A RE-EXAMINATION OF THE "VARIANCE BOUNDS" METHODOLOGY; A GENERAL EQUILIBRIUM SURPRISE

by

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ABSTRACT

Tests of “variance bounds” have usually been derived in the context of a model with a single representative firm. In the equilibrium framework of this paper we show that when the economy has multiple firms, the results of the classic variance bounds literature do not have any theoretical validity. The price volatility of a firm does not depend on the volatility of the firm’s dividends. As a consequence the “perfect foresight” price paths traditionally calculated in this literature do not contain correct information about the price volatility of the firm.

INTRODUCTION

In this paper we study the general equilibrium relation between the volatility of stock price "fundamentals" and stock prices. There is a large stock price volatility literature, which aims to document the empirical relation between stock prices and dividend volatility, and to provide tests of market efficiency by using the "variance bounds" methodology. A fundamental theoretical assumption in this methodology is that there is a direct relation between the volatilities of stock prices and their fundamentals. We develop a simple theoretical model to show that this assumption is incorrect. We also show that this result invalidates the inequality restrictions derived in this literature.

The "variance bounds" methodology develops tests of market efficiency that compare variances of market prices with variances of dividends and other stock "fundamentals."[1,2] Theoretical bounds on ratios of these variances (or functions of these variances) are derived by invoking market efficiency and rational expectations. The basic idea is that since the stock price is the present value of future dividends, the variance of stock prices must be related to the variance of the underlying dividends.[3]

In the classical variance bounds tests, the observed *ex-post* dividend path, starting at some past date *t-n* for a stock (or the market portfolio), is discounted to give P^*_{t-n}. Then a sequence of

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1 The variance bounds methodology has been applied to stock prices (Grossman and Shiller 1981, Shiller 1981a,b), to the term structure of interest rates (Shiller 1979, Singleton 1980) and to the foreign exchange markets (Evans 1986, Frankel and Meese 1987, Huang 1981).

2 In a recent contribution, Cochrane (1991) challenges the notion that variance bounds tests address new aspects of market efficiency, and he shows that these tests are identical to the traditional market efficiency tests. However, he does not challenge the validity of the variance bounds that are tested in the literature.

3 Bounds are frequently derived for changes or growth rates of prices and dividends. Shiller (1981a,b) and LeRoy and Porter (1981) assume constant discount factors. Grossman and Shiller (1981) allow the discount factors to be time-dependent. More recent work is Campbell and Shiller (1987, 1989), LeRoy and Parke (1988), Poterba and Summers (1986), Summers (1986), and West (1986, 1988). In most instances, the theoretical relations that have been derived appear to be violated decisively by the data.

prices is computed starting at t-n up to a more recent date t-k (n>k): P^*_{t-n}, P^*_{t-n+1} … P^*_{t-k}.

Under the assumption of perfect foresight, the *variance* of P^*_t, ought to be larger than the variance of the observed market price, P_t, in this interval.

Statistical shortcomings of the variance bounds tests have been widely documented in the literature, and the proposed remedies have concentrated on constructing stationary functions of the nonstationary variables involved, with statistical properties that are better suited to time series tests.[4] Other shortcomings of the tests include their vulnerability to dividend-smoothing policies of firms.[5]

In this paper we do not focus on the well-understood statistical inadequacies of the variance bounds tests. Instead, we deliberately choose an equilibrium framework —stationarity in all parameters and quantities— in which none of the real-life statistical or econometric difficulties exist. In this framework we examine a fundamental assumption of the above derivation, namely that the price volatility of a stock is related to its dividend volatility, and that higher dividend volatility will be associated with higher price volatility. In our simple multi-firm stationary economy everyone knows the discount factors exactly. Our basic theoretical result (Theorem 1) is that in equilibrium price volatility does not depend on dividend volatility (i.e., the dispersion or the distribution of dividends), but rather it is governed by the state-dependence of market

[4] Tests on stock returns have been criticized on empirical and theoretical grounds. Kleidon (1986a,b) shows that empirically these tests are flawed if dividends are nonstationary, because it is not possible to estimate the required cross-sectional variance of dividends from time series data. Flavin (1983) and Kleidon (1986a) show that many of these estimates also have serious small sample biases. West (1988) derives a test that is free of such small sample biases and which may be applied to any stationary process. His test also rejects these variance inequalities.

[5] Marsh and Merton (1986) examine the robustness of inequalities based on "perfect foresight" prices constructed from ex-post dividends, $p^*(t)$. They rely on Miller and Modigliani (1961) to propose a dividend-smoothing pattern that violates the Shiller inequalities. They show that certain dividend-smoothing patterns will reverse the variance bounds when dividends are nonstationary.

valuation.[6] In this connection we also show it is *not possible* to construct a rational economy that is stationary, has aggregate uncertainty, and also has state-independent state prices. Yet stationarity of state prices is assumed to derive the variance bounds results.

We derive our results in a representative consumer economy where uncertainty comes from productivity shocks to a nonperishable and non-growing capital stock. We use a Lucas general equilibrium state-preference model (Lucas 1978) of asset valuation, with a representative consumer, complete markets, and fully competitive firms. Consumption, firm values, and earnings are stationary. State prices may be state-dependent but they are stationary.[7]

Our results come from a straightforward observation: The price of a stationary dividend distribution will remain unchanged *regardless of its dispersion* as long as the pricing function is unchanged. State-dependent changes in the pricing function are the critical source of price volatility. The larger the state-dependent variations in the pricing function the more volatile the more volatile stock prices become, again *regardless of the distribution of their dividends*.

In particular, we show examples where —even though there is no dividend volatility— the stock price is volatile. Conversely, wide dispersions of dividends result in zero volatility if the pricing function is state-independent.

In section I we specify the model, derive its relevant equilibrium features, and obtain a valuation result. In section II we study the determinants of the variability of a firm's value, and in section III we show simulations that compare the volatility of P_t^* and P_t. Section IV is the conclusion.

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[6] This result also holds in a nonstationary economy with production and growth (see Benninga and Protopapadakis 1996).

[7] We choose such an economy in order to make our results directly comparable to the assumptions used in the important papers in this literature, such as Shiller, and LeRoy and Porter.

I. MODEL SPECIFICATION

Our model is a simple no-growth model with two future states possible from any date = t, and an infinitely-lived representative consumer. Markets are complete, and there is a single consumption good. There are two possible states of the world, labeled a and b. Transition probabilities depend on the current state of the world, and we assume these to be Markov:

From state	To state a	To state b
a	p_{aa}	p_{ab}
b	p_{ba}	p_{bb}

At each time-state pair, the representative consumer consumes the available consumption good. The consumption good is produced (or available) at each date, but there is no investment and no growth in this economy.

I.a Equilibrium With Two Perfectly Competitive Firms

We assume that there are two independent production technologies, i.e., production technologies are complete.[8] One can think of these technologies as two types of firms that operate in the fully competitive product market. At each date the x-technology (or x-firm) produces x_a output if the a-state obtains and x_b output if the b-state obtains. The z-technology (or z-firm) produces z_a output if the a-state obtains and z_b output if the b-state obtains. Both firms have shares outstanding that represent claims to their output in perpetuity. The consumer maximizes her expected utility:

[8] Production technologies are complete if there are at least as many independent technologies as states. This definition parallels the definition of complete markets. See Benninga and Protopapadakis (1991) for an extensive discussion of the implications of this specification.

(1) $$E\{u(c_n)\} = U(c_n) + d\langle p_{na}U(c_{na}) + p_{nb}U(c_{nb})\rangle + d^2\langle\ \rangle\ldots,$$

where d is the time-preference parameter, subject to,

(2) $c_a = x_a + z_a,$ and $c_b = x_b + z_b, \quad \forall\, t.$

We assume that the utility function is of the form:

(3) $$U(c) = \frac{c^{1-g}}{1-g}$$ for $g > 1$, where g is the relative risk aversion.

The first-order conditions for the consumer are,

(4) $p_{na}d\,\dfrac{U'(c_{na})}{U'(c_n)} = q_{na}$, $p_{nb}d\,\dfrac{U'(c_{nb})}{U'(c_n)} = q_{nb}$, for any state n.

Since consumption can take only one of two values, there are four possible state prices, as shown below.

From state	To state *a*	To state *b*
a	$q_{aa} = p_{aa}d\,\dfrac{U'(c_a)}{U'(c_a)}$	$q_{ab} = p_{ab}d\,\dfrac{U'(c_b)}{U'(c_a)}$
b	$q_{ba} = p_{ba}d\,\dfrac{U'(c_a)}{U'(c_b)}$	$q_{bb} = p_{bb}d\,\dfrac{U'(c_b)}{U'(c_b)}$

Since $\dfrac{U'(c_{nj})}{U'(c_n)} = \left(\dfrac{c_{nj}}{c_n}\right)^{-g}$, the state prices can be written more usefully as:

From state	**To state** *a*	**To state** *b*
a	$q_{aa} = p_{aa}d$	$q_{ab} = p_{ab}d\Phi$
b	$q_{ba} = p_{ba}d\Phi^{-1}$	$q_{bb} = p_{bb}d$

where $\Phi \equiv \left(\frac{c_b}{c_a}\right)^{-g} < 1$.

<u>Lemma 1:</u> In a stationary economy, uncertainty implies that state prices *must be* time-dependent.

The result is clear upon inspection of the above table. Even when probabilities are state-independent and equal ($p_{aa} = p_{ab} = p_{ba} = p_{bb}$), the state prices will be state-independent if and only if $\Phi = 1$. But this implies $c_b = c_a$, i.e., no uncertainty.[9] QED.

Equilibrium state prices make it possible to price the shares of the *x*- and *z*-firms. Consider being in state a at date = t. Since there is no growth over time and the transition probabilities are time-independent, time is irrelevant. The consumer will place the same value on the future output of, say, the *x*-firm in all the a states. The same holds for the b states. This leads to a pair of linear equations for the two values of the *x*-firm. Clearly a similar set of equations apply for the *z*-firm.

(5a) value of the *x*-firm in the a state: $VX_a = q_{ab}\left[VX_b + x_b\right] + q_{aa}\left[VX_a + x_a\right]$,

(5b) value of the *x*-firm in the b state: $VX_b = q_{ba}\left[VX_a + x_a\right] + q_{bb}\left[VX_b + x_b\right]$.

Denoting by Q the matrix of state prices, the analytic solution for this pair of equations is $\begin{pmatrix} VX_a \\ VX_b \end{pmatrix} = (I - Q)^{-1} \cdot Q \cdot \begin{pmatrix} x_a \\ x_b \end{pmatrix}$, where I is the *2 x 2* identity matrix. Since the state-dependence of the probabilities is not important for our main results, we study the solution for the case where the

[9] However, state-independent state prices *are* compatible with uncertainty in growth equilibria.

probabilities are state-independent, i.e., $p_{aa} \equiv p_{ba} = p$, and $p_{ab} \equiv p_{bb} = 1-p$. In this case $Q = \begin{pmatrix} pd & (1-p)d\Phi \\ pd\Phi^{-1} & (1-p)d \end{pmatrix}$, and the solution for the x-firm simplifies to:

(6a) $$VX_a = \left(\frac{d}{(1-d)[1+d(1-2p)]}\right)[px_a + (1-p)\Phi x_b],$$

(6b) $$VX_b = \left(\frac{d}{(1-d)[1+d(1-2p)]}\right)[p\Phi^{-1}x_a + (1-p)x_b].$$

Similarly, the z-firm valuation is:

(7a) $$VZ_a = \left(\frac{d}{(1-d)[1+d(1-2p)]}\right)[pz_a + (1-p)\Phi z_b],$$

(7b) $$VZ_b = \left(\frac{d}{(1-d)[1+d(1-2p)]}\right)[p\Phi^{-1}z_a + (1-p)z_b].$$

Note that the same macro-equilibrium—interest rates, state prices, consumption allocations, and the value of the market—is compatible with an infinite number of underlying micro-equilibria (i.e., production technologies and firm values) as long as the outputs add up to the same macro quantities, state by state. This is a very important feature of the model, because it allows us to examine comparative static questions, such as the relation between dividend and value volatility, within the same macro equilibrium.

II. THE DETERMINANTS OF DIVIDEND AND PRICE VOLATILITY

In this section first we show the determinants of value and price volatility, and state Theorem 1. We then turn our attention to the issues raised by Theorem 1 for the excess volatility literature.

II.a. Dividend and Price Volatility

We assume that firms pay out all their output as dividends, since there is no investment decision and we are not interested in examining possible dividend-smoothing strategies.

The standard deviation of the *x*-firm's dividends is,

$$Std(div_x) = \left|(x_b - x_a)\right|\sqrt{p(1-p)}\,. \tag{8}$$

Without loss of generality, each firm issues *one* share, so that "share price" and "firm value" are interchangeable. The standard deviation of share prices is,

$$\begin{aligned} Std(VX) &= \left|VX_b - VX_a\right|\sqrt{p(1-p)} \\ &= \left(\frac{d\sqrt{p(1-p)}}{(1-d)[1+d(1-2p)]}\right)\left[p\left(\Phi^{-1}-1\right)x_a + (1-p)(1-\Phi)x_b\right] \end{aligned} \tag{9}$$

In Theorem 1 we show that equation (9) can be used to conclude that there is no obvious equilibrium relation between the volatility of a firm's dividends and the volatility of its value. The intuition behind the theorem is that the state-dependence of state prices causes the volatility of stock prices, in a macro equilibrium that supports a variety of individual firm equilibria.

1. First we note from equation (9) that it is not possible to write the relation as a function of the standard deviation or variance of the *x*'s (the firm's dividends) on the right-hand-side. This precludes a direct relation between the standard deviations of *x* and *VX*.
2. Next, we note that by varying the outputs of both the *x*-firm and the *z*-firm, we can keep the macro equilibrium constant *and* get same *Std(VX)* for different variances of the *x*'s. This is easily seen when equation (9) is rewritten as:

$$Std(VX) = \left(\frac{d\sqrt{p(1-p)}}{(1-d)\left[1+d(1-2p)\right]}\right)\left[p\left(\Phi^{-1}-1\right)(c_a - z_a) + (1-p)(1-\Phi)(c_b - z_b)\right].$$

Keeping the macro equilibrium identical (i.e., d, p, g, c_a, c_b, and thus Φ, constant) still allows infinite choices for x_a, x_b, that would result in the same *Std*(*VX*).[10]

3. The above includes the infinite number of special cases where *Std(VX)* > *0* but the volatility of the *x*-dividends is zero. To see this set $c_a - z_a = c_b - z_b$.

4. Finally, holding the *x*-dividends fixed and varying the macro equilibrium will give different *STD(VX)*. In the special case Φ = 1, *Std(VX)* = *0*, regardless of the volatility of the dividends.

We summarize these four points in the following theorem:

<u>Theorem 1</u>: The price volatility of the firm does not depend on the volatility of its dividends.

Equation (9) shows that even when dividends are equal, price volatility exists. Indeed, price volatility is *never* zero, as long as there is macro uncertainty. Price volatility depends mainly on Φ, a macro quantity that measures macro uncertainty by the dispersion of consumption across states; Φ=1 when there is no macro uncertainty (see Lemma 1).

Theorem 1 may be a surprising result. In all equilibrium models in finance, share values are always the present value of payoffs. It seems only natural to conclude that the *volatility* of these values must depend on the *volatility* of the payoffs! We are not aware of any research in which this proposition has been examined in equilibrium. This apparently self-evident proposition turns out to be false.

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[10] A small example may be useful: Take the following arbitrary but plausible values: p=0.30, d = 0.99, g = 2.0, c_a = 2.10, c_a = 2.30. The values of xs that follow all result in the same *Std*(*VX*) = 6.00 but different *Std*(x).

x_a	x_b	z_a	z_b	***Std*(*x*)**
0.8000	0.4975	1.3000	1.8025	0.1386
1.0000	0.2578	1.1000	2.0422	0.3401
1.1000	0.1380	1.0000	2.1620	0.4408

In Figures 1 and 2 we specify a reasonable macro equilibrium and examine the behavior of the price volatility for both the x-firm and the z-firm as x_a varies between 0 and c_a.[11] As Figure 1 shows, varying the dividends of the x-firm affects the value volatility of both firms; however, as can be seen in Figure 2, there is no clear relation between dividend volatility and value volatility. In fact, as discussed in Theorem 1, dividend volatility of the x-firm can be zero even as the value volatility of the firm is positive.

Figure 1

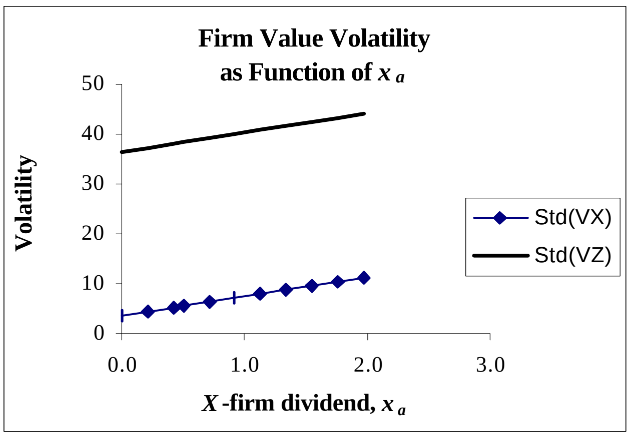
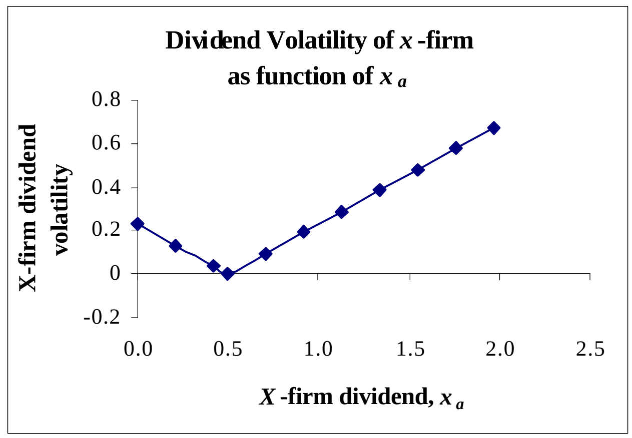


Figure 2



II.2 The Excess Volatility Literature

Theorem 1 raises fundamental questions about the underpinnings of the variance bounds literature. A fundamental quantity in this literature is the "perfect foresight" price of the firm, P_t^*. This is the discounted value of the realized path of dividends. Shiller (1981a,b), LeRoy and Porter (1981), and others have shown that Var(P_t^*) > Var(P_t) where P_t is the observed price, when markets are efficient, dividends stationary, and discount factors constant. However, this proposition relies on the premise that price volatility depends on dividend volatility.

We summarize briefly the theory. Let X_t^f be an efficient forecast of X_t^*. It follows that $X_t^* = X_t^f + e_t$, where e_t is i.d. The immediate result is that $\mathrm{var}\left(X_t^*\right) > \mathrm{var}\left(X_t^f\right)$. Leroy & Porter and Shiller & Grossman use an ingenious approach to take advantage of this inequality. They note that the current price of a stock, P_t, is an efficient forecast, because it is the present value of *expected* dividends. They construct a price series, P_t^*, from the actual ex-post dividends, X_t^*. Assuming the researcher prices dividends correctly, it follows that $\mathrm{var}\left(P_t^*\right) > \mathrm{var}(P_t)$, since the actual dividends must be more volatile than the expectations of these dividends. We will refer to this inequality as the LPSG inequality.

We address the following question: Can this result be true if it is also true that price volatility is independent of dividend volatility?

Consider first the "constant discount factor" approach, which has been used often in this literature. If the actual dividends are discounted by a constant discount factor by the researcher, the LPSG inequality cannot hold.[12] To see why, consider the case of equal dividends. When dividends are equal, $\mathrm{var}\left(P_t^*\right) = 0 < \mathrm{var}(P_t)$. By continuity, it follows that the LPSG inequality is violated at least over some range of dividend volatilities. Note, however, that using a constant discount factor is theoretically incorrect, as shown in Lemma 1.

The only hope is volatility in the discount factor. We have already shown that in such a stationary economy with uncertainty, the discount factor is state-dependent. And as we have seen, it is the variation in the volatility of state prices that produces price volatility.

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11 The macro equilibrium is: p=0.30, d=0.99, g=2.0, c_a=2.10, c_b =2.30.

12 We have already shown that a constant discount factor is inconsistent with a stationary equilibrium. However, the econometrician may choose to use a constant discount factor.

It is difficult to obtain analytic results for a path-dependent quantity like P_t^*. Therefore we resort to numerical simulations. Our main goal is to find legitimate counter-examples to the LPSG inequality rather than to study the statistical properties of P_t^*. Since the LPSG inequality is a general proposition, any well-specified counterexample is sufficient to falsify it.

In the next section we discuss in detail the two method we use to construct P_t^*. The marginal utility of consumption approach, which is precisely that used by Grossman and Shiller (1981), and a state-dependent cost of capital that is used in the standard valuation formula of finance. In these simulations we have the advantage over real-life situations that the pricing factors we use are "correct" because we know the model. In our brief numerical analysis, the LPSG inequality *fails* in all the cases we compute.

III. THE SIMULATIONS

Throughout the simulations, the underlying macro equilibrium is unchanged. This equilibrium is the same as in Figure 1, and it is characterized as follows:

Macro Equilibrium		**States**	
		a	***b***
Probability		0.30	0.70
Time Preference	***d***	0.99	
RRA	***g***	2.00	
Consumption	***c***	2.10	2.30
Expected Consumption		2.24	
Volatility of Consumption		0.0917	
Market Value		195.24	234.21
Expected Market Value		222.52	
Volatility of the Market		17.85	
Risk Free Rate		14.32%	-4.70%
Expected Risk Free Rate		1.01%	
Market Div/Price Ratio		0.011	0.010

The micro equilibrium we use for the simulations we report is:

Micro Equilibrium	**States**	
	a	***b***
Firm Dividends		
x-firm	1.25	1.15
z-firm	0.85	1.15
Stock Price Volatility		
x-firm	9.47	
z-firm	8.38	

We first specify all the necessary steps to generate simulated data for P^*. Then we compute the volatility of P^* for each path and compare it to the volatility of the market price, P. Three steps are required to construct P^*.

1. Determine "sample" paths analogous to the "actual" dividend data in the empirical literature,
2. Compute:
   a. The state-dependent Cost of Capital --*coc*-- that is needed to discount dividends.
   b. The relevant marginal utilities of consumption for each dividend.
3. Construct the time series of P^* by discounting appropriately each dividend along the sample path.

III.1. The Determination of the Sample Path

1. We draw at random 500 numbers, *u*, from a uniform distribution over the [0,1] interval. Each draw of 500 *u*s is a sample path. For the simulations we draw 250 such sample paths.
2. We assign the sequence of states in the following way: $\forall\ u > p$, assign the beta state; to all others we assign the alpha state.

Date=500 is the *last* (and nearest) date of the sample path, while the first observation (date=0) is the furthest.

For the results we present we use the same 250 sample paths for all the calculations in order to make direct comparison possible.

III.2.a. The State-Dependent Cost of Capital, *coc*

The discount factors, or the two costs of capital, *coc*, are derived from the firm values. Consider the *x*-firm: Since expected dividends are constant, it follows that $E(div_x \mid a) = E(div_x \mid b) = E(div_x)$.

1. Let $\Gamma \equiv \left(\dfrac{d}{(1-d)[1+d(1-2p)]}\right)$. Also define $\Theta \equiv \dfrac{x_a}{x_b}$. Since expected dividends are constant, the perpetuity relation gives: $VX_a = \dfrac{E(div_x)}{coc_a}$, $VX_b = \dfrac{E(div_x)}{coc_b}$.

2. It follows then that, $coc_a = \Gamma^{-1} \dfrac{p + (1-p)\Theta^{-1}}{p + (1-p)\Phi\Theta^{-1}}$, $coc_b = \Gamma^{-1} \dfrac{p\Theta + (1-p)}{p\Theta\Phi^{-1} + (1-p)}$.

III.2.b The Marginal Utility of Consumption

The marginal utilities of consumption are the corresponding probability-adjusted state price for each state, and they are given by,

From state	To state a	To state b
a	$mu_{aa} = d$	$mu_{ab} = d\Phi$
b	$mu_{ba} = d\Phi^{-1}$	$mu_{bb} = d$

III.3.a. The Calculation of P_t^* Using *coc*

1. We present value the 500 "actual" dividends of each sample path to the initial date (date=0), using the appropriate *coc* for each state along the sample path.
2. We also present value the theoretical price of the *nearest* observation (P_{500}), to take into account the present value of the dividends after date=500, that have been necessarily truncated in the simulation. This also helps eliminate any biases in P_t^* associated with the increasing importance of the truncation as we progress forward through the observations.

$$P_0^* = \sum_{t=1}^{500}\left(\frac{div_t}{\prod_{t=0}^{t}(1+coc_t)}\right) + \left(\frac{P_{500}}{\prod_{t=0}^{500}(1+coc_t)}\right).$$

P_0^* is the first and furthest observation.

3. We use the recursive nature of pricing to calculate the time series of P_t^*.

$P_t^* = \dfrac{div_{t+1}}{1+coc_t} + \dfrac{P_{t+1}^*}{1+coc_t}$ can be rewritten as, $P_{t+1}^* = (1+coc_t)P_t^* - div_{t+1}$. Equivalently, one can use the method in (2) above for each observation.

III.3.b. The Calculation of P_t^* Using Marginal Utilities

We use the same method as above. The only difference is that we apply the state-specific marginal utility to each dividend during the present valuing process.

$$P_0^* = \sum_{t=1}^{500}\left(div_t \prod_{t=0}^{t}\left(mu_t \right)\right) + P_{500}\prod_{t=0}^{500}\left(mu_t \right).$$

III.4. Computing the Volatility of P^* and the Results

For each sample path drawn above, we compute the volatility (measured by the standard deviation) of P^*.[13] We show typical results from these simulations in Figures 3 and 4. Figures 3A and 3B show the simulation results for both the *x*- and *z*-firms when the *coc* is used to discount the dividend path. Figures 4A and 4B show similar simulation results that use the marginal utilities to do the discounting, while figure 4C shows the results for the market volatility. The horizontal lines in each graph are the theoretical volatilities. The same 250 sample paths are used in all four figures.

Using the *coc* for discounting produces very low volatilities compared to the theoretical ones. We have run several simulations with this method, and it is a rare event when a P^* volatility exceed the theoretical one. In Figures 3A and 3B there are no instance of such an event. The reason for these typically low volatilities is that the *coc*s are too "smooth" by construction. There is no evidence of average mispricing, however.

[13] The computed standard deviation of P^* can be influenced unduly by the "last" observation, i.e., the theoretical P. Because P becomes a larger part of P^* as we approach the nearest observation, it will bias the computed Std downwards if P^* happens to be near P at this stage, and it will bias it upwards if P^* happens to be far from P at this stage. Therefore, we omit the nearest 50 observations when we compute the path standard deviations.

Using the marginal utilities produces markedly different results. In figures 4A, 4B, and 4C the volatilities of P^* exceed the theoretical ones a little more than half the time. The increased volatility of P^* should not be surprising, since the marginal utility method mirrors the theoretical valuation. These results hold both for the individual securities (figures 4A, 4B) and for the market portfolio as a whole.

From the point of view of the "volatility bounds" literature, these figures are conclusive counterexamples to the volatility inequalities in the literature. The $\mathrm{Var}(P_t^*) > \mathrm{Var}(P_t)$ inequality simply *does not hold*. We have run many simulations that we do not report here, and we have generated all sorts of results, including cases in which the volatility of P^* is mostly below or mostly above the theoretical volatility. This shows that not only it is not possible to sign the inequality theoretically, but also that the volatility of any given sample path is a *very imprecise estimator* of the volatility of P.

Our simulations are also consistent with the conclusion in Grossman and Shiller (1981), that as the volatility of the discount factor increases (in their case the marginal utility), the volatility of P^* will eventually exceed that of P. Even though our simulations are computed with the "correct" discount factors, the volatility of P^* is heavily path-dependent. Thus, one cannot rely on such a method to infer marginal utilities and implicitly relative risk aversion.

IV. CONCLUSION

We study the relation between the volatilities of stock "fundamentals" and stock prices, using a multi-firm Lucas, stationary, state-preference economy, with a representative consumer who maximizes a CRRA utility function. There is one good produced by two perfectly competitive firms that possess distinct production technologies. Capital is fixed for each firm.

Uncertainty exists because the productivity of each firm is state-dependent. We assume that only two states are possible at each date and that markets are complete.

First we show that in such a time-stationary economy, the volatility of stock prices does *not* depend on the volatility of dividends or earnings. Rather, it depends on the volatility of the state-dependent pricing function. The intuition here is that, regardless of the distribution of dividends, their pricing will change over time only if the pricing function varies. We also show that uncertainty in a stationary economy *necessarily* implies a state-dependent pricing function.

Next we turn to the "volatility bounds" literature, which derives inequalities between the volatilities of "perfect foresight" prices and market prices. Since these inequalities are based on the assumption that stock price volatility depends on dividend volatility, it is likely that they will fail. We present simulation results which conclusively show that the inequalities in this literature are *incorrect*.

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FIGURE 3A

The Volatility of P_x^* Compared to P_x, Using the *CoC* For Valuation; 250 Simulations

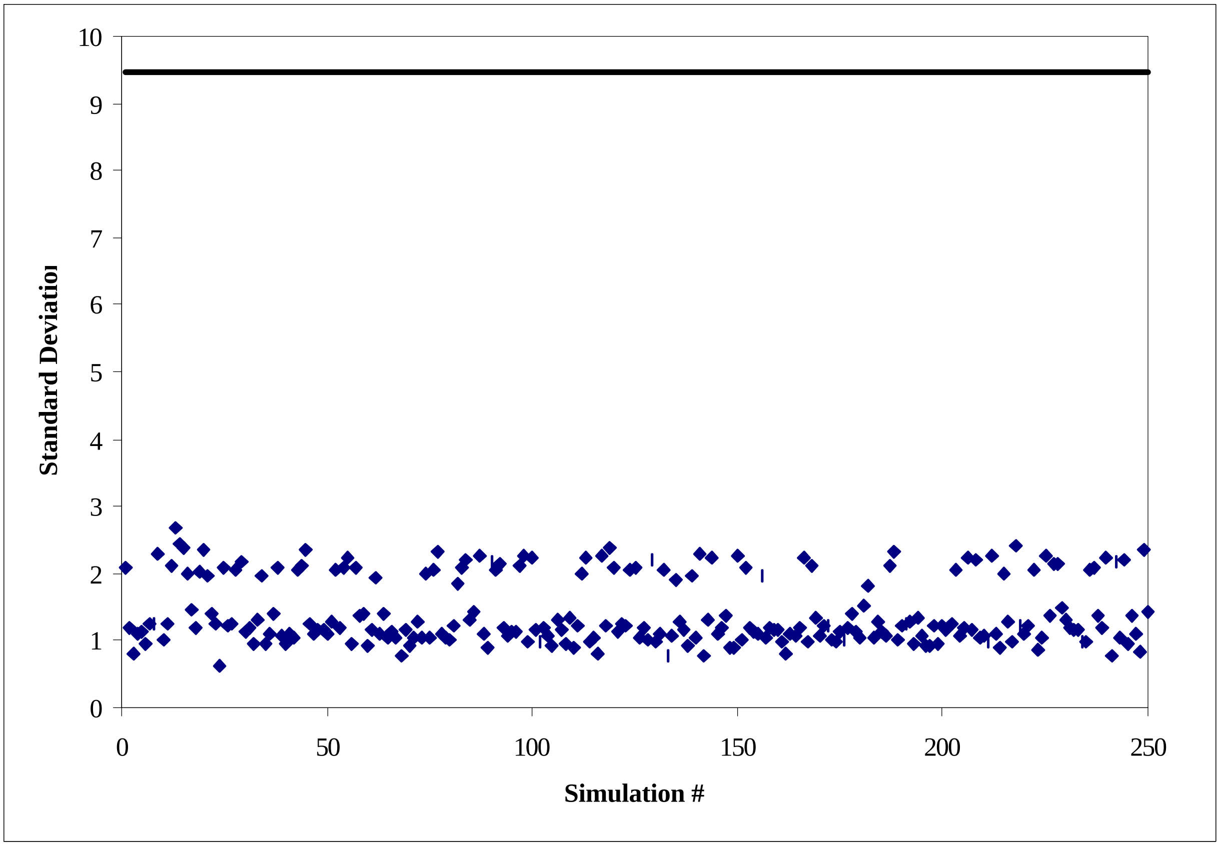

FIGURE 3B

The Volatility of P_z^* Compared to P_z, Using the *CoC* For Valuation; 250 Simulations

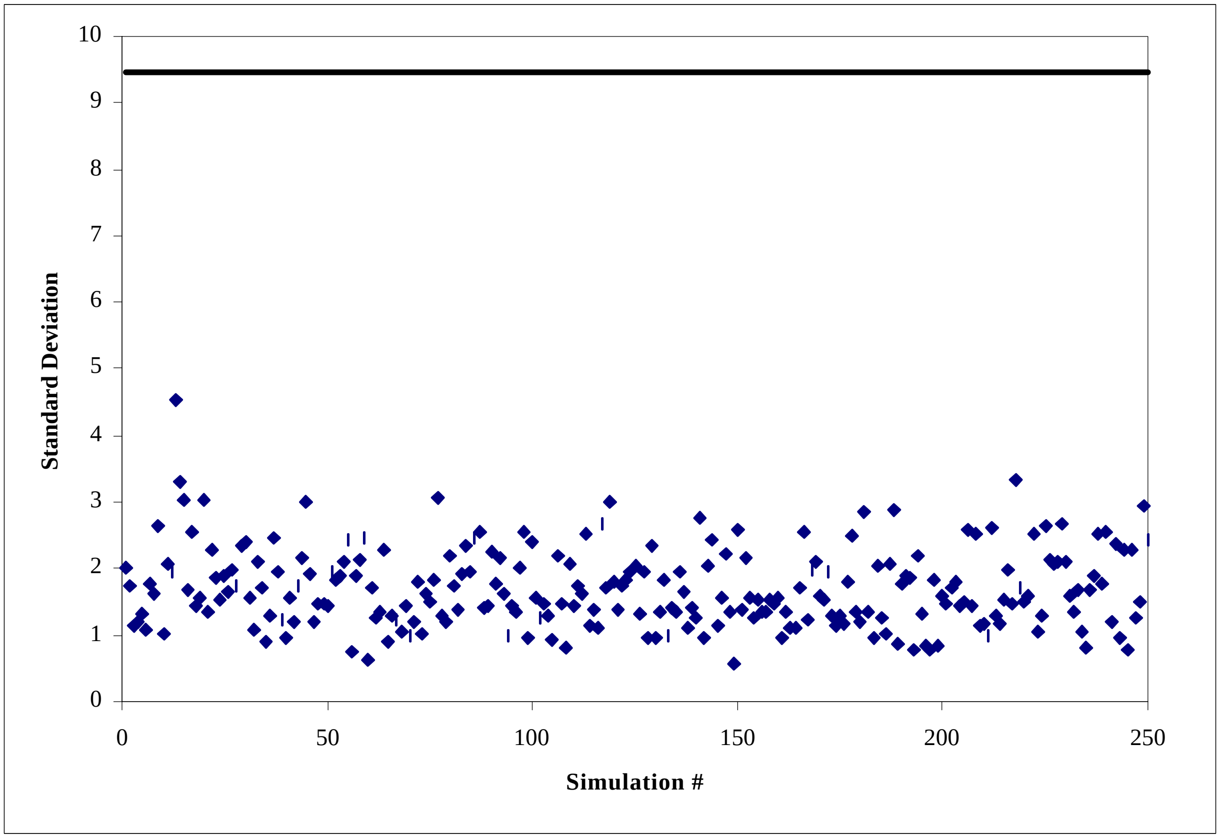

FIGURE 4A

The Volatility of P_x^* Compared to P_x, Using Marginal Utility For Valuation; 250 Simulations

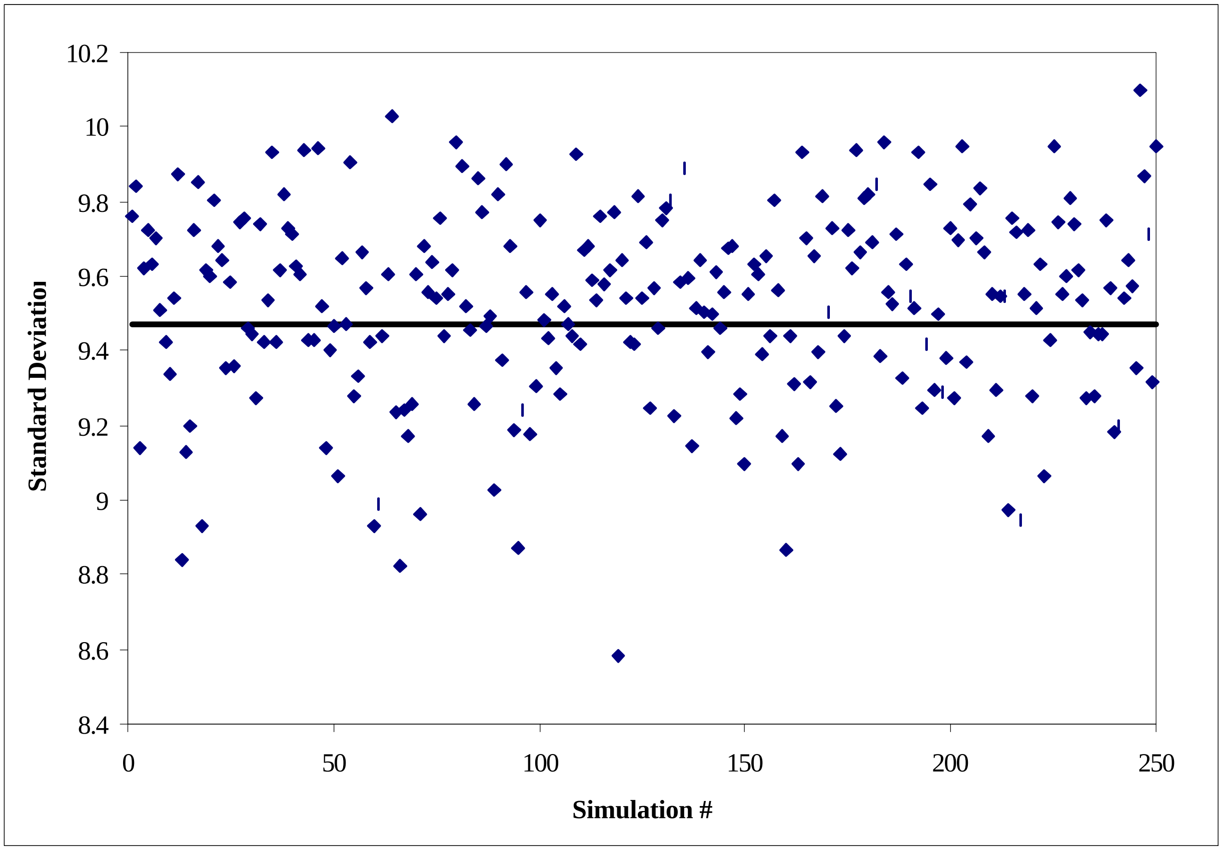

FIGURE 4B

The Volatility of P_z^* Compared to P_z, Using Marginal Utility For Valuation; 250 Simulations

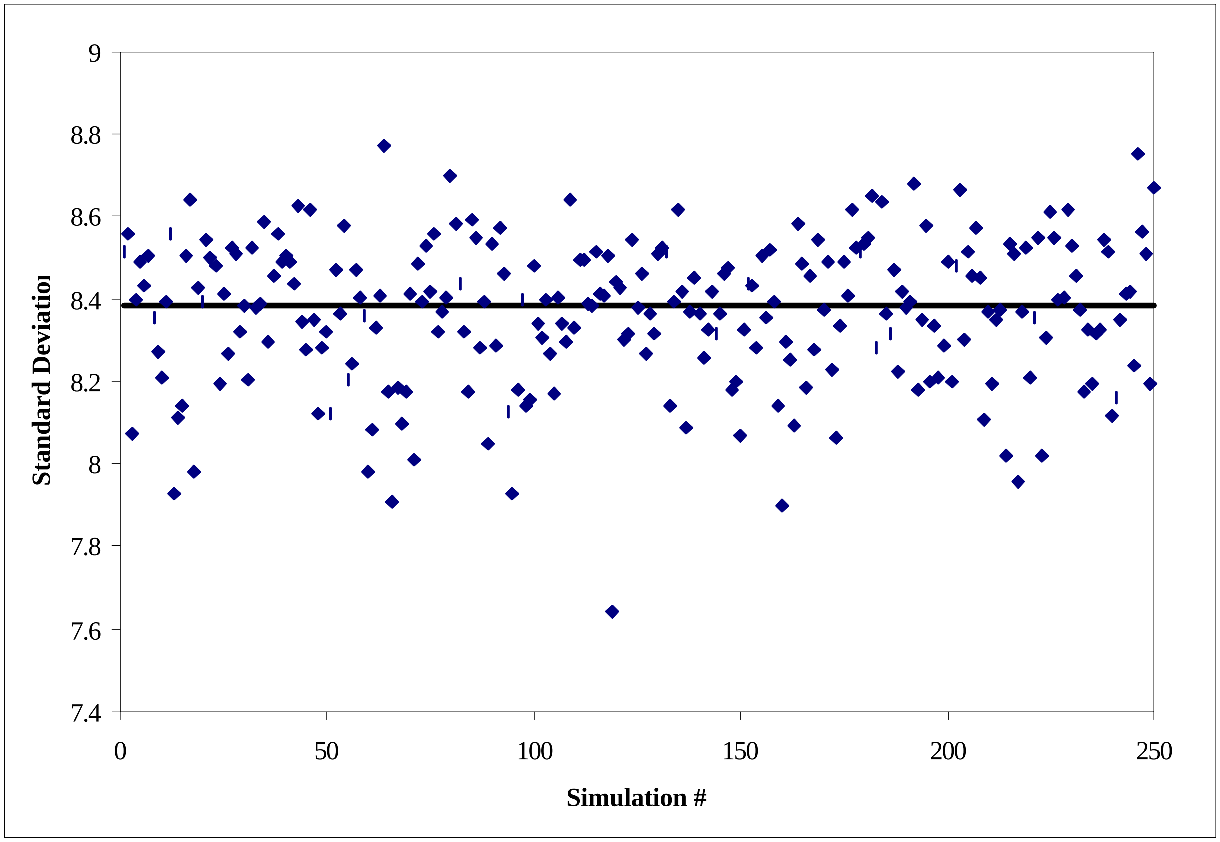

FIGURE 4C

The Volatility of P^*_{market} Compared to P_{market}, Using Marginal Utility For Valuation; 250 Simulations