r^e_{eq,t}$ and $rfx^e_{eq,t}$ as well. However, the current FX rate is affected by these equilibrium values only to the extent that the expected values at which convergence will be achieved are affected. For example, if $r^e_{eq,t}$ and $rfx^e_{eq,t}$ are i.i.d., then the FX rate will not respond directly at all to current shocks to r_{eq} and rfx_{eq}, because their expected values do not change. For the RFX to be I(0) –and long run PPP to hold– the processes for r_{eq} and rfx_{eq} must be I(0), so that only a fraction of a current shock will affect expected values at convergence and therefore the current FX rate.

E. The Expected FX Rate, the Forward Rate, and the Standard UIRP Regression Errors

Expectations of equation (7) give the general relation for the expected FX rate:

$$\begin{aligned} s^{e,t}_{t+1} - s_t &= \Phi_h\left({}_{t+1}r^{e,t}_{h,t+2} - {}_t r_{h,t+1}\right) - \Phi_f\left({}_{t+1}r^{e,t}_{f,t+2} - {}_t r_{f,t+1}\right) + {}_t\pi^{e,t}_{f,t+1} - {}_t\pi^{e,t}_{h,t+1} \\ &\quad + \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right), \end{aligned} \tag{8}$$

Further simplification gives,

[19] Flannery and Protopapadakis (2012) find that macro announcements' surprises that influence the daily FX rate for the most part simultaneously influence real yields.

$$\begin{aligned} s^{e,t}_{t+1} - s_t &= \left(r^{e,t}_{eq,t} - {}_t r_{h,t+1}\right) - \left(r^{e,t}_{eq,t} - {}_t r_{f,t+1}\right) + \pi^{e,t}_{f,t+1} - \pi^{e,t}_{h,t+1} \\ &+ \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right) \end{aligned}. \tag{9}$$

This rearrangement shows how the FX rate accounts for the reversion to the mean property of the real yields and of changes in the equilibrium values of r^e_{eq} and rfx^e_{eq}. Equation (9) can also be written in terms of nominal yields,

$$s^{e,t}_{t+1} - s_t = {}_t i_{f,t+1} - {}_t i_{h,t+1} + \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right). \tag{10a}$$

Furthermore, even though the equilibrium values of r_{eq} and rfx_{eq} change over time, $r^{e,t}_{eq,t+1} = r^{e,t}_{eq,t}$, and $rfx^{e,t}_{eq,t+1} = rfx^{e,t}_{eq,t}$, because these are equilibrium values and the information set is for time = t for both expectations. So equation (10a) further simplifies to the UIRP statement,

$$s^{e,t}_{t+1} - s_t = {}_t i_{f,t+1} - {}_t i_{h,t+1}. \tag{10b}$$

It is no surprise that the model forecast is UIRP, since the model is based on UIRP holding ex ante.

However, equation (10b) seems to contradict the model; why do the forecasts of real yields and inflation predict FX movements in the same direction while in equation (6) they have opposite effects? The answer is that just as a high inflation predicts depreciation a high real yield also predicts depreciation because the real yield is expected to fall, and the two variables have identical impacts on expectations. A high current real yield causes the *FX* rate to appreciate while simultaneously predicting its future depreciation.

The regression residuals of the standard UIRP law of motion can be expressed as,

$$\begin{aligned} s_t - s_{t-1} &= \beta_1\left({}_{t-1} i_{f,t} - {}_{t-1} i_{h,t}\right) \\ &+ \Phi_h \; {}_t r_{h,t+1} - \left(\Phi_h + 1\right)_{t-1} r_{h,t} - \left\{\Phi_f \; {}_t r_{f,t+1} - \left(\Phi_f + 1\right)_{t-1} r_{f,t}\right\} \\ &+ \Theta\Delta r^{e,t}_{eq,t} + \Delta rfx^{e,t}_{eq,t} + \left({}_{t-1}\pi_{f,t} - {}_{t-1}\pi^{e,t-1}_{f,t}\right) - \left({}_{t-1}\pi_{h,t} - {}_{t-1}\pi^{e,t-1}_{h,t}\right) \end{aligned} \tag{11}$$

The second and third lines of the equation show the variables that end up in the error term of such a regression. These errors are complicated functions of real yields quoted at time *t*-1 and *t*, changes in the r_{eq} and rfx_{eq} expectations, and inflation forecasting errors. These regression errors are correlated with the explanatory variable (the interest rate differential) so that estimates of β_1 are almost surely going to be heavily biased.[20]

III. The Data and Estimation of the Real Yields

The broad features of simulation results are compared to the FX stylized facts to assess how well the model helps explain FX behavior and thus the puzzles. For the simulations to be useful, parameter values have to be informed by the data. The data are described in section A, the estimation of real yields and inferences about their statistical properties and those of inflation are discussed in section B.

A. The Data

Datastream makes available daily data for FX rates and one-month eurocurrency yields (among others) for 17 countries from Barclay's: Australia, Canada, Denmark, Finland, France, the Eurozone, Germany, Hong Kong, Italy, Japan, Netherlands, New Zealand, Norway, Singapore, Sweden, Switzerland, the U.K., and the U.S, from 1986 or later. I drop Finland and Hong Kong, and concatenate the DM and euro data into one series.[21] Empirical tests are generally done with monthly data, so end-of-month values are used. CPI data are from Datastream's IFS database. U.S. Inflation Indexed Bond (TIPS) yields are from the St. Louis Fed. The data are available only for long-term bonds; the 5-year bond (the shortest available maturity) is used. International

[20] Regressions run on simulated data from the model have standard UIRP regression coefficients very close to and indistinguishable from zero. The simulations use data-based parameters, as explained in the section that follows.

[21] Finland has a very small sample (1995-1999). Hong Kong has a currency board, so the bulk of the yield differentials with the U.S. consist of risk premia related to the possible abandonment of its currency board arrangement. Concatenation of the DM and euro data is standard in the literature, see Cialenco and Protopapadakis (2011).

Inflation Indexed Bonds (IIB) price data are available from Datastream from 2003 but these IIB data exist only for Australia (1 bond), France (4), Germany (1), Italy (3), Japan (1), and the U.K. (5), for the countries in the sample; the issuance of these bonds is sparse. I use all but the Australian and Italian data in the analysis that follows.[22] The data are daily and for specific bonds; enough information is supplied to convert prices into yields.

Datastream also provides nominal bond data for the countries in our sample. These data are needed to estimate inflation expectations from the IIB bonds. However, the duration of available bonds doesn't match closely that of the IIBs; I select the best fitting bonds for the purpose. Finally, the Survey of Professional Forecasters (SPF) from the Philadelphia Fed provides quarterly one-year inflation forecasts for the U.S.

B. The Estimation of Real Yields and the Statistical Parameters

The critical requirements for simulating and estimating the new FX model is to obtain real yields and inflation expectations and to ascertain their statistical properties. To do this, it is necessary to estimate real yields and inflation expectations.

It is clear from the data section that it is not feasible to use IIB data to construct market-based short term real yields.[23,24] Thus, real yields have to be estimated from inflation expectations, for which there are also no direct market-based estimates or widely available survey estimates. The subsections that follow describe the procedures I use to compute real yields and inflation expectations and to determine their statistical properties.

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[22] There appear to be serious data errors in the Australian bond prices. Based on the data provided, the 4% bond with 2 years to maturity is priced at 180, implying a yield below -20%! The Italian bond was issued only in June 2008.

[23] There are other difficulties in addition to the lack of sufficient data. The prices of the IIB bonds are for specific issues so that the maturity of the implied real yield varies through the bond's lifetime. Critically, these are long-term bonds while UIRP tests are done at monthly or maybe quarterly frequencies. Finally, IIB real yields also include risk premia, as well as premia associated with the precise method of indexing in each country.

B.1 Inflation Expectations

The standard approach to computing inflation expectations is to estimate a forecasting relation, typically an autoregressive model of the inflation rate, and use the "predicted" part as expected inflation.[25] Unfortunately, such expectations estimated from monthly CPI data are very volatile so they are likely to result in poor real yield estimates; a large part of the difficulty is that the CPI is measured with errors that self-correct over time. Table 3 shows the volatility of inflation, as well as the volatility of alternative estimates of expected inflation. Column 2 shows the annualized standard deviations of the monthly CPI-based inflation rates for each country. Column 3 shows the standard deviations of forecasts from the best-fitting AR model of this monthly inflation, while Column 4 shows analogous standard deviations derived from IIB data, where they can be computed.[26] Compared to the IIB and the SPF inflation expectations the AR forecasts from monthly CPI massively overstate expected inflation volatility.[27]

Column 5 of Table 3 shows the standard deviations of "year-over-year" (YoY) inflation rates; YoY inflation is a 12-month moving average of monthly CPI inflation.[28] Clearly the standard deviations of this measure are lower, in many cases substantially so, compared to the monthly estimates. Column 6 shows the standard deviations of inflation forecasts computed from best-fitting AR processes for YoY inflation. Though the estimated volatilities are considerably higher than those in column 4, they are decidedly smaller that those in column 3. For constructing the real yields and for the tests that follow, I use inflation expectations derived from

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[24] For some applications researchers use the ex post real yield as a proxy for the expected one. Rational expectations guarantee that such estimates are measured with error and are more volatile than the ex ante real yields.
[25] In some instances researchers also use last period's inflation rate for expected inflation. Complex VAR models have been used in other applications.
[26] Combinations of AR1 through AR12 regressions are estimated for each series. The results shown are for the "best-fitting" model.
[27] Ang, Bekaert, and Wei (2007) show that the SPF survey forecasts consistently outperform model-based ones.
[28] The BoG uses this construct regularly because it smooths out the largely self-correcting monthly fluctuations.

the YoY measure of CPI inflation. Column 7 shows the means and standard deviations of the estimated real yields.

B.2 Autocorrelations

A very important parameter for simulating the model is the persistence of the real yields. AR1 coefficients (in AR1 regressions) of the estimated real yields for the 14 currencies and the U.S. average 0.95 (0.997-0.877). The lowest AR1 coefficient for the IIBs and TIPS is 0.96 (for a French IIB that matures in 2032); if these coefficients are adjusted to a one-month time interval, they are all above 0.999.[29] In the simulations that follow, the AR1 coefficients for the real yields are set to $\rho_h = \rho_f = 0.99$, unless otherwise stated; the sensitivity of the simulation results to the ρs is investigated.

The autocorrelation of the monthly CPI inflation is low for most of the currencies (mean: 0.60, range: 0.95 – 0.30) because the monthly CPI is a noisy estimator. In contrast, the autocorrelations of the YoY inflation rates is quite high (mean: 0.96, range: 0.999-0.924) but this is partly due to the autocorrelation induced by the YoY procedure. However, the autocorrelation of non-overlapping annual data from the YoY series also shows high autocorrelations (mean: 0.95, range: 0.99-0.83). In the simulations that follow the AR1 coefficients for the inflation processes are set to $\rho_{\pi,h} = \rho_{\pi,f} = 0.90$.

[29] Adjusting to a one-month interval is as follows: suppose the AR1 coefficient of five-year maturity real yields is 0.90, then the equivalent coefficient for the one-month frequency would be $0.90^{1/60} = 0.998$.

B.3 Volatility and Other Parameters

From Table 3, the real yields' average standard deviation 2.2% (range: 3.5%-1.4%).[30] For the simulations, real yield volatility is set to 2%. The YoY inflation standard deviations average 1.8% (range: 3.7%-1.2%).[31] For the simulations inflation volatility is set to 3.0%; this volatility affects the fluctuations of PPP-FX but not those of the RFX. Finally, the equilibrium real yield (r_{eq}) is set to 2.0%, and the equilibrium inflation rates ($\pi_{eq,h}$, $\pi_{eq,f}$) at 3.0%, somewhat higher than the inflation rate targets in many developed countries; RFX_{eq} is set to 1.0.[32]

IV. Simulation Results

The parameter values summarized in the table are used for all the simulations presented below, unless otherwise stated.

Simulation Parameter Values

ρ_h	ρ_f	$\rho_{\pi,h}$	$\rho_{\pi,f}$	$\pi_{eq,h}$	$\pi_{eq,f}$
0.990	0.990	0.900	0.900	3.00%	3.00%
r_{eq}	**RFX_{eq}**	**Std. Dev. (r_h)**	**Std. Dev. (r_f)**	**Std. Dev. (π_h)**	**Std. Dev. (π_f)**
2.00%	1.00	2.00%	2.00%	3.00%	3.00%

In all the simulations, rfx_{eq} and r_{eq} are constant and there are no "error terms" or risk premia.[33] Furthermore, the cross-correlations of the real yields are set to zero, though there are substantial correlations in the data.[34] The nominal yields are computed from ${}_t i_{j,t+1} = {}_t r_{j,t+1} + {}_t \pi^e_{j,t+1};\ \ j = h, f$. Four random variables are needed for each simulation: one for each real yield and one for each inflation rate.

[30] The IIB and TIPS data suggest somewhat lower standard deviations.
[31] For comparison, the monthly CPI inflation rate standard deviations average 4.22% with a 6.19% - 2.50% range.
[32] In the simulations, the CPIs are built up from the inflation rates.
[33] These assumptions make it possible to focus on the mechanism that drives UIRP. In the empirical section, it is shown that only a fraction of the FX volatility comes from the real yields.
[34] The average correlation of U.S. real yields with those of the other 14 countries is 0.41. Across all the countries, the highest real yield correlation is between the French Franc and the Danish Krone (0.87). Of the 105 independent

There are two types of simulations. One simulation is a panel of 16 currencies (one currency is designated as the home currency), with 325 observations for each "currency," essentially the number of months in the available data. The 325 observation limit is intended to mimic random departures of sample averages from their population values, as in the data. This panel of currencies is simulated 10 times to produce the cross-currency results. The "currencies" in these simulations are identical ex ante, and all the starting values are set to their equilibrium values. Any differences in the simulated behaviors arise from the random disturbance sequences. The second type of simulation is for 10,000 observations, and is used to assess population values.

A. Properties of the Simulations and the FX Puzzles

The top panel of Figure 3 (analogous to Figure 1) shows the relation between the growth of the FX rate (*GFX*) and the yield differential for two simulated currencies relative to the home currency. These "currencies" are picked for illustrative purposes; all the currencies in the simulations exhibit near-identical behavior. The pattern closely matches the stylized facts of FX behavior; the simulated *GFX* is much more volatile than the nominal yield differential. The average standard deviation for *GFX* across all the simulations is 41%, compared to 36% in the data.[35]

The bottom panel of Figure 3 (analogous to Figure 2) shows the relations between FX, RFX, and PPP-FX rates, for the same two "currencies," again relative to the home currency. The simulated patterns are again remarkably similar to the FX stylized facts. As in the data, FX and PPP-FX rates seem to be only remotely connected (the "disconnect" with fundamentals), the

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correlations in this dataset, only eight are above 0.80; the highest U.S. correlation is with the Singapore dollar (0.80).

RFX wanders away from its equilibrium value (1.0), and it is highly correlated with the FX rate. And as in the data, the FX rate broadly follows PPP-FX. Across all the simulations, the ratio of the maximum to the minimum FX rate is 2.2, compared to the data's 1.9.

Figure 4 shows the same simulation "currencies" (same random disturbances) but with much lower persistence of the real yields (ρ_j = 0.80). There is a striking change in behavior. These figures resemble the world imagined by PPP models. Departures from fundamentals are minor and short-lived, and the RFX never strays far from its equilibrium value of 1.0. Clearly, even an apparently modest reduction in the persistence of the real yields dramatically changes the behavior of the FX rate.

The FX model presented here is able to reproduce remarkably well key stylized facts in the FX data that to-date are viewed as puzzles, while delivering UIRP in expectations. The high persistence of real yields is responsible for the large departures from "fundamentals," and in fact, the observed behavior is evidence of adherence to and not departures from fundamentals.

B. Forecastability of the FX Rate

Equation (8) implies that there ought to be a particular kind of predictability in the FX rate, because of the mean reversion of the real yields. The table below summarizes the model's prediction:

	r_f high	**r_f low**	**r_h high**	**r_h low**
***FX* Rate (*fc*/*hc*)**	**will rise**; depreciation probability of *fc* > 50%	**will fall**; depreciation probability of *fc* < 50%	**will fall**; depreciation probability of *fc* < 50%	**will rise**; depreciation probability of *fc* > 50%

[35] The average standard deviation of the nominal yield differential is 4.1% across all simulations, compared to 2.5% for the data; the higher standard deviations in the simulations likely result from the assigned higher-than-in-the-data inflation volatility.

One way to assess the degree of forecastability implied by the model is to compute the transition probabilities of the subsequent moves (fall or rise) of the FX rate when current real yields are very high and very low. According to the new model there ought to be mean reversion in such instances, while the random walk requires all such transition probabilities to be 50%. This mean reversion can be seen clearly in Table 4 that shows simulation results from 10,000 observations with different real yield persistence parameters (values of ρ_j). The table shows the probability of appreciation and depreciation when the respective real yields are unusually "high" or "low." The unusually high and low values are defined as approximately the 10% largest and the 10% lowest real yields in the sample, respectively, for each currency. All the transition probabilities are statistically different from 50%, at the 5% confidence level. Table 4 also shows that lower ρs result in more predictability because the real yields revert to mean faster. Experimentation not reported reveals that lower real yield volatility also increases predictability in the simulations.[36]

C. An Explanation of the Carry Trade Stylized Facts

"…picking up pennies in front of a steamroller…" The Economist's characterization of carry trade profits.

The "carry trade" strategy is based on statistical patterns in the data which show that currencies with high local yields generate higher average returns when converted to the $US uncovered ($-equivalent return) than currencies with low local market yields. Also, a portfolio long on a high-local-yield currency and short in a low-local-yield currency generates substantial returns, as discussed earlier in the paper and shown in Table 2. These stylized facts are an unexplained anomaly because the traditional UIRP formulation predicts that all uncovered $-equivalent returns ought to equal the U.S. yields in expectation, up to a risk premium; there

[36] Similar tests on RFX rates or using nominal interest rates as the trigger show very similar results.

should be no relation between the average local currency yield and the average $-equivalent return.[37]

The FX model presented here shows how correlation between local and $-equivalent returns arises ex post in a well-behaved equilibrium. Let each currency start with identical parameters and starting values. After several periods some currencies will have high and some low real yields. As the model shows, an increase in the currency's real yield appreciates the currency. For a "currency" to have high average real yields means that there were more frequent (or larger) than expected increases in the currency's real yields in the sample. As a result, when returns over a relatively small time period are tallied, currencies with higher real yields will also have an appreciated currency and will have produced higher-than-average $-equivalent returns.

This process manifests itself strongly in the currency simulations. The results in Table 5 are from the same 10 simulations of 16 "currencies" as before. The ordinary correlation coefficient is somewhat lower but remarkably close to that in the data. The average Hi-Lo return is very nearly the same but the statistical significance of Hi-Lo is higher in the simulations than in the data; in 6 of the 10 trials, Hi-Lo is statistically significant at the 5% level or lower. The table shows that the simulations reproduce closely the so-far-unexplained stylized facts in the data, over similar sample sizes.

This finding establishes that ex post these patterns are consistent with equilibrium behavior. However, it does not imply that a trader can earn excess returns on a Hi-Lo portfolio created in real time, as it is often claimed. Studying the empirical success of the carry trade and how it would fare in simulations is well beyond the scope of this paper.

[37] The reference to a "steamroller" reflects the observation that the average standard deviation of the $-equivalent returns is 31.9%, compared to the 2.4% average standard deviation of the local yields.

V. Initial Tests of the New Model

A. Empirical Implementation

The tests of the model and hence UIRP provided here are confined to estimating the relation between real yields and the RFX. The RFX is stationary under almost all FX models, including this one, so equation (5b) can be implemented directly. The empirical version of equation (5b) is:

$$rfx_t = \beta_0 + \beta_h \; _t r_{h,t+1} + \beta_f \; _t r_{f,t+1} + \varepsilon_t \, , \tag{12}$$

where the model predicts $\beta_h > 1.0$, and $\beta_f < -1.0$, with no prediction for β_0. The error term could be autocorrelated as it contains $r_{eq,t}$ and $rfx_{eq,t}$ for which the model supplies no information. To take advantage of error correlations between the RFX and the real yields, equation (13) is estimated jointly with the relevant real yield equations, specified as AR3 processes. The procedure is just-identified GMM with lagged real yields as instruments and a 12-lag Newey-West autocorrelation correction.[38]

Table 6 reports the test results for equation (12) only, for brevity. The results offer strong support for UIRP. The real yield coefficients are all larger than unity, showing that the effects of real yields are indeed amplified. Many of the results are consistent with UIRP but some are oddly inconsistent. Of the 14 β_h coefficients, 11 are positive, as predicted by the model, and of the positive ones, 8 are significant, many with *P*-values well below 5%. Similarly, of the 14 β_f coefficients, 10 are negative, as predicted by the model, and of the negative ones, 6 are

[38] There are other possible statistical remedies for autocorrelated residuals. One could be to add the lagged dependent variable to the RHS. It turns out that its coefficient estimates are very close to (but below) 1.0, so that the regression is essentially one in which the LHS is the first difference of the *rfx*; however this is inconsistent with the model specification. Another remedy could be to run the regressions in first differences; this of course is over-differencing. Since real yields are estimated with (possibly substantial) error, differencing is likely to take out most of the information in the data. Indeed even though in principle one ought to obtain similar coefficient values but different (and possibly better) standard errors, the parameter estimates of these regressions are qualitatively different from the ones in the levels.

significant, many with *P*-values well below 5%. Encouragingly, several of the significant coefficients are in double digits, consistent with high levels of persistence in the real yields.[39]

There are three currencies for which the estimates do not support the model. Singapore is the most puzzling: both βs have the wrong sign *and* are statistically significant.[40] The second is Norway, for which, according to the estimates, the Krone reacts *the same way* to both real yields, and strongly so. The third exception is the lack of any explanatory power of the regression in the case of the Germany/Eurozone; the coefficients are highly insignificant and the R-squared is virtually zero. Examining the results of the two currencies separately (not reported) does not resolve the difficulty; for the DM alone, both coefficients are significant but positive; for the euro alone, the coefficients are significant only at the 10% level, and again they are both positive; the R-squared in both cases is 0.15.

To put these results in perspective one must bear in mind that the model describes the FX market in the absence of complete arbitrage. In effect the model asserts that there is enough arbitrage to value the FX rate at the ex ante UIRP level but not enough to drive the real yields together. In such an incomplete arbitrage environment, determined central bank intervention, changed "market sentiment," and many other market forces may to force departures from UIRP not accounted for in the model.

B. Can FX Rates Be Forecasted?

> *"No model projecting directional movements in exchange rates is significantly superior to tossing a coin."* (Alan Greenspan, March 2, 2004).

Greenspan's remark reflects the standard position of the profession since Meese and Rogoff's (1983) demonstration that a variety of FX rate models that rely on "fundamentals" fail to forecast

[39] Since the real yields are measured with (possibly substantial) error, the coefficient estimates are likely to be biased downward.

out-of-sample; filter rule based speculation models fare no better.[41] Hence, the random walk is the best forecast by default.[42] Yet, theoretical models connect fundamentals to the FX rates in ways that require some predictability. Engel and West (2005) and Engel, Mark and West (2007) show that the present value of future expectations embedded in rational expectations FX rate models mean that FX rates are likely to be near-random walks and thus likely to be empirically indistinguishable from a random walk but there will still be some forecastability.[43]

The model presented here predicts a particular type of reversion to the mean because of the mean reversion of the real yields. As demonstrated in the simulations results in Table 4, this reversion manifests itself strongly when real yields are unusually high or low. Table 7 shows the analogous empirical transition probabilities for the 14 currencies in the sample; the $US is the home currency. As in Table 4 high and low are defined as approximately the 10% largest and the 10% lowest real yields for each currency (details are in Table 7). The results show that 77% of the transition probabilities (104 in the table) are statistically different from 50% at a 5% confidence interval. This is a major departure from random walk and strong evidence that there is predictability in the FX rates. However, while almost 46% of the estimated probabilities are significant and in the direction predicted by the model, 30% are significant and *contrary* to the direction predicted by the model.

The category with the most violations is "r_f high." For the currencies whose data start in 1986, their Hi real yields coincide with U.S. high real yields; almost all yield highs are from that period (01/1986 – 02/1987). Of these currencies, only the U.K. pound exhibits the expected behavior. Since both real yields are Hi, the model prediction depends on relative magnitudes of

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[40] It is tempting to suppose that the FX rate is defined incorrectly as ($/SP$), but unfortunately that is not the case.
[41] See Cialenco and Protopapadakis (2011) and the references therein.
[42] For a discussion of the unfortunate economics and policy implications of FX rates being random walks, see Alvarez, Atkeson, and Kehoe (2007).

the real yields and the specific nuance of expectations during that period, so it is not surprising that violations occur. The remaining currencies' (AU, NZ, NO, SD) their yield highs do not closely coincide with those of the U.S., and their behavior is consistent with the model.[44]

The category with the next most violations is "r_{US} low." For the currencies not absorbed by the Euro, Lo in U.S real yields occurs exclusively at the end of the sample, starting on July 2009. But this is the same time that Lo yields occur for the corresponding currencies, except for the Yen (its Lo is July 2002 – January 2006). When both currencies are simultaneously Lo, once again the model doesn't have clear predictions. The Lo values of the currencies absorbed by the Euro (FF, Lira, G) do not coincide with the corresponding Lo U.S. and the results are consistent with the model's predictions.

Finally, the differences between the pairs of transition probabilities in the data (that add up to 100%) are in some cases much larger than those of the simulation result in Table 6, even when ρ is 0.80. For example, for Switzerland the difference in probabilities is 48% when r_{SF} is high and for New Zealand it is 60% for when $r_{NZ\$}$ is low; this compares with 24% and 25% for transition probabilities for the corresponding conditions when $\rho = 0.80$. More elaborate real yield prediction models, risk premia, or central bank interventions may help explain these discrepancies.

VI. Conclusions

The primary contribution of this paper is to derive a model that when supplied with data-based parameters mirrors closely the puzzling "disconnect" of FX rates from their fundamentals as well

[43] See also Evans (2012) for a different analysis that finds some predictability in FX rates.
[44] The requirement that 10% of the "Hi" and "Lo" yields are selected, along with the variation in sample size and sample starting dates means that U.S. Hi and Lo yields are not for the same dates for all the currencies.

as other puzzling FX rate behavior. The related empirical UIRP tests provide very encouraging support for the model and therefore for the UIRP proposition.

Simulations show that UIRP-driven FX rate responses alone can account for the volatility of FX rates relative to the relevant yields. When real yields (and inflation) vary stochastically across currencies, the requirement that UIRP hold ex ante results in a law of motion that reproduces several yet-unexplained stylized facts of exchange rate (FX) behavior –labeled puzzles in the literature– and provides strong empirical support for UIRP. The law of motion derived here incorporates the opposing effects of real yields and inflation premia, so that it cannot be reconciled with the traditional law of motion derived from UIRP.

The model shows why the traditional tests fail to support UIRP, why the volatility of exchange rates is so much higher than that of the "fundamentals," why the exchange rates and real exchange rates seem disconnected from their "fundamentals," and why short run PPP doesn't hold. The unifying feature of these explanations is that real yields vary stochastically, they are highly autocorrelated, and only modestly cross-correlated. These are precisely the empirical characteristics of real yields in the 14-country sample examined here. Using data-based parameter values I show that the puzzling exchange rate behaviors can be readily reproduce by the model, qualitatively and quantitatively in simulations.

The effect of the real yields is amplified in the model because a high autocorrelation implies a slow return to real yield parity. The exchange and real exchange rates violate PPP because (for instance) a decrease in the real yield of the home currency requires it to depreciate so that it can be expected to appreciate in order to make up the expected future shortfall compared to the foreign currency yield. The higher the autocorrelation the longer it takes for the real yield to return to its equilibrium, and the further the FX rate has to depart from its PPP value in order to deliver UIRP along its expected path. Only if real yields of a pair of currencies are perfectly correlated will the model be observationally equivalent to the traditional UIRP formulation, and short-run PPP will hold.

Initial empirical tests that use data for 14 currencies and the US$ provide strong support for the new model, and therefore for UIRP. The estimated coefficients for the real yields are all larger than unity, as required by the model, and most are significant and of the correct sign; the R-squared values are often substantial.

The model, along with many others, requires FX rates to have some predictability, rather than be random walks, because the real yields are mean-reverting. This prediction is not supported by the empirical evidence to-date. I show that estimates of the FX rate's transition probabilities when real yields take on extreme values are very often far different than the 50% predicted by the random walk and for the most part are consistent with the model.

Finally the model provides an equilibrium explanation for the statistical patterns "carry trade" strategies are based on. One is that local currency returns are positively correlated with their $-equivalent returns. The other is that a portfolio long in the highest local yield currency and short in the lowest local yield currency produces economically significant excess returns. These stylized facts are in direct contradiction to the standard UIRP formulation.

The explanation for these observations lies again in the variation and autocorrelation of real yields. In the simulations shown all "currencies" are ex-ante identical but small-sample random variation results in some currencies ending up with higher average real yields and therefore appreciated currencies and some with lower average real yields and therefore depreciated currencies. As a result, the $-equivalent returns are high for the high average real yield currencies and low for the low average real yield ones. The statistics computed from the simulations match remarkably well with the data.

The results here suggest that it is important to establish theoretical GE underpinnings of the variation in real yields beyond risk premia, so that it is possible to combine the law of motion derived here with general equilibrium risk premia. This would make it possible to devise tests that measure the relative contribution of risk premia to the behavior of FX rates and possibly account for departures from strict UIRP. The high autocorrelation of the UIRP regression residuals suggest that real yield behavior alone is not a complete explanation of FX behavior.

Additionally, results using alternative statistical techniques need to be investigated, and alternative definitions of real yields devised and tried before it is known how well the data conform to the theory and what departures from the theory might imply.

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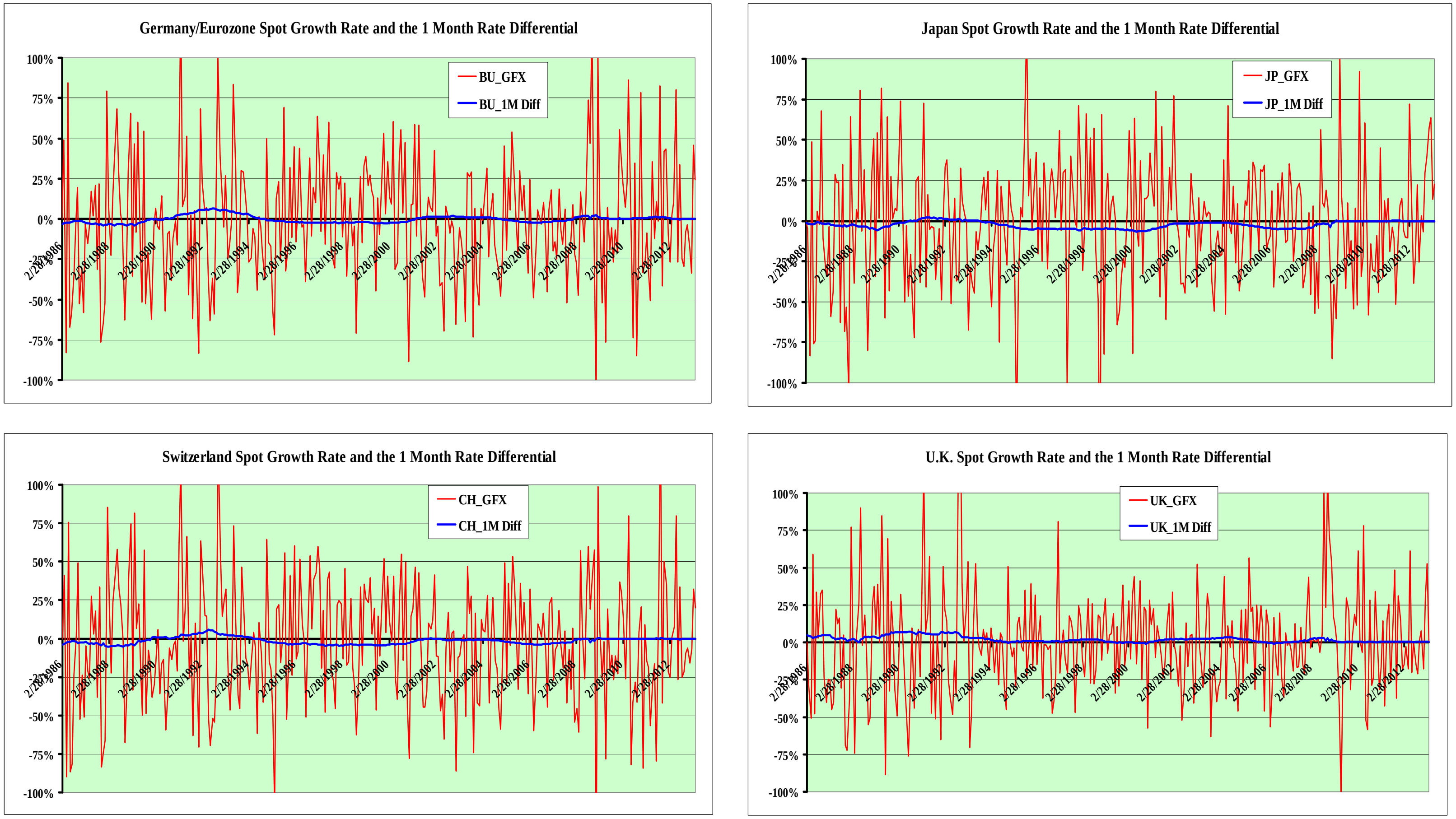
FIGURE 1

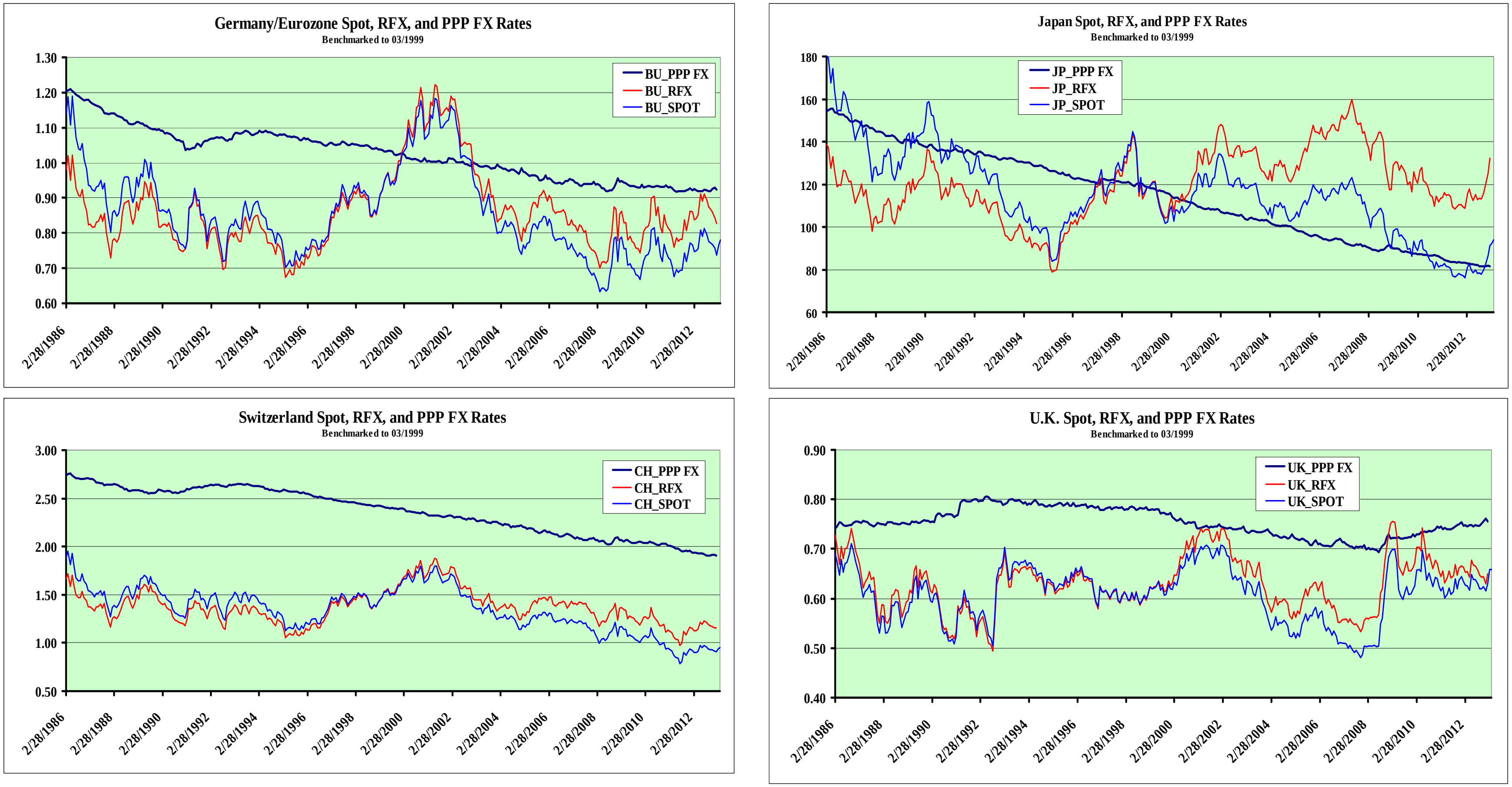
FX Volatility Compared to Yield Differentials for Selected Currencies



Notes:
The figure shows the annualized growth rates of the fc/$ rates (blue lines), and the corresponding annualized one-month yield differential relative to the U.S. (red lines), for DM/Euro (BU), Yen (JP), Swiss Franc (CH), and the U.K.Pound (UK).

FIGURE 2

FX, RFX, and PPP-FX Rates for Selected Currencies

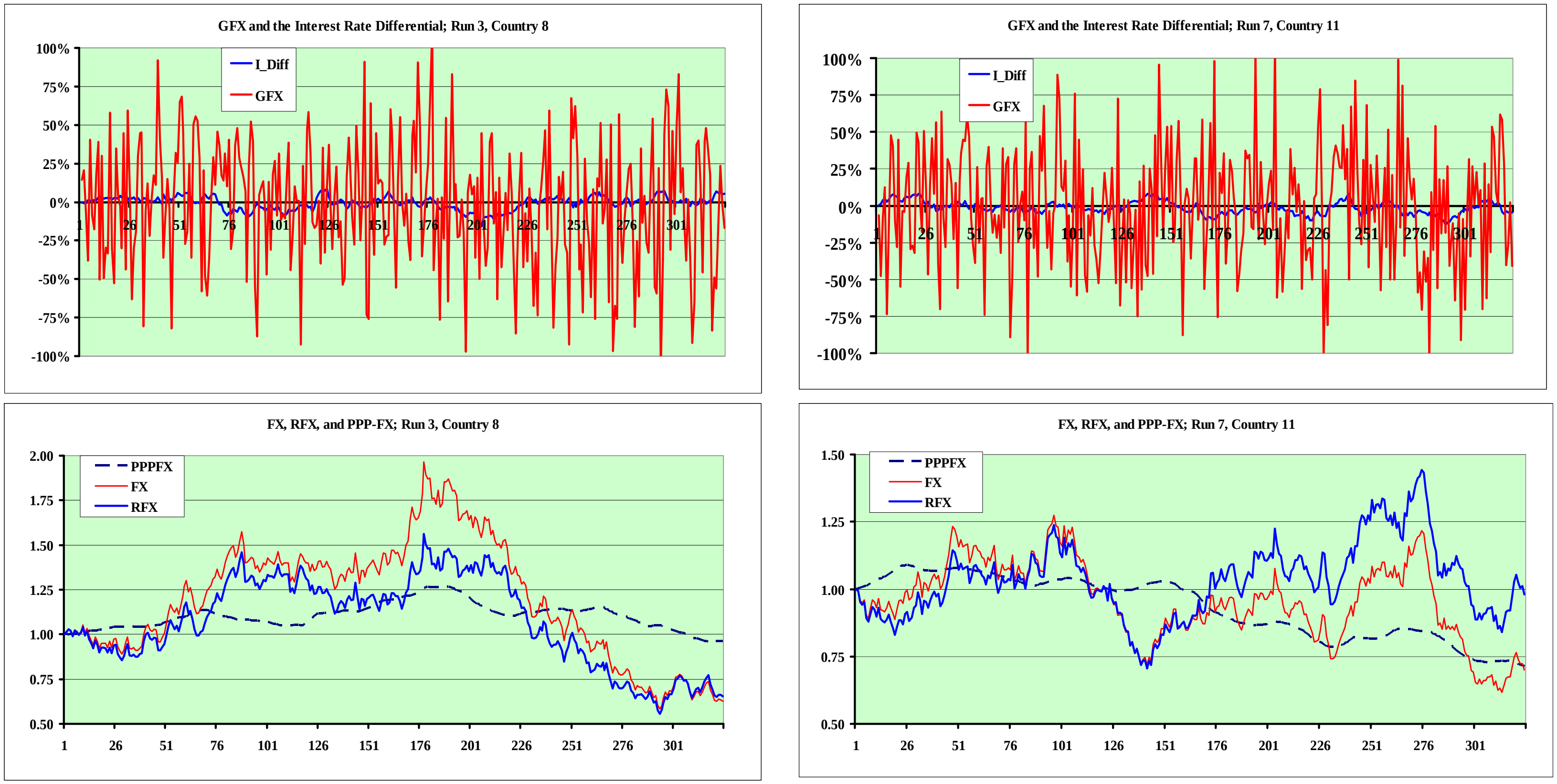


Notes:
The figures show fc/$ FX rates (red lines) along with the associated real *F*X rate (RFX –light blue lines) and the PPP-FX rate (dark blue lines), for DM/Euro (BU), Yen (JP), Swiss Franc (CH), and the U.K. pound (UK), relative to the U.S. The RFX rate is the spot rate adjusted for price differences and the PPP-FX rate is the rate that would prevail if the PPP held continuously. The PPP-FX and RFX rates are benchmarked to the March 1999 Big Mac Index.

FIGURE 3

Selected Simulation Results

Currency 8 is from Run 3, and Currency 11 is from Run 7 ($\rho_h = \rho_f = 0.99$)



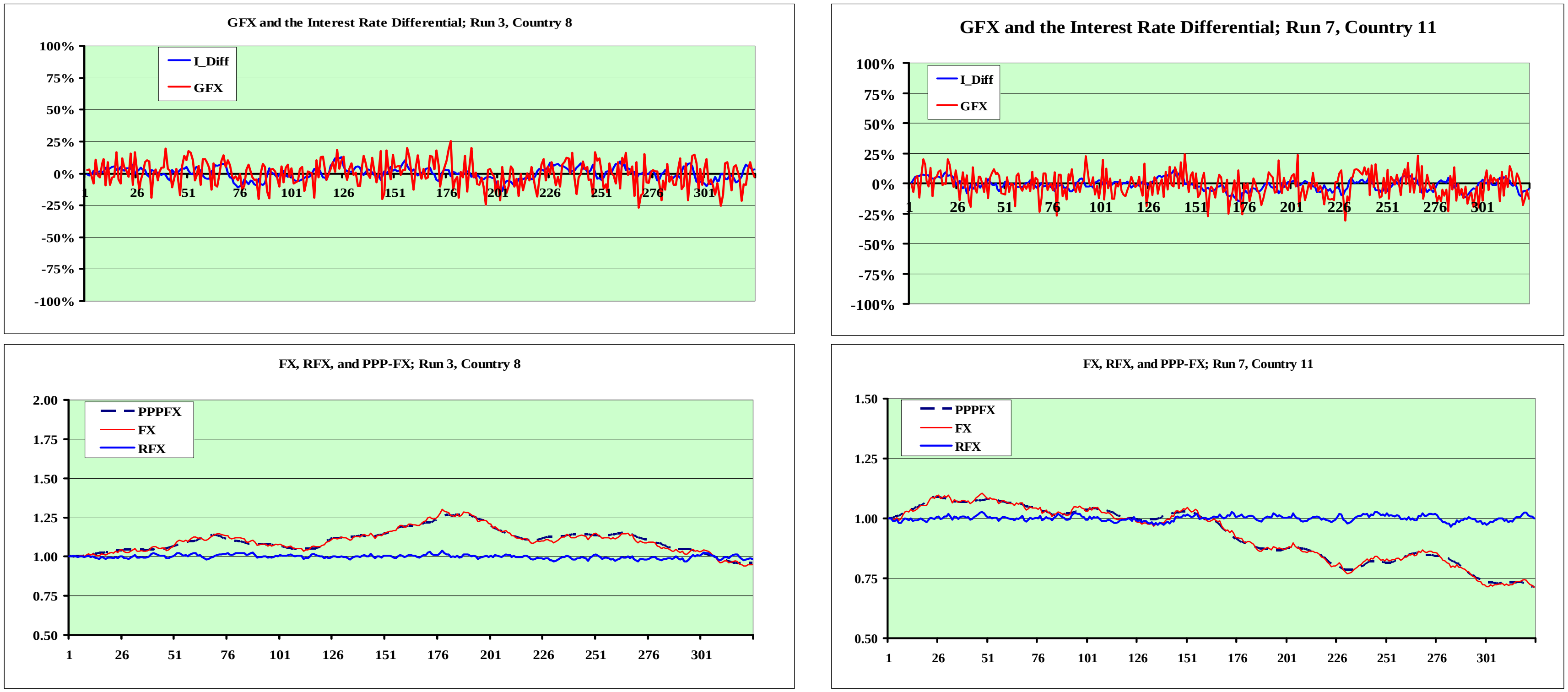
Notes:

1. The figures show simulated fc/hc FX rates and their properties. The top two figures show the relation between the annualized growth rates of FX rates (blue lines), and the corresponding annualized one-month yield differential (red lines). The two figures at the bottom show the FX rates (red lines), and the associated real FX rate (RFX –light blue lines) and PPP-FX rates (dark blue lines), for two currencies from the simulation. The figure is analogous to Figure 2 above. The patterns are representative of all the simulation results.
2. rfx_{eq} and r_{eq} are held constant, so the only source of uncertainty are the yield and inflation rate processes.

FIGURE 4

Selected Simulation Results:

Currency 11 is from Run 7 and Currency 8 is from Run 3 ($\rho_h = \rho_f = 0.80$)



Notes:
The figures show the identical information as Figure 3, except that the real yields' persistence is lower ($\rho = 0.80$). The two currencies are the same as in Figure 3, and the random variables for the simulation are the same. The scale in the figure is kept unchanged to make it easer to see the effects of the smaller ρ.

TABLE 1

Results From Standard Uncovered Interest Rate Parity (UIRP) Tests

	β_0	β_1	NOBS/RSQ		β_0	β_1	NOBS/R^2
A$ Australia			191	**G Netherlands**			155
Coeff	0.002	-2.143	0.008	**Coeff**	-0.0023	-0.146	0.0001
P-Value from 0	*[.528]*	*[.229]*		**P-Value from 0**	*[.384]*	*[.907]*	
P-Value from +1	n.a.	*[.078]* *		**P-Value from +1**	n.a.	*[.360]*	
C$ Canada			326	**NZ$ N. Zealand**			191
Coeff	0.000	-0.891	0.003	**Coeff**	-0.0003	-0.329	0.0002
P-Value from 0	*[.722]*	*[.225]*		**P-Value from 0**	*[.940]*	*[.835]*	
P-Value from +1	n.a.	*[.010]*		**P-Value from +1**	n.a.	*[.400]*	
DK Denmark			326	**NK Norway**			191
Coeff	-0.0009	-0.428	-0.002	**Coeff**	-0.0003	-0.730	0.0015
P-Value from 0	*[.602]*	*[.499]*		**P-Value from 0**	*[.891]*	*[.564]*	
P-Value from +1	n.a.	*[.024]* ✞		**P-Value from +1**	n.a.	*[.172]*	
FF France			155	**SP$ Singapore**			302
Coeff	-0.0014	-0.218	0.0003	**Coeff**	-0.003	-1.908	0.017
P-Value from 0	*[.587]*	*[.863]*		**P-Value from 0**	*[.023]* ✞	*[.053]* *	
P-Value from +1	n.a.	*[.337]*		**P-Value from +1**	n.a.	*[.003]* ✞	
€ Germany/EU			326	**SK Sweden**			191
Coeff	-0.001	-0.173	0.0001	**Coeff**	-0.0011	-1.346	0.0035
P-Value from 0	*[.434]*	*[.868]*		**P-Value from 0**	*[.663]*	*[.381]*	
P-Value from +1	n.a.	*[.261]*		**P-Value from +1**	n.a.	*[.127]*	
Lira Italy			155	**Sf Switzerland**			326
Coeff	-0.0057	1.654	0.0142	**Coeff**	-0.004	-1.242	0.005
P-Value from 0	*[.290]*	*[.338]*		**P-Value from 0**	*[.116]*	*[.264]*	
P-Value from +1	n.a.	*[.705]*		**P-Value from +1**	n.a.	*[.044]* ✞	
¥ Japan			326	**£ U.K.**			326
Coeff	-0.005	-1.572	0.008	**Coeff**	0.000	-0.126	0.0001
P-Value from 0	*[.025]* ✞	*[.102]*		**P-Value from 0**	*[.997]*	*[.916]*	
P-Value from +1	n.a.	*[.008]* ✞		**P-Value from +1**	n.a.	*[.348]*	

Notes:

1. The results are from regressions, $s_t - s_{t-1} = \beta_0 + \beta_1\left({}_{t-1}i_{f,t} - {}_{t-1}i_{h,t}\right) + \varepsilon_t$, where s is defined as fc/$. The traditional UIRP formulation predicts that $\beta_0 = 0, \quad \beta_1 = +1.0$.
2. *P*-values are given both relative to zero and to 1.0 in brackets for β_1.
3. ✞ is for significance at the 5% confidence level.
4. * is for significance at the 10% confidence level.
5. The 4^{th} and 8^{th} columns show the number of observations in the regression (NOBS) and immediately below the R^2 of the regression.

TABLE 2

Average Local Currency Yields and Carry Trade Returns[1]

	Local Yield	$-Equivalent Yield		Local Yield	$-Equivalent Yield		Local Yield	$- Equivalent Yield
1986-2013			**1997-2013**			**1986-1999**		
£	**6.24%**	**5.12%**	**NZ$**	**5.50%**	**2.55%**	**Lira**	**10.19%**	**8.47%**
DK	**5.27%**	**7.41%**	**A$**	**5.07%**	**7.75%**	**£**	**9.13%**	**11.07%**
C$	**4.97%**	**6.49%**	**NK**	**4.10%**	**3.86%**	**DK**	**8.09%**	**11.27%**
$	***4.25%***	***4.25%***	**£**	**4.00%**	**3.93%**	**FF**	**7.41%**	**10.74%**
€	**4.00%**	**6.27%**	**C$**	**3.00%**	**5.28%**	**C$**	**7.33%**	**6.79%**
SP$	**2.92%**	**7.03%**	***$***	***2.94%***	***2.94%***	***$***	***6.03%***	***6.03%***
SF	**2.76%**	**5.01%**	**SK**	**2.86%**	**4.71%**	**G**	**5.82%**	**9.09%**
¥	**1.81%**	**6.36%**	**DK**	**2.82%**	**7.22%**	**DM**	**5.66%**	**10.14%**
			€	**2.58%**	**3.95%**	**SP$**	**4.92%**	**6.89%**
			SP$	**1.73%**	**2.92%**	**SF**	**4.57%**	**8.40%**
			SF	**1.14%**	**4.40%**	**¥**	**3.57%**	**9.27%**
			¥	**0.24%**	**5.99%**			
Avg Std	**2.9%**	**30.0%**	**Avg Std**	**1.5%**	**32.7%**	**Avg Std**	**2.6%**	**32.9%**
Hi-Lo	**4.43%**	**3.16%**	**Hi-Lo**	**5.27%**	**5.19%**	**Hi-Lo**	**6.61%**	**5.24%**
T-Stat	***17.7***	***1.1***	***T-Stat***	***36.1***	***1.4***	***T-Stat***	***22.4***	***1.1***
Correlation	**0.98**		**Correlation**	**0.96**		**Correlation**	**0.96**	
T-Stat	***2.79***		***T-Stat***	***3.34***		***T-Stat***	***3.20***	
Spearman	**0.17**		**Spearman**	**0.40**		**Spearman**	**0.28**	
T-Stat	***0.42***		***T-Stat***	***1.50***		***T-Stat***	***0.90***	

Notes:

1. There are two distinct but barely overlapping subperiods in the data: the 1986-1999 period, after which France, Italy, and the Netherlands adopted the Euro, and the 1997-2013 period, for which data for Australia, New Zealand, Norway, and Sweden are available. Data are available for the whole period for Canada, Denmark, Germany/Euro, Japan, Singapore (starts in 1988), Switzerland, the U.K. and the U.S.
2. There are three sets of results (three column each), the first three columns are for the currencies with data for the whole period, and the other two sets of three column are for the two subperiods.
3. For each sample period, the table shows the currency, the average local yield, and the average $-Equivalent yield for that sample. The data are sorted by the local yields.
4. $-Equivalent returns are calculated by assuming a U.S. resident buys the foreign currency, earns the foreign yield, and converts the total value back to $s at the then-prevailing FX rate; no bid-ask spread is "charged" for these notional transactions.
5. The bottom panels of the table display statistics for (i) average standard deviation of the relevant yield (Avg Std), (ii) the difference between the highest and lowest return (Hi-Lo), (iii) the *T*-Statistic for Hi-Lo (assuming independence), (iv) the simple correlation between Local and $-Equivalent yields (Correlation) and its *T*-Statistic below, and (v) the Spearman Rank Correlation coefficients (Spearman) and its *T*-Statistic below.

TABLE 3
Relevant Statistics of Actual and Expected Inflation Measures and Real Yields

Annualized %

1 Currencies	2 CPI Inflation	3 AR Expected Inflation from CPIs	4 IIB Implied Expected Inflation[1]	5 CPI YoY Inflation	6 AR YoY Expected Inflation	7 Real Yields from YoY Expected Inflation
A$ Australia	2.55%	2.27%	n.a.	2.46%	2.27%	2.46% (*1.43*)
C$ Canada	4.22%	1.25%	n.a.	1.46%	1.39%	2.45% (*2.54*)
DK Denmark	4.58%	2.88%	n.a.	1.35%	0.88%	2.74% (*3.52*)
FF France	2.50%	1.22%	0.67%	1.21%	0.89%	4.97% (*2.18*)
€ Germany/Euro	3.97%	1.70%	0.94%	1.39%	1.10%	2.13% (*2.09*)
Lira Italy	2.62%	1.53%	n.a.	1.40%	1.43%	5.40% (*1.65*)
¥ Japan	4.51%	2.79%	0.22%	1.31%	1.22%	1.26% (*1.67*)
G Netherlands	4.45%	3.50%	n.a.	1.25%	1.09%	3.91% (*2.17*)
NZ$ New Zealand	3.81%	3.25%	n.a.	3.68%	3.36%	3.26% (*2.18*)
NK Norway	5.41%	2.63%	n.a.	2.05%	1.80%	2.08% (*1.99*)
SP$ Singapore	5.08%	2.52%	n.a.	2.04%	1.70%	0.91% (*2.10*)
SK Sweden	6.19%	3.24%	n.a.	2.70%	2.61%	1.54% (*1.39*)
SF Switzerland	4.37%	2.87%	n.a.	1.67%	1.60%	1.22% (*1.51*)
£ U.K.	5.09%	3.81%	1.04%	1.75%	1.66%	3.21% (*3.13*)
$ U.S.	3.93%	2.03%	0.31%[2]	1.53%	1.26%	1.32% (*2.18*)
U.S. SPF	- - -	- - -	0.65%[3]		- - -	- - -
Mean	**4.22%**	**2.50%**	**0.64%**	**1.82%**	**1.62%**	**2.6% (*2.2*)**
Max	**6.19%**	**3.81%**	**1.04%**	**3.68%**	**3.36%**	**5.4% (*3.5*)**
Min	**2.50%**	**1.22%**	**0.22%**	**1.21%**	**0.88%**	**0.9% (*1.4*)**

Notes:
Column 2 shows the standard deviations for monthly inflation measured by the CPIs. Column 3 shows the standard deviations of "expected" inflation, derived from the best-fitting AR model from combinations of AR specifications up to AR12, for each currency. Column 4 shows the standard deviations of implied expected inflation from IIB data, where available, as well as that of the SPF forecast for the U.S. Column 5 shows the standard deviations of the "year-over-year" (YoY) version of inflation. Column 6 shows the standard deviations of "expected" inflation, using the same procedure as for column 3, applied to the YoY inflation measure. The "year-over-year" (YoY) inflation measure is a 12-month moving average of the monthly CPI inflation rates. Column 7 shows the means and standard deviations (in parentheses) of the real yields computed using the AR YoY CPI expected inflation rates.

[1] IIB stands for Inflation-Indexed Bonds; TIPS is the U.S. version. These data are not available for most of the countries in the sample. The nominal yields the IIB-derived real yields are compared to are not from exactly similar bonds. At best, these data ought to be thought of as indicators of expected inflation volatility.

[2] These data are from the five-year maturity TIPS yields in the St. Louis Fed's database.

[3] These data are from the Survey of Professional Forecasters of the Philadelphia Fed. The data are quarterly and they are one-year inflation expectations.

TABLE 4

Reversion to the Mean of *FX* Rates[1]

Results from Simulated 10,000 Observations

Foreign Currency	r_f high[2] Deprec	r_f high[2] Apprec	r_f low[2] Deprec	r_f low[2] Apprec	r_h high Deprec	r_h high Apprec	r_h low Deprec	r_h low Apprec	ρ_f / ρ_h
Prediction	***> 50%***	***< 50%***	***< 50%***	***> 50%***	***< 50%***	***> 50%***	***> 50%***	***< 50%***	
Probability	**53.7%**	**46.3%**	**48.0%**	**52.0%**	**46.1%**	**53.9%**	**53.0%**	**47.0%**	0.99
95%	*54.1%*	*46.8%*	*48.5%*	*52.5%*	*46.6%*	*54.4%*	*53.5%*	*47.4%*	0.99
Interval	*53.2%*	*45.9%*	*47.5%*	*51.5%*	*45.6%*	*53.4%*	*52.6%*	*46.5%*	
Probability	**59.2%**	**40.8%**	**41.1%**	**58.9%**	**40.5%**	**59.5%**	**58.5%**	**41.5%**	0.90
95%	*59.7%*	*41.2%*	*41.5%*	*59.4%*	*41.0%*	*59.9%*	*59.0%*	*42.0%*	0.90
Interval	*58.8%*	*40.3%*	*40.6%*	*58.5%*	*40.1%*	*59.0%*	*58.0%*	*41.0%*	
Probability	**61.9%**	**38.1%**	**37.3%**	**62.7%**	**37.4%**	**62.6%**	**61.9%**	**38.1%**	0.80
95%	*62.4%*	*38.6%*	*37.8%*	*63.2%*	*37.9%*	*63.1%*	*62.4%*	*38.6%*	0.80
Interval	*61.4%*	*37.6%*	*36.8%*	*62.2%*	*36.9%*	*62.1%*	*61.4%*	*37.6%*	

Notes:

[1] The table displays the probability of a depreciation or appreciation of the foreign currency (row labeled Probability) when one of the four conditions listed in the top row occur. The 95% interval for that probability is shown in the two consecutive rows that follow.

[2] High and low are defined so that approximately the largest 10% of the yields are "high" and the smallest 10% are "low," separately for both r_f and r_h.

TABLE 5
Carry Trade Simulation Results[1]

1	2	3	4	5
Trial[2]	**Ordinary Correlation**	**Spearman Correlation**	**$-Equivalent Hi-Lo[3]**	**T-Stats[4]**
1	86.8%	81.5%	5.16%	1.74
2	75.3%	76.8%	6.75%	2.14
3	60.6%	30.6%	6.16%	2.07
4	27.5%	22.1%	3.24%	0.96
5	76.0%	71.8%	6.75%	2.11
6	89.3%	85.6%	6.64%	2.05
7	84.0%	81.2%	4.40%	1.37
8	85.9%	80.9%	7.12%	2.14
9	82.5%	86.5%	4.40%	1.32
10	83.8%	82.4%	6.91%	3.44
Mean of Simulations[5]	**75%**	**70%**	**5.8%**	**1.93**
Data[6]	**97%**	**28%**	**6.0%**	**0.94**

Notes:
[1] The table shows results from 10 trials in which data for 15 currencies plus a home currency are simulated each time for 325 observations, as before.
[2] For each trial the table shows the ordinary and Spearman Correlation coefficients between the local currency and the $-equivalent returns. According to the traditional UIRP formulation this correlation ought to be 0.0.
[3] Column 4 shows the difference between the $-equivalent highest and lowest return for each trial.
[4] The *T*-Statistics are for the Hi-Lo values.
[5] Average values across the ten trials.
[6] The corresponding statistics for the 14 currencies in the data.

TABLE 6

Results From the New Uncovered Interest Rate Parity Tests From Joint GMM Regressions

		fc	*US$*	**RSQ**			*fc*	*US$*	**RSQ**
	β_0	β_f	β_h	**NOBS**		β_0	β_f	β_h	**NOBS**
A$ Australia					**G Netherlands**				
Coeff	0.52	-89.4✞	95.1✞	0.46	**Coeff**	0.51	6.1	16.4*	0.09
StdError	0.041	15.481	18.532	183	**StdError**	0.032	6.8	9.5	141
P-Value	*[.000]*	*[.000]*	*[.000]*		**P-Value**	*[.000]*	*[.372]*	*[.084]*	
C$ Canada					**NZ$ N. Zealand**				
Coeff	0.22	-37.4✞	52.9✞	0.16	**Coeff**	0.31	-7.9	54.7✞	0.22
StdError	0.031	9.554	11.979	310	**StdError**	0.050	14.063	15.831	186
P-Value	*[.000]*	*[.000]*	*[.000]*		**P-Value**	*[.000]*	*[.575]*	*[.001]*	
DK Denmark					**NK Norway**				
Coeff	1.81	-9.7✞	18.5✞	0.06	**Coeff**	1.84	39.9✞	24.5✞	0.34
StdError	0.022	3.643	7.804	311	**StdError**	0.012	8.4	7.6	188
P-Value	*[.000]*	*[.008]*	*[.017]*		**P-Value**	*[.000]*	*[.000]*	*[.001]*	
FF France					**S$ Singapore**				
Coeff	1.57	-4.2	23.5✞	0.15	**Coeff**	0.37	43.6✞	-36.9✞	0.25
StdError	0.036	5.524	8.386	141	**StdError**	0.017	9.142	13.101	298
P-Value	*[.000]*	*[.446]*	*[.005]*		**P-Value**	*[.000]*	*[.000]*	*[.005]*	
€ Germany/EU					**SK Sweden**				
Coeff	*-0.23*	*-2.7*	*-2.1*	0.00	**Coeff**	2.01	12.8	9.9	0.04
StdError	*0.017*	*5.621*	*9.508*	311	**StdError**	0.023	20.727	11.164	188
P-Value	*[.000]*	*[.635]*	*[.823]*		**P-Value**	*[.000]*	*[.536]*	*[.375]*	
Lira Italy					**SF Switzerland**				
Coeff	7.48	-37.0✞	3.6	0.24	**Coeff**	0.18	-14.1	28.7✞	0.10
StdError	0.045	5.661	11.173	141	**StdError**	0.022	10.960	11.568	311
P-Value	*[.000]*	*[.000]*	*[.744]*		**P-Value**	*[.000]*	*[.197]*	*[.013]*	
¥ Japan					**£ U.K.**				
Coeff	4.61	-30.5✞	-10.1	0.07	**Coeff**	-0.52	-18.9✞	10.6	0.15
StdError	0.024	9.946	7.857	310	**StdError**	0.012	5.5	8.1	311
P-Value	*[.000]*	*[.002]*	*[.198]*		**P-Value**	*[.000]*	*[.001]*	*[.193]*	

Notes:

1. For each currency a system of three equations is estimated: the *RFX* equation $rfx_t = \beta_0 + \beta_{f\ t} r_{f,t+1} + \beta_{h\ t} r_{h,t+1} + \varepsilon_t$ and two autoregressive equations for the real yields, one for each currency; $r_{j,t} = \gamma_{j,0} + \rho_{j1} r_{j,t-1} + \rho_{j2} r_{j,t-2} + \rho_{j3} r_{f,t-3}$, for *j* = *f*, *h*. The results in the table are for the *RFX* regressions only, where *rfx* is the real *FX*, defined from *s* in fc/$. The model predictions are: $\beta_f < -1.0$, $\beta_h > 1.0$, and no prediction for β_0.
2. The real yields are constructed from YoY inflation forecasts.
3. The GMM regressions use the lag values of the real yields and a constant as instruments, the Newey-West autocorrelation correction is for 12 lags.
4. *P*-values are relative to zero.
5. ✞ signifies statistical significance at the 5% level or less, and * at the 10% level. Shaded cells indicate that the coefficient sign is contrary to that predicted by the model.

TABLE 7

Empirical Evidence for Reversion to the Mean of *FX* Rates
Results from 14 Currencies and the US$

	r_f *high*		r_f *low*		r_{US} *high*		r_{US} *low*		**NOBS**
Foreign Currency	**Deprec**	**Apprec**	**Deprec**	**Apprec**	**Deprec**	**Apprec**	**Deprec**	**Apprec**	
Prediction	*> 50%*	*< 50%*	*< 50%*	*> 50%*	*< 50%*	*> 50%*	*> 50%*	*< 50%*	
A$ Australia	**57.9%†**	**42.1%†**	**22.2%†**	**77.8%†**	**64.7%†**	**35.3%†**	**50.00%**	**50.00%**	**186**
95%	*61%*	*46%*	*25%*	*81%*	*68%*	*39%*	*54%*	*54%*	
Interval	*54%*	*39%*	*19%*	*75%*	*61%*	*32%*	*46%*	*46%*	
C$ Canada	**41.9%†**	**58.1%†**	**46.4%†**	**53.6%†**	**41.5%†**	**58.5%†**	**42.5%†**	**57.5%†**	**312**
95%	*45%*	*61%*	*49%*	*56%*	*44%*	*61%*	*45%*	*60%*	
Interval	*39%*	*55%*	*44%*	*51%*	*39%*	*56%*	*40%*	*55%*	
DK Denmark	**29.3%†**	**70.7%†**	**48.4%**	**51.6%**	**27.9%†**	**72.1%†**	**57.1%†**	**42.9%†**	**312**
95%	*32%*	*73%*	*51%*	*54%*	*30%*	*75%*	*60%*	*46%*	
Interval	*27%*	*68%*	*46%*	*49%*	*25%*	*70%*	*54%*	*40%*	
FF France	**32.0%†**	**68.0%†**	**50.00%**	**50.00%**	**36.0%†**	**64.0%†**	**60.0%†**	**40.0%†**	**143**
95%	*36%*	*72%*	*54%*	*54%*	*40%*	*68%*	*64%*	*44%*	
Interval	*28%*	*64%*	*46%*	*46%*	*32%*	*60%*	*56%*	*36%*	
€ DM/Euro	**42.2%†**	**57.8%†**	**45.2%†**	**54.8%†**	**26.8%†**	**73.2%†**	**47.5%**	**52.5%**	**313**
95%	*45%*	*60%*	*48%*	*58%*	*29%*	*76%*	*50.3%*	*55%*	
Interval	*40%*	*55%*	*42%*	*52%*	*24%*	*71%*	*45%*	*49.7%*	
Lira Italy	**40.0%†**	**60.0%†**	**35.7%†**	**64.3%†**	**36.0%†**	**64.0%†**	**60.0%†**	**40.0%†**	**143**
95%	*44%*	*64%*	*40%*	*68%*	*40%*	*68%*	*64%*	*44%*	
Interval	*36%*	*56%*	*32%*	*60%*	*32%*	*60%*	*56%*	*36%*	
¥ Japan	**44.4%†**	**55.6%†**	**47.2%†**	**52.8%**	**48.7%**	**51.3%**	**50.00%**	**50.00%**	**312**
95%	*47%*	*58%*	*49.98%*	*56%*	*51%*	*54%*	*53%*	*53%*	
Interval	*42%*	*53%*	*44%*	*50.02%*	*46%*	*49%*	*47%*	*47%*	
G Netherlands	**38.5%†**	**61.5%†**	**64.3%†**	**35.7%†**	**33.3%†**	**66.7%†**	**60.0%†**	**40.0%†**	**143**
95%	*42%*	*66%*	*68%*	*40%*	*37%*	*71%*	*64%*	*44%*	
Interval	*34%*	*58%*	*60%*	*32%*	*29%*	*63%*	*56%*	*36%*	
NZ$ New Zealand	**60.0%†**	**40.0%†**	**20.0%†**	**80.0%†**	**68.4%†**	**31.6%†**	**42.1%†**	**57.9%†**	**188**
95%	*63%*	*43%*	*23%*	*83%*	*72%*	*35%*	*46%*	*61%*	
Interval	*57%*	*37%*	*17%*	*77%*	*65%*	*28%*	*39%*	*54%*	
NK Norway	**52.6%**	**47.4%**	**41.7%†**	**58.3%†**	**63.2%†**	**36.8%†**	**52.6%**	**47.4%**	**189**
95%	*56%*	*51%*	*45%*	*62%*	*67%*	*40%*	*56%*	*51%*	
Interval	*49%*	*44%*	*38%*	*55%*	*60%*	*33%*	*49%*	*44%*	
SP$ Singapore	**31.3%†**	**68.8%†**	**37.1%†**	**62.9%†**	**33.3%†**	**66.7%†**	**38.2%†**	**61.8%†**	**300**
95%	*34%*	*71%*	*40%*	*66%*	*36%*	*69%*	*41%*	*65%*	
Interval	*29%*	*66%*	*34%*	*60%*	*31%*	*64%*	*35%*	*59%*	
SK Sweden	**70.0%†**	**30.0%†**	**42.9%†**	**57.1%†**	**72.2%†**	**27.8%†**	**70.0%†**	**30.0%†**	**190**
95%	*73%*	*33%*	*46%*	*61%*	*75%*	*31%*	*73%*	*33%*	
Interval	*67%*	*27%*	*39%*	*54%*	*69%*	*25%*	*67%*	*27%*	
SF Switzerland	**25.8%†**	**74.2%†**	**41.4%†**	**58.6%†**	**34.4%†**	**65.6%†**	**42.5%†**	**57.5%†**	**312**
95%	*28%*	*77%*	*44%*	*61%*	*37%*	*68%*	*45%*	*60%*	
Interval	*23%*	*72%*	*39%*	*56%*	*32%*	*63%*	*40%*	*55%*	
£ U.K.	**51.4%**	**48.6%**	**50.00%**	**50.00%**	**48.6%**	**51.4%**	**60.0%†**	**40.0%†**	**312**
95%	*54%*	*51%*	*53%*	*53%*	*51%*	*54%*	*637%*	*43%*	
Interval	*49%*	*46%*	*47%*	*47%*	*46%*	*49%*	*57%*	*37%*	

Notes:

1. The table shows the empirical probability that the FX rate will depreciate or appreciate when r_f or r_h are "high" or "low." The table's first row describes the condition of the yields (high or low), the second row shows whether FX depreciation or appreciation is measured, while the third row lists the predictions of the model. The home currency is the US$.
2. To compute the transition probabilities, I select roughly the highest and lowest 10% of real yields for each currency along with the complementary data from the U.S. Then I record the subsequent movement of the FX rate as "Up" –depreciation– and "Down" –appreciation. The average proportion of observations selected are: 10.4% for r_f Hi, 10.6% for r_f Lo, 9.9% for r_{US} Hi, and 11.1% for r_{US} Lo. These numbers are not exactly 10% because of the lumpiness of the data (several yields are nearly identical). The lowest proportion of observations selected for any of the currencies is 9.0% and the highest is 13.2%
3. The subsequent rows show the results for each currency. The first row for each curency shows the probability, while the second and third rows show its 5% confidence interval.
4. "✞" denotes that the probability is significantly different from 50%, at the 5% confidence interval. Shaded cells indicate that the empirical probability is contrary to the model's prediction.

APPENDIX

A. *The Full Derivation of Equation (5b)*

The derivation is done for one real yield because the two yield processes are fully symmetric; also for simplicity, ${}_n r_{n+1} \equiv r_n$. Expand equation (2) from the text:

$$rfx_0 - rfx_{eq,0}^{e,0} = r_0 + r_1^{e,0} + r_2^{e,0} + r_3^{e,0} \ldots + r_j^{e,0} \ldots + r_n^{e,0} . \qquad \text{(A1)}$$

Substitute the forecasts for *r*, made at date=0.

$$\begin{aligned} rfx_0 - rfx_{eq,0}^{e,0} &= r_0 + (r_c + \rho r_0) + (r_c(1+\rho) + \rho^2 r_0) + (r_c(1+\rho+\rho^2) + \rho^3 r_0) \\ &\quad + (r_c(1+\rho+\rho^2+\rho^3) + \rho^4 r_0) \ldots + r_j \ldots + (r_c(1+\rho+\rho^2+\rho^3+\ldots+\rho^{n-1}) + \rho^n r_0) \end{aligned} \qquad \text{(A2)}$$

There are two components to this sum. The first is:

$$r_0(1+\rho+\rho^2+\ldots+\rho^n) = r_0\left(\frac{1-\rho^n}{1-\rho}\right). \qquad \text{(A3a)}$$

The second component is:

$$r_c\left[1+(1+\rho)+(1+\rho+\rho^2)+(1+\rho+\rho^2+\rho^3)+\ldots+(1+\rho+\rho^2+\rho^3+\ldots+\rho^{n-1})\right]. \qquad \text{(A3b)}$$

This second expression can be rewritten as:

$$\begin{aligned} &= r_c\left[n+(n-1)\rho+(n-2)\rho^2+(n-3)\rho^3+(n-4)\rho^4+\ldots+(n-j)\rho^j+\ldots+\rho^{n-1}\right] \\ &= r_{eq,0}^{e}(1-\rho)\left[n+(n-1)\rho+(n-2)\rho^2+(n-3)\rho^3+(n-4)\rho^4+\ldots+(n-j)\rho^j+\ldots+\rho^{n-1}\right], \end{aligned}$$

Or more compactly as:

$$\begin{aligned} r_{eq,0}^{e,0}(1-\rho)\sum_{j=0}^{n-1}(n-j)\rho^j &= r_{eq,0}^{e,0}(1-\rho)\left(n\sum_{j=0}^{n-1}\rho^j - \sum_{j=0}^{n-1} j\rho^j\right), \\ &= r_{eq,0}^{e,0}(1-\rho)\left\{n\left(\frac{1-\rho^n}{1-\rho}\right) - \sum_{j=0}^{n-1} j\rho^j\right\}, \end{aligned} \qquad \text{(A4)}$$

Consider the second term inside the brackets:

$$\sum_{j=0}^{n-1} j\rho^{j} = \rho + 2\rho^{2} + 3\rho^{3} + 4\rho^{4} + \ldots + (n-1)\rho^{n-1} = S_{n}. \tag{A5}$$

Consider now ρS_n.

$$\rho S_{n} = \rho^{2} + 2\rho^{3} + 3\rho^{4} + 4\rho^{5} + \ldots + (n-1)\rho^{n}. \tag{A6}$$

Subtract to get:

$$\begin{aligned} S_{n} - \rho S_{n} &= \rho + \rho^{2} + \rho^{3} + \rho^{4} + \rho^{5} + \ldots + \rho^{n-1} - (n-1)\rho^{n}, \\ S_{n}(1-\rho) &= \rho\left(1 + \rho + \rho^{2} + \rho^{3} + \rho^{4} + \ldots + \rho^{n-2}\right) - (n-1)\rho^{n}, \end{aligned} \tag{A7}$$

$$\begin{aligned} S_{n}(1-\rho) &= \rho\frac{1-\rho^{n-1}}{1-\rho} - (n-1)\rho^{n}, \\ S_{n} &= \rho\frac{1-\rho^{n-1}}{(1-\rho)^{2}} - \frac{(n-1)\rho^{n}}{1-\rho}. \end{aligned} \tag{A8}$$

Substitute the value of the sum back into (A4),

$$\begin{aligned} & r_{eq,0}^{e,0}(1-\rho)\sum_{j=0}^{n}(n-j)\rho^{j} \\ &= r_{eq,0}^{e,0}(1-\rho)\left\{ n\left(\frac{1-\rho^{n+1}}{1-\rho}\right) - \rho\frac{1-\rho^{n}}{(1-\rho)^{2}} + \frac{n\rho^{n+1}}{1-\rho} \right\} \\ &= r_{eq,0}^{e,0}\left\{ n\left(1-\rho^{n+1}\right) - \rho\frac{1-\rho^{n}}{(1-\rho)} + n\rho^{n+1} \right\}. \end{aligned} \tag{A9}$$

Collect terms,

$$\begin{aligned} rfx_{t} - rfx_{t+n}^{e,t} =& \left(\frac{1-\rho_{h}^{n}}{1-\rho_{h}}\right)_{t} r_{h,t+1} - \left(\frac{1-\rho_{f}^{n}}{1-\rho_{f}}\right)_{t} r_{f,t+1} \\ & + r_{eq,t}^{e,t}\left\{ n\left(1-\rho_{h}^{n+1}\right) - \rho_{h}\frac{1-\rho_{n}^{n}}{1-\rho_{h}} + n\rho_{h}^{n+1} - n\left(1-\rho_{f}^{n+1}\right) - \rho_{f}\frac{1-\rho_{f}^{n}}{1-\rho_{f}} - n\rho_{f}^{n+1} \right\} \end{aligned} \tag{A10}$$

and simplify,

$$rfx_t - rfx_{t+n}^{e,t} = \left(\frac{1-\rho_h^n}{1-\rho_h}\right)_t r_{h,t+1} - \left(\frac{1-\rho_f^n}{1-\rho_f}\right)_t r_{f,t+1}$$
$$+ r_{eq,t}^{e,t}\left\{\left(\rho_f^{n+1} - \rho_h^{n+1}\right) + \frac{\rho_f\left(1-\rho_f^n\right)}{1-\rho_f} - \frac{\rho_h\left(1-\rho_n^n\right)}{1-\rho_h} + n\left(\rho_h^{n+1} - \rho_f^{n+1}\right)\right\} \quad \text{(A11)}$$

Let $n \to \infty$. Then the expression simplifies to (ρ^n becomes small faster than n grows),

$$r_{eq,0}^{e,0}(1-\rho)\sum_{j=0}^{n}(n-j)\rho^j = r_{eq,0}^{e,0}\left\{n - \frac{\rho}{(1-\rho)}\right\}. \quad \text{(A12)}$$

Next, bring together the *home* and *foreign* components to get,

$$r_{eq,0}^{e,0}\left\{n - \frac{\rho_h}{(1-\rho_h)}\right\} - r_{eq,0}^{e,0}\left\{n - \frac{\rho_f}{(1-\rho_f)}\right\}$$
$$= r_{eq,0}^{e,0}\left\{\frac{\rho_f}{(1-\rho_f)} - \frac{\rho_h}{(1-\rho_h)}\right\}, \quad \text{(A13)}$$

because the $n * r_{eq,0}^{e,0}$ terms cancel out.[45] The full expression then is:

$$rfx_t = \left(\frac{1}{1-\rho_h}\right)_t r_{h,t+1} - \left(\frac{1}{1-\rho_f}\right)_t r_{f,t+1} + \left\{\frac{\rho_f}{(1-\rho_f)} - \frac{\rho_h}{(1-\rho_h)}\right\} r_{eq,t}^{e,t} + rfx_{eq,t}^{e,t}. \quad \text{(A14)}$$

The term, $\left\{\frac{\rho_f}{(1-\rho_f)} - \frac{\rho_h}{(1-\rho_h)}\right\} r_{eq,t}^{e,t}$ "evens out" the contributions of the two yields if the ρs are not equal. Say $\rho_h > \rho_f$. Then $\frac{1}{1-\rho_h} > \frac{1}{1-\rho_f}$. The effect of the term in (A11) is to make the total contribution of ρ_f be the same as that of ρ_h. This means that the total contribution to the change in *rfx* is governed by the larger ρ.

[45] This also shows that if the two real yields do not converge to a common value in probability, the FX rate is undefined in this model.

Proof:

$$
\begin{aligned}
&-\frac{1}{\left(1-\rho_f\right)}+\frac{\rho_f}{\left(1-\rho_f\right)}-\frac{\rho_h}{\left(1-\rho_h\right)} \\
&=-\frac{\rho_h}{\left(1-\rho_h\right)}-\left\{\frac{1-\rho_f}{1-\rho_f}\right\}=-\left\{1+\frac{\rho_h}{\left(1-\rho_h\right)}\right\} \qquad \text{(A15)} \\
&=-\frac{1}{\left(1-\rho_h\right)}
\end{aligned}
$$

B. *The Expected FX Rate and the Forward Rate*

The forward rate is given by ${}_t f_{t+1} = s_t + {}_t i_{f,t+1} - {}_t i_{h,t+1}$ or, ${}_t f_{t+1} = s_t + {}_t r^{e,t}_{f,t+1} - {}_t r^{e,t}_{h,t+1} + \left(\pi^{e,t}_{f,t+1} - \pi^{e,t}_{h,t+1}\right)$. The yields are quoted at the same time as the forward rate, time = *t*.

In the model, the current real yield determines the current spot, so

$$
s_t = \Phi_h \ {}_t r_{h,t+1} - \Phi_f \ {}_t r_{f,t+1} + \Theta r^{e,t}_{eq,t} + rfx^{e,t}_{eq,t} - \left(p_{h,t} - p_{f,t}\right), \qquad \text{(A16)}
$$

where $\Phi_j \equiv \dfrac{1}{1-\rho_j}$, $\Theta = \left\{\dfrac{\rho}{1-\rho_f} - \dfrac{\rho}{1-\rho_h}\right\}$.

The expected *FX* rate at time *t* for time = *t*+1 is:

$$
s^{e,t}_{t+1} = \Phi_h \ {}_{t+1} r^{e,t}_{h,t+2} - \Phi_f \ {}_{t+1} r^{e,t}_{f,t+2} + \Theta r^{e,t}_{eq,t+1} + rfx^{e,t}_{eq,t+1} + \left(p^{e,t}_{f,t+1} - p^{e,t}_{h,t+1}\right). \qquad \text{(A17)}
$$

Rewrite in term of the current *FX* rate as:

$$
\begin{aligned}
s^{e,t}_{t+1} = s_t &+ \Phi_h\left({}_{t+1} r^{e}_{h,t+2} - {}_t r_{h,t+1}\right) - \Phi_f\left({}_{t+1} r^{e}_{f,t+2} - {}_t r_{f,t+1}\right) \\
&+ \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right) + \left(\pi^{e,t}_{f,t+1} - \pi^{e,t}_{h,t+1}\right).
\end{aligned} \qquad \text{(A18)}
$$

The expected *FX* rate can be also written in terms of the processes of the expected real yields. The real yield forecasts are, ${}_{t+1} r^{e,t}_{t+2} = r_{eq}\left(1-\rho\right) + \rho \ {}_t r_{t+1}$, from equation (3a). Then from equation (A18),

$$s^{e,t}_{t+1} = s_t + \Phi_h(1-\rho_h)\left(r^{e,t}_{eq,t+1} - {}_t r_{f,t+1}\right) - \Phi_f(1-\rho_f)\left(r^{e,t}_{eq,t+1} - {}_t r_{h,t+1}\right)$$
$$+ \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right) + \left(\pi^{e,t}_{f,t+1} - \pi^{e,t}_{h,t+1}\right). \quad \text{(A19)}$$

$$s^{e,t}_{t+1} = s_t - {}_t r_{h,t+1} + {}_t r_{f,t+1} + \Theta\left(r^{e,t}_{eq,t+1} - r^{e,t}_{eq,t}\right) + \left(rfx^{e,t}_{eq,t+1} - rfx^{e,t}_{eq,t}\right) + \left(\pi^{e,t}_{f,t+1} - \pi^{e,t}_{h,t+1}\right). \quad \text{(A20)}$$

C. *The Difference Between the realized FX rate and the Forward Rate That Predicts It*

$$s_{t+1} - s_t = \Phi_h\left({}_{t+1} r_{h,t+2} - {}_t r_{h,t+1}\right) - \Phi_f\left({}_{t+1} r_{f,t+2} - {}_t r_{f,t+1}\right)$$
$$+ \Theta\left(r^{e,t+1}_{eq,t+1} - r^{e}_{eq,t}\right) + \left(rfx^{e,t+1}_{eq,t+1} - rfx^{e}_{eq,t}\right) + {}_t\pi_{f,t+1} - {}_t\pi_{h,t+1}\,, \quad \text{(A21)}$$

Also, ${}_t f_{t+1} - s_t = {}_t i_{f,t+1} - {}_t i_{h,t+1} \equiv {}_t r_{f,t+1} - {}_t r_{h,t+1} + {}_t\pi^{e,t}_{f,t+1} - {}_t\pi^{e,t}_{h,t+1}$. (A22)

Recall the notation-simplifying assumption that there exists a market real yield.

$$s_{t+1} - {}_t f_{t+1} = \Phi_h\left({}_{t+1} r_{h,t+2} - {}_t r_{h,t+1}\right) - \Phi_f\left({}_{t+1} r_{f,t+2} - {}_t r_{f,t+1}\right)$$
$$+ \Theta\left(r^{e,t+1}_{eq,t+1} - r^{e}_{eq,t}\right) + \left(rfx^{e,t+1}_{eq,t+1} - rfx^{e}_{eq,t}\right) + {}_t\pi_{f,t+1} - {}_t\pi_{h,t+1}\,, \quad \text{(A23)}$$
$$- \left({}_t r_{f,t+1} - {}_t r_{h,t+1} + {}_t\pi^{e,t}_{f,t+1} - {}_t\pi^{e,t}_{h,t+1}\right)$$

$$s_{t+1} - {}_t f_{t+1} = \Phi_h\left({}_{t+1} r_{h,t+2}\right) - \left(\Phi_h - 1\right){}_t r_{h,t+1} - \Phi_f\left({}_{t+1} r_{f,t+2}\right) - \left(\Phi_f - 1\right){}_t r_{f,t+1}$$
$$+ \Theta\left(r^{e,t+1}_{eq,t+1} - r^{e}_{eq,t}\right) + \left(rfx^{e,t+1}_{eq,t+1} - rfx^{e}_{eq,t}\right) + \left({}_t\pi_{f,t+1} - {}_t\pi^{e,t}_{f,t+1}\right) - \left({}_t\pi_{h,t+1} - {}_t\pi^{e,t}_{h,t+1}\right), \quad \text{(A24)}$$