Wealth Effects and Endogenous Uncertainty *

Elias Albagli

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Abstract

This paper studies the impact of wealth effects in generating endogenous macroeconomic uncertainty. I embed a rational expectations equilibrium (REE) environment into an OLG economy in which households with CRRA preferences delegate investment decisions to informed intermediaries with CARA utility. As current output contracts and dividends fall, households allocate less funding into intermediaries, leading to a contraction in their number and a deterioration in the informational content of prices. The combination of these elements –wealth-sensitive households and a varying mass of wealth-insensitive intermediaries– introduces wealth effects into a REE framework that retains closed form solutions for prices and their informational content. Following a negative first-moment shock in dividends, less informative prices increase the uncertainty about forthcoming payoffs, and the consequent risk premium in stocks. Moreover, a persistent dividend process links the informational content of prices intertemporally, and for certain parameter values, uncertainty can spike with a lag following negative dividend innovations. Economically, the mechanism explains time-varying pricing moments without relying on habit formation and/or exogenous volatility processes, which suggests fluctuations in expected returns can be sub-optimal if they arise from disaggregating information in malfunctioning asset markets. Methodologically, the framework allows exploring wealth effects in an asymmetric information framework while still retaining the tractability in asset prices that characterize standard REE models.

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* University of Southern California, Marshall FBE. F: 213-7406418. albagli@marshall.usc.edu.

1 Introduction

It has long been recognized that asset prices play a key informational role. By aggregating information originally dispersed among agents, prices reduce uncertainty about future prospects of firms and help guide both portfolio allocations as well as real production decisions. Some stylized facts of economic recessions and financial crises, however, suggest that the process of information aggregation in financial markets might be less effective during downturns. These facts include i) a tightening of funding constraints of financial intermediaries in the form of capital outflows and margin calls; ii) steep declines in the price of risky assets and sharp increases in expected returns and volatility; and iii) spikes in market- and survey-based measures of economic uncertainty, such as the VIX volatility index and the dispersion and mean forecast errors of professional analysts.

A renewed interest to link these facts has spawned a large literature in asset pricing and macroeconomic models with some role for the financial sector. The emphasis in these studies, however, is mostly quantitate in nature and uncertainty is modeled as another exogenous economic variable whose driving process is backed out from maximizing the fit with observed data.[1] This paper takes the opposite approach by proposing a framework where fluctuations in uncertainty arise endogenously in a macroeconomic model with financial intermediation. The economy I propose is a dynamic rational expectations equilibrium (REE) in which prices aggregate information about underlying fundamentals to an extent that depends on the capacity of intermediaries to trade upon superior information. In a nutshell, an economy where the informativeness of prices ebbs and flows as conditions in financial markets reflect the underlying state of the macroeconomy.

In general terms, the mechanism proposed works as follows. As economic conditions deteriorate, negative wealth shocks lead households to shift away from risky assets. Money allocated to risky intermediation grows scarce, leading to a reduction in asset prices across the board. But tight funding also limits the ability of intermediaries to capitalize superior information about future asset returns through trading, and knowledge that is originally dispersed across financial intermediaries *remains* dispersed. In equilibrium, prices are not only lower but they are also less informative, leading to an increase in economic uncertainty. Heightened uncertainty leads to an even larger drop in prices with respect to expected dividends, enhancing the fluctuations in risk premia.

Delivering time-varying uncertainty in the context of standard REE frameworks is, however, a difficult task. The central point in these models is that prices are endogenous signals about fundamentals that enter a Bayesian filtering problem. For the analysis to remain tractable, it is necessary to impose restrictive assumptions on preferences and payoffs. First, CARA utility is assumed in order to eliminate wealth effects and deliver demands which are linear in private signals. Second, dividends are assumed to follow a normal distribution. This particular combination allows solving for prices that inherit a normal distribution that can be easily death with in the filtering problem.

Naturally, these assumptions are problematic when one tries to incorporate the REE framework into

[1] Bansal and Yaron (2004) model time-varying risk premium in stock markets assuming a highly persistent, heteroskedastic process for dividends. Wachter (2012) and Gourio (forthcoming) study time-variation in the probability of rare disasters as the driver of asset prices and macroeconomic aggregates. Ilut and Schneider (2012) fit a DSGE model with shocks to ambiguity aversion (Knightian uncertainty).

a macroeconomic model where the main drivers are, indeed, wealth effects. To overcome this problem, I assume that the economy consists on two "layers" of risk-averse agents. The first layer are households with CRRA preferences, whose tolerance for risk is therefore sensitive to wealth.[2] The second layer are informed traders that exhibit CARA preferences. These traders receive a limited amount of funding from households which they use to invest in risky assets, and profits are split according to a linear sharing rule. Importantly, the intermediary can invest in a continuum of assets. This assumption allows a trader to take an unbounded position in a particular asset as long as the fund's budget constraint is satisfied, with then makes funding constraints manageable in the context of an REE framework.

Variations in price informativeness arise from fluctuations in the mass of intermediaries that can operate in the economy. I assume that to run shop, intermediaries must manage a minimum level of funding. As the economy contracts and households allocate less funding to intermediaries, the mass of traders that can operate grows thinner. The market clearing condition then implies that exogenous innovations in the supply of each asset (noise trading) must be borne by a smaller mass of informed traders, leading to a reduction in the signal to noise ratio of equilibrium prices and to the consequent increase in economic uncertainty. The assumption of a shrinking mass of operating intermediaries during crisis, while crucial for delivering time-variation in stock price informativeness, is consistent with ample evidence about investor flow behavior into multiple types of wealth management firms including hedge funds and mutual funds operating in diverse investment strategies.[3]

Of course, time-varying uncertainty leads to a second problem which makes the standard techniques used in dynamic REE environments innaplicable –namely, value functions in the dynamic optimization problem are not closed form, making the implied filtering problem intractable.[4] To circumvent this issue, I assume an overlapping generations framework where both households and traders live for two periods and invest in two-period lived assets. While this assumption limits the applicability of the model to study the quantitative contribution of time-varying uncertainty in asset pricing moments, it is necessary to retain tractability and keep the filtering problem manageable.

Moreover, the persistent process in dividends implies that learning is a dynamic phenomenon, and that the informativeness of prices is linked intertemporally. In particular, uncertainty can spikes with a lag after the initial fall in dividends. This gradual build up of uncertainty arises through the following mechanism: a fall in the informativeness of prices about the current realization of the persistent dividend component implies that in the future, both households and traders inherit an economy with more uncertainty about the value of underlying assets. Hence, risk-averse traders become more reluctant to invest in the (riskier) asset, which further deteriorates the informational content of future prices, bequesting even more uncertainty to future trading rounds. The evolution of traders's beliefs is captured by Kalman filtering techniques which describe posterior first and second moments that vary as a function of the aggregate level of dividends in the economy.

The structure of the paper is as follows. Section 2 presents the main ingredients of the model.

[2]CRRA is assumed because they are the simplest preferences in which risk tolerance is an increasing function of wealth.

[3]For instance, Hedge Fund Research, Inc. (2009) documents that Investor redemptions were widespread and indiscriminate across investment strategies, affecting even hedge funds that delivered positive returns.

[4]Wang (1994) solves a dynamic REE with infinitely lived trades and assets. Because second moments are deterministic in his model, the value function has a closed form which leads to linear demands and normally distributed prices.

Section 3 characterizes the equilibrium. Section 4 discusses the main asset pricing implications of time-varying uncertainty and presents a comparison of the results with an economy in which uncertainty (price informativeness) is set to an exogenous constant. Section 5 concludes.

2 Model Setup

2.1 Securities and Payoffs

Time is discrete: t = 0, 1, ∞. There is a continuum of firms $i \in [0,1]$, each paying a dividend given by

$$D_{t+1}^i = f_{t+1}^i + \varepsilon_{t+1}^{i,d} + \varepsilon_{t+1}^d, \tag{1}$$

where f_{t+1}^i is a firm-specific persistent dividend component, $\varepsilon_{t+1}^{i,d}$ is a firm-specific transitory disturbance, and ε_{t+1}^d is an economy-wide transitory disturbance. The firm-specific persistent component f_{t+1}^i loads on an aggregate process y_{t+1} (i.e., trend GDP), plus a firm-specific disturbance η_{t+1}^i, according to

$$f_{t+1}^i = y_{t+1} + \eta_{t+1}^i. \tag{2}$$

The aggregate process y_{t+1} is itself an mean-reverting series described by

$$y_{t+1} = (1-\rho_y)\bar{y} + \rho_y y_t + \varepsilon_{t+1}^y, \tag{3}$$

while we allow the firm-specific disturbance η_{t+1}^i to also exhibit persistence,

$$\eta_{t+1}^i = \rho_\eta \eta_t^i + \varepsilon_{t+1}^{i,\eta}, \tag{4}$$

where $0 < \rho_y, \rho_\eta < 1$.

There is a risk-free asset with a constant gross return normalized at $R = 1$, and a continuum of stocks, each providing claims to its underlying firm's dividends. While each firm pays an infinite stream of dividends, ownership of the stock of firm i at time t entitles to the next dividend D_{t+1}^i only, and expires after that, being replaced by a new stock i at t+1 which entitles to dividend D_{t+2}^i, and so on.[5] The net supply of the stock of firm i is $1+\theta_t^i$. The random supply component θ_t^i loads on an economy-wide supply innovation θ_t, and a firm-specific supply innovation ϕ_t^i according to

$$\theta_t^i = \theta_t + \phi_t^i, \tag{5}$$

where

$$\theta_t = (1-\rho_\theta)\bar{\theta} + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta \tag{6}$$

[5] As explained below, this assumption is crucial for tractability, and can be found in related work (Veldkamp, 2005).

and

$$\phi_t^i = \rho_\phi \phi_{t-1}^i + \varepsilon_t^{i,\phi}, \tag{7}$$

are also autoregressive processes.

2.2 Households

A representative household of unit mass is born every period t, and dies in period t+1. Households are endowed with 1 stock of each firm, with a value at market prices of $W_t^y = P_t$, where the superscript y (o) denotes a young (old) household in period t, and $P_t \equiv \int P_t^i di$. Households have CRRA preferences over consumption at time t+1 with a risk aversion coefficient of γ^h. They can invest on the risk free asset by themselves, but can only access risky securities through financial intermediaries. As there is no consumption in period t, the household's choice amounts to the fraction of wealth invested with intermediaries: α_t. Wealth at t+1 is then given by:

$$W_{t+1}^o = W_t^y \cdot (1 + \alpha_t \cdot \tilde{r}_{t+1}) + w^o, \tag{8}$$

where w^o is exogenous income received at period t+1, and $\tilde{r}_{t+1}$ is the endogenous return of investing with intermediaries.[6] It is assumed that households split their risky savings equally among the mass of active intermediaries.

2.3 Intermediaries

There is a continuum of financial intermediaries $j \in [0, J]$. Intermediary j receives an amount F_t^j from households to fund the trading of securities at period t. From the profits generated by the intermediary, π_{t+1}^j, a fraction $0 < c < 1$ is retained by the intermediary and the complement $1 - c$ is reimbursed to the household at period t+1. Each intermediary j employs a continuum of traders, each in charge of trading the asset of firm i (henceforth labeled trader (i, j)). Traders are born in period t and die in period t+1, with exponential preferences over consumption at t+1 characterized by a constant absolute risk aversion coefficient γ. The contract between trader (i,j) and the intermediary takes the following form. Each trader can invest unlimited amounts in the stock i, and retains as profits the amount

$$\pi_{t+1}^{i,j} = c \cdot x_t^{i,j} \cdot (D_{t+1}^i - P_t^i \cdot (1 + \kappa)), \tag{9}$$

where κ is an internal interest charged by the intermediary to the trader for each dollar borrowed. This rate is determined endogenously in the equilibrium, such that the budget constraint for each intermediary that receives F_t^j from households is satisfied:[7]

[6] It is straightforward to add a labor income process which covaries with the aggregate state y_{t+1}.

[7] Equation (10) implicitly assumes that short positions alleviate the budget constraint. While in practice short positions are capital-consuming, this assumption delivers a simple form for the asset demand that is essential for tractability. In addition, the mean net supply of risky assets can be made large enough such that the probability of any trader taking a short

$$\int x_t^{i,j}(\cdot;\kappa)\cdot P_t^i di \leq F_t^j. \tag{10}$$

Free entry into intermediation is assumed, but each intermediary must attain a minimum size for operation to be viable. In particular, intermediaries must draw a minimum amount of funding from households:

$$F_t^j \geq \underline{F}. \tag{11}$$

Expression (11) could result, for instance, from fixed costs of setting and/or running an intermediary, but these are not explicitly modeled here. The net trading profit generated by each intermediary, π_{t+1}^j, is the sum of net profits generated by the individual traders,[8]

$$\pi_{t+1}^j = \int x_t^{i,j}\cdot(D_{t+1}^i - P_t^i)di,$$

which by the law of large numbers, is equal among all intermediaries: $\pi_{t+1}^j = \pi_{t+1}, \forall j$.[9] Given a profit share (1-c) accruing to households, the realized risky return for households is given by:

$$\tilde{r}_{t+1} = \frac{(1-c)\cdot\pi_{t+1}}{F_t^j}. \tag{12}$$

Traders born in period t observe the complete history of prices and dividends, $\{\underline{D}_t^i; \underline{P}_t^i\}$, for all $i \in [0,1]$. In addition, each trader observes a private signal of the future dividend's persistent component, f_{t+1}^i, given by

$$s_t^{i,j} = f_{t+1}^i + \varepsilon_t^{i,j}, \tag{13}$$

where the disturbance vector $\epsilon_{t+1}^{i,j} \equiv \begin{bmatrix}\varepsilon_{t+1}^y & \varepsilon_t^d & \varepsilon_t^{i,d} & \varepsilon_{t+1}^{i,\eta} & \varepsilon_t^\theta & \varepsilon_t^{i,\phi} & \varepsilon_t^{i,j}\end{bmatrix}^T$ is assumed to follow a joint normal distribution with mean zero and variance-covariance matrix Σ:

$$\epsilon_{t+1}^{i,j} \sim \mathcal{N}(0,\Sigma), \qquad \Sigma = \text{Diag}(\sigma_y^2, \sigma_d^2, \sigma_{i,d}^2, \sigma_\eta^2, \sigma_\theta^2, \sigma_\phi^2, \sigma_s^2).$$

position in a given stock is arbitrarily small.

[8]The total interest κ collected by the intermediary from traders could either be reimbursed as a lump-sum payment to each trader equal to the average collection, or it could be retained by the intermediary. Both assumptions yield identical results for the equilibrium mass of operating intermediaries, as well as the asset demand of traders, which are the relevant objects of the analysis.

[9]Equalization of profits across intermediaries follows from the the fact that each trader (i,j)'s signal error is uncorrelated across traders belonging to the same intermediary j. This also implies that the shadow cost of capital κ is the same for each intermediary.

3 Equilibrium Characterization

To characterize the rational expectations equilibrium of the model it is useful to write the evolution of the main state variables in a recursive form. Let $\Psi^i_{t+1} \equiv \left[y_{t+1} \;\; \eta^i_{t+1} \;\; \theta_t \;\; \phi^i_t\right]^T$. Given equations (2) through (7), the evolution of Ψ^i_{t+1} can be written as

$$\Psi^i_{t+1} = C_\psi \bar{\Psi} + A_\psi \Psi^i_t + B_\psi \epsilon^{i,j}_{t+1}, \tag{14}$$

where C_ψ, A_ψ and B_ψ are matrices of proper order (see Appendix A). Most of the equations of interest, including the evolution of beliefs, optimal demands, and equilibrium prices, can be expressed in terms of this recursive representation. In particular, we will look for a rational expectations equilibrium in which the price of each security is a linear combination of the expectations of the state vector Ψ^i_{t+1} which condition on public information up to time t, the realization of the persistent future dividend component f^i_{t+1}, and the firm-specific random supply θ^i_t:

$$P^i_t = p_{0,t} + P_{c,t} \cdot \mathbb{E}[\Psi^i_{t+1} \mid \Omega^c_t] + p_{f,t} \cdot f^i_{t+1} + p_{\theta,t} \cdot \theta^i_t, \tag{15}$$

where $P_{c,t}$ is a time-dependent matrix of proper order.

The solution approach builds on the standard technique used in CARA-normal REE setups (see Vives, 2008 for a textbook discussion). These models apply a guess-verify procedure which consists of 3 steps. First, conjecture that prices are linear in the underlying shocks at each period, as in equation (15) above. Based on these conjectures, update beliefs (posterior means and variance) of future dividends conditional on all available information. Second, derive the optimal demands of traders. Third, impose market clearing and solve for the conjectured coefficients in the price functions in terms of the underlying model parameters.

In the present context however, demands for stocks by each trader (i,j) depend on the overall funding available for the intermediary F^j_t (through the shadow price κ), which in turn depends on households' portfolio decisions. But the portfolio decision of households –namely, the amount of risky savings allocated to intermediaries– depends on both the exogenous processes determining dividends and the endogenous prices of securities. In other words, prices must satisfy two layers of a fixed-point problem. First, they must satisfy the standard fixed-point problem of REE setups –clear markets and be consistent with the beliefs and demands which lead to those prices. Second, prices must lead to household's risky savings decisions which are consistent with the demand for securities which lead to such prices.

More formally, let F_t be the total risky savings allocated by households born at time t to intermediaries. Let $\underline{h}_t \equiv \{h_{t-s}\}^{+\infty}_{s=0}$ denote the complete history of a variable h up to time t. Let $\Omega^c_t = \{\underline{D}^i_t, \underline{P}^i_t\}_{i\in[0,1]}$ denote the history of public information in the economy up to time t, and $\Omega^{i,j}_t = \{\Omega^c_t \bigcup s^{i,j}_t\}$ denote trader's (i,j) information at time t. For any information set Ω, let $H(x|\Omega) : R \to [0, 1]$ denote the conditional posterior cdf of a random variable x. The equilibrium concept is as follows:

A competitive sequential equilibrium is defined by:
1. a sequence of price functions $P_t^i(\mathbb{E}[\Psi_{t+1}^i \mid \Omega_t^c], f_{t+1}^i, \theta_t^i; F_t)$*; 2. risky savings decisions by households* $F_t = \alpha_t(\cdot;\Omega_t^c, P_t)\cdot W_t^y$*; 3. demand for asset i by trader (i,j),* $x_t^{i,j} = x(P_t^i, \Omega_t^{i,j}, \kappa(\cdot;F_t))$*, and; 4. a set of prior beliefs* $H(\Psi_{t+1}^i|\Omega_{t-1}^c)$ *for all agents, posterior beliefs* $H(\Psi_{t+1}^i|\Omega_t^c)$ *for households, and* $H(\Psi_{t+1}^i|\Omega_t^{i,j})$ *for trader (i,j) such that* $\forall\ i \in [0,1]$*,* $j \subset J$*, and t:*
(i) risky savings by households are optimal given stock prices and posterior beliefs; (ii) stock demands by traders are optimal given stock prices, interest cost κ *and posterior beliefs; (iii) the securities markets clear for all stocks at all times; and (iv) Posterior beliefs satisfy Bayes law.*

The equilibrium characterization below first solves the problem of intermediaries for an exogenous level of risky savings by households at period t. The equilibrium prices are then fed to to household's problem, which choose their optimal level of risky savings into the intermediaries sector. The economy is at an equilibrium when we find the fixed point level of risky savings that satisfy both problems.

3.1 Traders' Problem

3.1.1 Evolution of Beliefs

To characterize the asset market equilibrium for an exogenous level of risky savings by households, the first step is finding posterior beliefs of each trader (i,j), conditional on $\Omega_t^{i,j} = \{\Omega_t^c \bigcup s_t^{i,j}\}$.[10] This can be achieved by first finding the recursive representation of posterior beliefs at period t which condition on the vector of public signals, and then updating posterior beliefs within period t after the observation of private signals.

Note that although traders have access to an infinite (continuum) of signals of different stocks, information about stock h is only relevant for predicting the future dividend of stock i to the extent that it helps predict the common underlying dividend component y_t. In other words, shocks affecting D_t^h and P_t^h which are not correlated with the aggregate processes y_t and θ_t are orthogonal to the projection of y_{t+1}, and hence D_{t+1}^i, for all $i \neq h$. It follows that for projecting D_{t+1}^i, the infinitely dimensional vector $\{\underline{D}_t^i, \underline{P}_t^i\}_{i\in[0,1]}$ is informationally equivalent to $\{\underline{D}_t^i, \underline{P}_t^i; \underline{D}_t, \underline{P}_t\}$, where $D_t \equiv \int D_t^i di$.

According to the price conjecture in (15), the two endogenous signals P_t and P_t^i are informationally equivalent to $p_t = f_{t+1} + \Delta_t \cdot \theta_t$ and $p_t^i = f_{t+1}^i + \Delta_t \cdot \theta_t^i$, respectively. The coefficient Δ_t multiplying the noisy supply innovation corresponds to the ratio of the price conjecture coefficients: $\Delta_t = p_{\theta,t}/p_{f,t}$. Using the recursive representation in (14), the vector of public period t signals can then be expressed as:

$$S_t^{i,c} = \begin{bmatrix} D_t & D_t^i & p_t & p_t^i \end{bmatrix}^T = C_s\bar{\Psi} + A_{s,t}\Psi_{t+1}^i + B_s\epsilon_{t+1}^{i,j}, \tag{16}$$

and we can apply Kalman filtering techniques to describe the evolution of posterior beliefs (means and variances) conditional on the public history of dividends and prices. Specifically, let $O_t^c \equiv \mathbb{E}[(\Psi_{t+1}^i - \mathbb{E}[\Psi_{t+1}^i \mid \Omega_t^c]) \cdot (\Psi_{t+1}^i - \mathbb{E}[\Psi_{t+1}^i \mid \Omega_t^c])^T \mid \Omega_t^c]$,

[10] It can be shown that the interest cost κ is a non-linear function of the aggregate state variables y_{t+1} and θ_t. To keep the filtering problem tractable, we assume agents do not update information from the observation of this cost.

Theorem 1 (Beliefs with common information):
i) The posterior expectation of Ψ^i_{t+1}*, conditional on public information* Ω^c_t*, is given by*

$$\mathbb{E}[\Psi^i_{t+1} \mid \Omega^c_t] = C_\psi \bar{\Psi} + A_\psi \mathbb{E}[\Psi^i_t \mid \Omega^c_{t-1}] + K^c_t \cdot (S^{i,c}_t - \mathbb{E}[S^{i,c}_t \mid \Omega^c_{t-1}]), \tag{17}$$

ii) The variance of Ψ^i_{t+1}*, conditional on public information* Ω^c_t*, is given by*

$$O^c_t = (I_4 - K^c_t A_{s,t}) \cdot (A_\psi O^c_{t-1} A^T_\psi + B_\psi \Sigma B^T_\psi) \tag{18}$$

Proof: In Appendix A.

Equations (17) states that the expectations of the state vector Ψ^i_{t+1} correspond to its unconditional mean ($C_\psi \bar{\Psi}$) plus the expectations of the lagged state vector adjusted by time decay ($A_\psi \mathbb{E}[\Psi^i_t \mid \Omega^c_{t-1}]$), plus an update corresponding to the surprise component of the signal vector $S^{i,c}_t$ with respect to its prediction conditional on information up to time t -1. This update is determined by the matrix K^c_t, which is the linear projection of the signal vector $S^{i,c}_t$ into the state vector Ψ^i_{t+1}. Expression (18) shows that the posterior variance O^c_t equals the lagged variance O^c_{t-1} adjusted by time decay ($A_\psi O^c_{t-1} A^T_\psi$) plus the variation coming from the impact on the state vector of the new disturbances ($B_\psi \Sigma B^T_\psi$), scaled down by the information contained in the new vector of public signals ($I_4 - K^c_t A_{s,t}$), where $K^c_t A_{s,t}$ denotes the Kalman gain.

We now express the posterior mean and variance of the state vector, conditional on traders' information $\Omega^{i,j}_t$, in terms of a similar pair of transition equations. These equations describe how the beliefs in Theorem 1 are updated with the additional private signal $s^{i,j}_t$, which written in the recursive form can be expressed as $s^{i,j}_t = a_s \Psi^i_{t+1} + b_s \epsilon_{t+1}$ (see Appendix A). Specifically, let $O_t \equiv \mathbb{E}[(\Psi^i_{t+1} - \mathbb{E}[\Psi^i_{t+1} \mid \Omega^{i,j}_t]) \cdot (\Psi^i_{t+1} - \mathbb{E}[\Psi^i_{t+1} \mid \Omega^{i,j}_t])^T \mid \Omega^{i,j}_t]$,

Theorem 2 (Beliefs with private information):
i) The posterior expectation of Ψ^i_{t+1}*, conditional on trader (i,j)'s information* $\Omega^{i,j}_t$*, is given by*

$$\mathbb{E}[\Psi^i_{t+1} \mid \Omega^{i,j}_t] = \mathbb{E}[\Psi^i_{t+1} \mid \Omega^c_t] + K^p_t \cdot (s^{i,j}_t - \mathbb{E}[s^{i,j}_t \mid \Omega^c_t]) \tag{19}$$

ii) The variance of Ψ^i_{t+1}*, conditional on trader (i,j)'s information* $\Omega^{i,j}_t$*, is given by*

$$O_t = (I_4 - K^p_t a_s) \cdot O^c_t \tag{20}$$

Proof: In Appendix A.

The intuition behind equations (19) and (20) is similar to that discussed in Theorem 1. Traders' expectations of the state vector Ψ^i_{t+1} correspond to their prior expectation conditional on public information up to time t, plus the update corresponding to the surprise component of the private signal $s^{i,j}_t$ with respect to its prediction conditional on public information. Since there is no time lag between the object being updated with different information sets, there is no time decay (A_ψ) in this expression. Similarly, the variance of Ψ^i_{t+1} given all information is just the variance given public information (O^c_t), scaled down by the Kalman gain associated with the private signal ($K^p_t a_s$).

3.1.2 Optimal demands

Trader (i,j) chooses $x^{i,j}_t$ to maximize expected utility of consuming terminal wealth in expression (9), or

$$\max_x \ \mathbb{E}[-\exp\{-\gamma \cdot c \cdot x \cdot (D^i_{t+1} - P^i_t \cdot (1+\kappa))\} \mid \Omega^{i,j}_t]. \tag{21}$$

From the results in Theorem 2, terminal wealth is conditionally normally distributed. Together with exponential preferences this yields the following optimal demand of trader (i,j) for stock i:

$$x^{i,j}_t = \frac{\mathbb{E}[D^i_{t+1} \mid \Omega^{i,j}_t] - P^i_t(1+\kappa)}{\gamma \cdot c \cdot \mathbb{V}[D^i_{t+1} \mid \Omega^{i,j}_t]}, \tag{22}$$

where the dividend D^i_{t+1} can be rewritten in terms of the state vector Ψ^i_{t+1} and the innovation $\epsilon^{i,j}_{t+1}$,

$$D^i_{t+1} = a_d \Psi^i_{t+1} + b_d \epsilon^{i,j}_{t+1}, \tag{23}$$

with $a_d = [1 \ \ 1 \ \ 0 \ \ 0]$, and $b_d = [0 \ \ 1 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0]$.

3.1.3 Market clearing

The free entry condition together with the minimum size constraint of expression (11) imply that in equilibrium all intermediaries will have the minimum size $\underline{F}$. In turn, this implies that the total mass of operating intermediaries for an exogenous level of risky savings by households $F_t = \alpha_t \cdot W^y_t$ is given by

$$m_t = F_t / \underline{F}. \tag{24}$$

By the law of large number, the aggregate demand for stock i coming from the measure m_t of traders is equal to the average trader demand over all $i \in [0, J]$, times m_t.[11] This allows writing the following market clearing condition:

[11] It is assumed that J is large enough so that $m_t < J$ almost surely.

$$m_t \int x_t^{i,j} dj = 1 + \theta_t^i \tag{25}$$

Using the characterization of posterior beliefs from Theorems 1 and 2 in the trader's demand equation (22), together with the recursive representation of the dividend in (23), we can solve for the price P_t^i as a function of the mass of operating intermediaries m_t:

$$P_t^i = \frac{a_d(I_4 - K_t^p a_s)\mathbb{E}[\Psi_{t+1} \mid \Omega_t^c]}{1+\kappa} + \frac{a_d K_t^p f_{t+1}^i}{1+\kappa} - \frac{\gamma \cdot c \cdot (a_d O_t a_d^T + b_d \Sigma b_d^T)(1 + \theta t^i)}{m_t(1+\kappa)}, \tag{26}$$

from which it directly follows that the conjectured coefficients of expression (15) are consistent with an equilibrium in the asset markets (for an exogenous level of m_t) if

$$\begin{aligned} p_{0,t} &= \gamma \cdot c \cdot (a_d O_t a_d^T + b_d \Sigma b_d^T)/(1+\kappa), \quad P_{c,t} = \frac{a_d(I_4 - K_t^p a_s)}{1+\kappa} \\ p_{f,t} &= a_d K_t^p/(1+\kappa), \qquad p_{\theta,t} = -\gamma \cdot c \cdot (a_d O_t a_d^T + b_d \Sigma b_d^T)/m_t(1+\kappa), \text{ and} \\ \Delta_t &= -\frac{\gamma \cdot c (a_d O_t a_d^T + b_d \Sigma b_d^T)}{m_t \cdot a_d K_t^p}. \end{aligned} \tag{27}$$

Proposition 1 (Existence and uniqueness)
For every exogenous value $m_t \in \Re^+$, there is a unique price function with the form of equation (15), where the coefficients are given by equation (27).

A proof of uniqueness can be found on static REE setups with dispersed information, such as Diamond and Verrecchia (1980) (see Vives 2008 for a textbook discussion). Although the present model analyzes information aggregation into prices in a dynamic setup, traders live for two periods and trade a two period-lived asset, which reduces decisions to a repeated static optimization problem where the prior beliefs of future dividend's moments are history dependent.[12]

Expression (27) shows the main parameters affecting the informativeness of prices. In particular, prices will be more informative about the underlying persistent dividend component f_{t+1}^i of each stock i, and about the aggregate process $y_{t+1} = \int f_{t+1}^i di$, when the ratio Δ_t is small (in absolute magnitude). This ratio is increasing in the effective risk aversion of traders (the product between risk aversion and the share of trading profits, $\gamma \cdot c$), as well as the posterior variance of dividends $(a_d O_t a_d^T + b_d \Sigma b_d^T)$. This last term, as well as the product $a_d K_t^p$ in the denominator are endogenous objects which must be determined through numerical methods.

The last element in the expression is m_t, the mass of active intermediaries. The intuition for this expression is simple: as the mass of intermediaries is reduced, there are fewer traders operating in the market for stock i that are able to bear the risk. For the same level of net supply to be absorbed by a

[12]The situation is of course different if the asset is longer-lived, case in which multiple linear price equilibria can arise in an overlapping generations model (see Spiegel, 1998).

smaller group of traders, each will require a larger price concession to be willing to hold the other side of the trade. This makes prices fluctuate strongly to innovations in the noisy supply of the asset, which reduces the signal to noise ratio of the endogenous public signals in the model.

3.2 Household Problem

We now close the equilibrium characterization by solving the household problem, taking as given the aggregate stock price level of the economy P_t. Households maximize expected utility of consumption of terminal wealth, which is given by equation (8). Their choice variable is the fraction α_t of their current wealth to be invested with intermediaries. With CRRA preferences, and an information set of Ω_t^c, we can write the household problem as

$$\max_{\alpha} \ \mathbb{E}[\frac{(W_t^y \cdot (1+\alpha_t \cdot \tilde{r}_{t+1}) + w^o)^{1-\gamma^h}}{1-\gamma^h} \mid \Omega_t^c], \tag{28}$$

which gives rise to the following f.o.c. that α_t must satisfy:

$$\begin{aligned} \mathbb{E}[(W_t^y \cdot (1+\alpha_t \cdot \tilde{r}_{t+1}) + w^o)^{-\gamma^h} \mid \Omega_t^c] \cdot \mathbb{E}[W_t^y \cdot \tilde{r}_{t+1} \mid \Omega_t^c] = \\ -Cov[(W_t^y \cdot (1+\alpha_t \cdot \tilde{r}_{t+1}) + w^o)^{-\gamma^h} \ ; \ W_t^y \cdot \tilde{r}_{t+1} \mid \Omega_t^c]. \end{aligned} \tag{29}$$

Replacing $W_t^y = P_t$, equation (29) allows to solve implicitly for α_t^*, which in turn pins down the equilibrium level of m_t from expression (24).

3.3 Discussion of Main Assumptions

This section discusses the rationale behind the main model assumptions, and whether the results are likely to hold under more general conditions.

3.3.1 Short lived assets and traders

Traders and households are assumed to live two periods. The OLG structure, in particular for the case of traders, is necessary for retaining tractability. Indeed, the methodology for solving dynamic optimization developed for REE setups, such as Wang (1994), is inapplicable here as future prices have stochastic first *and* second moments. This leads to a value function problem, asset demands and equilibrium prices which cannot be solved in closed form, resulting in an intractable filtering problem.[13]

A related assumption concerns the short life of the assets. The methodology used in models with OLG traders but infinitely lived assets[14] is not applicable for the same reason: stochastic second moments imply price functions whose coefficients are time-varying, which makes terminal wealth of the OLG traders have

[13]While some authors have used numerical procedures (Bernardo and Judd, 2000) or log-linear approximations (Peress, 2004) to deal with learning problems where equilibrium prices are non-normal, these methods have only been developed so far in static contexts.

[14]See Spiegel (1998), and Watanabe (2008).

a non-normal distribution, conditional on information. For these reasons, the present model assumes an OLG structure for both assets and traders. When assets last one period, future payoffs include future dividends only (and not future prices), which are normally distributed conditional on traders' information. This allows the equilibrium to be solved in closed-form with the standard guess-verify procedure of CARA-normal REE models.

My conjecture is that extending the life span of the assets would enhance the contribution of wealth effects discussed here to the fluctuation of uncertainty. In the current setup, an increase in uncertainty about the underlying aggregate process y_{t+1} affects traders in period t by increasing the conditional variance of their payoffs D^i_{t+1}. Although the increase in uncertainty also propagates into future rounds of trading (i.e., increasing the uncertainty about D^i_{t+2}, D^i_{t+3}, and so on), this effect is not internalized by traders at time t. Were these traders to live and trade further into the future, the dynamic propagation of uncertainty would make their future stream of payments more risky. This would make asset demands less responsive to information (more cautions trading), further decreasing the signal to noise ratio of current prices.

3.3.2 Intermediation contracts

The contracts between households, intermediaries and traders assumed in the model are not claimed to be optimal. They are chosen because they lead to an optimization problem by traders which imply linear demands for the asset, which is essential for tractability. Optimal contracting in portfolio delegation problems where households and intermediaries are heterogenous in their information and/or preferences is the topic of a large body of literature studying principal-agent problems more generally, and portfolio delegation more specifically, but is out of the scope of the present paper.[15]

3.3.3 Multiple assets

Most of the model's main insights could be recast in single asset version of the current setup. The reason for studying a multiple asset setup –in particular, one with a continuum of assets– is that this structure allows to deal with funding constraints at the intermediary level, and still retain simple, closed-form solutions for traders' asset demands. With normally distributed posterior beliefs and CARA preferences, there is no guarantee that optimal traders' demand for all assets will remain bounded. With a finite number of securities, an unbounded position on an asset which has strictly positive mass in the intermediaries portfolio will lead to a violation of the resource constraint of the intermediary (expression (10)). With a continuum of securities, each has zero mass in the total portfolio and unbounded positions in individual assets are allowed. In this case, it is the cost of the average position what matters, which remain bounded for a finite value of the interest cost κ that makes the inequality in (10) hold.

[15] Cadenillas, Cvitanić and Zapatero (2007) study the optimal contract in a portfolio delegation problem under general concave preferences for both households and intermediaries.

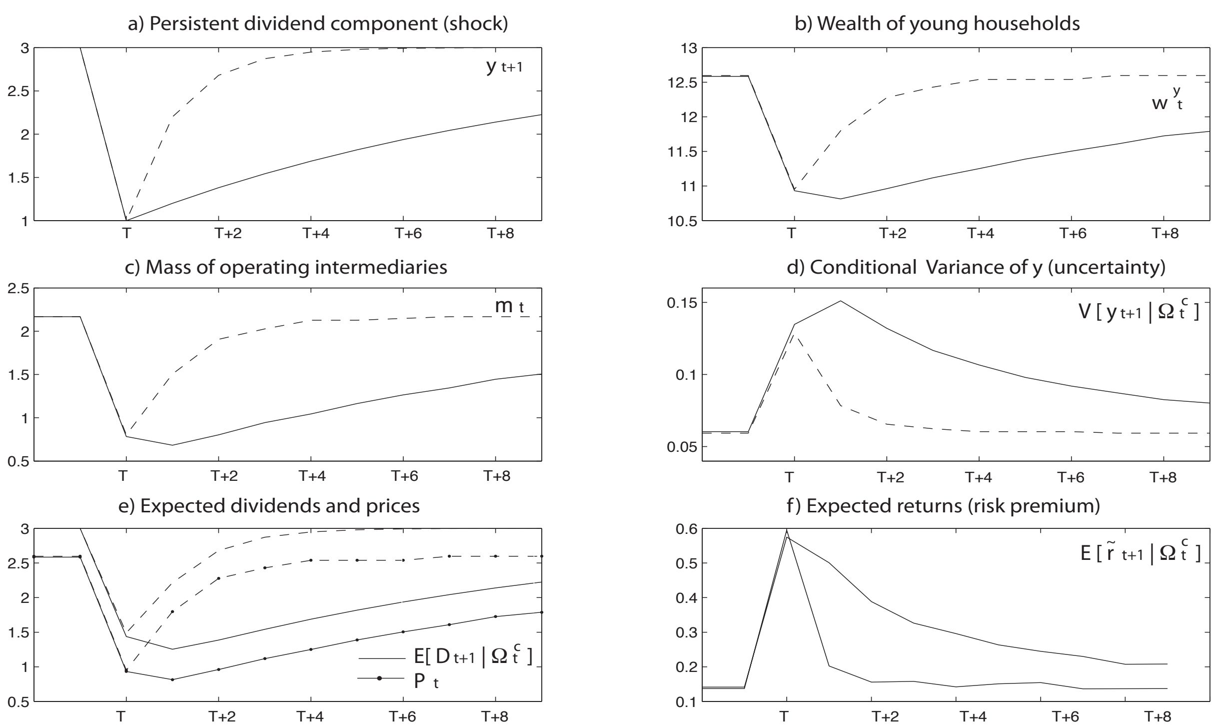
4 Wealth and Uncertainty Dynamics

This section describes the evolution of wealth, uncertainty, and its asset pricing implications through two numerical exercises. The first exercise is a simple impulse-response characterization of the dynamic evolution of uncertainty in response to a negative wealth shock (low dividend realization). The second exercise is a random simulation of two alternative economies: the baseline model described above, and a fictitious economy in which the informativeness of asset prices is exogenously given. This last exercise allows to disentangle the marginal contribution of endogenous uncertainty to time-varying expected returns (risk premium).

4.1 Dynamic Evolution of Uncertainty

Figure 1 displays a simple impulse-response exercise simulated from the model developed above (see Appendix B for details). The economy starts at the long-run level for the main state variables: $\Psi_0 = \bar{\Psi}$, and the model is run setting all shocks to zero for 20 periods, more than enough to reach a steady state under the parameters assumed. In period 21, an innovation of -2 standard deviations of ε^y is fed into the model, and we set all other shocks to zero for all other state variables and periods. The solid lines represent a parametrization in which shocks are relatively persistent ($\rho_y = 0.9$), while the dashed lines represent an economy with relatively transitory shocks ($\rho_y = 0.4$).

Figure 1: Impulse response: -2 st.dev. shock ε^y_{t+1}



The figure illustrate the main mechanisms at work in the model. The negative shock in y_{t+1} (panel a)) is anticipated, as it is partially revealed by average stock prices P_t, through traders private signals about the persistent component of dividends. This causes an immediate wealth effect (panel b)) that increases the effective risk aversion of households who decide to allocate a smaller fraction of wealth as risky savings into intermediaries. Because both wealth is lower and the fraction of it allocated to the risky portfolio falls, the aggregate effect is a steep decline in the mass of intermediaries that can remain operative (panel c)). This in turn lowers the demand for the risky stock.

The effects of this fall in demand are several and interrelated. First, lower demand for stocks due to the reduced funding in the economy brings stock prices further down from the expected future dividend (panel e)). But as stock prices fall, the present wealth of households falls further and so does the availability of funding for risky intermediation. Of course, as stock prices fall back from future expected dividends, expected returns increase, which stabilizes demand for stocks at an interior level with a positive price of securities.

Furthermore, the persistence nature of dividends implies that price informativeness in one period affects uncertainty in future periods, and hence the actions of traders and the informativeness of prices in future periods. The propagation of uncertainty under alternative parameter assumptions can be appreciated by plotting the variance of y_{t+1}, conditional on public information up to period t (panel d)). Although uncertainty rises steeply at impact, it does not peak until the following period in the economy with high shock persistence. The intuition is as follows. In anticipation of the shock in period t+1, prices at period t become less informative about the realization of the future persistent component y_{t+1} (through lower m_t). This implies that future households and traders inherit an economy in which not only wealth is below its steady state value (as dividends still need a few period to mean-revert), they also inherit a less precise prior about the current value of the state variable y_{t+1}. This constitutes an additional incentive to lower asset demands for any given expectation of future dividends, which further deteriorates the informativeness of prices in period t+1. This makes the conditional variance $\mathbb{V}[y_{t+2} \mid \Omega^c_{t+1}]$ even larger than $\mathbb{V}[y_{t+1} \mid \Omega^c_t]$, even though dividends have already began to recuperate (on average) towards their steady state value. The potential for hump-shape dynamics in uncertainty depend of course on parameters assumptions. In the economy with relatively short-lived disturbances, uncertainty about past values of the state variable y_{t+1} have a lesser contribution in the current uncertainty for y_{t+2}, and the impulse-response functions look standard.

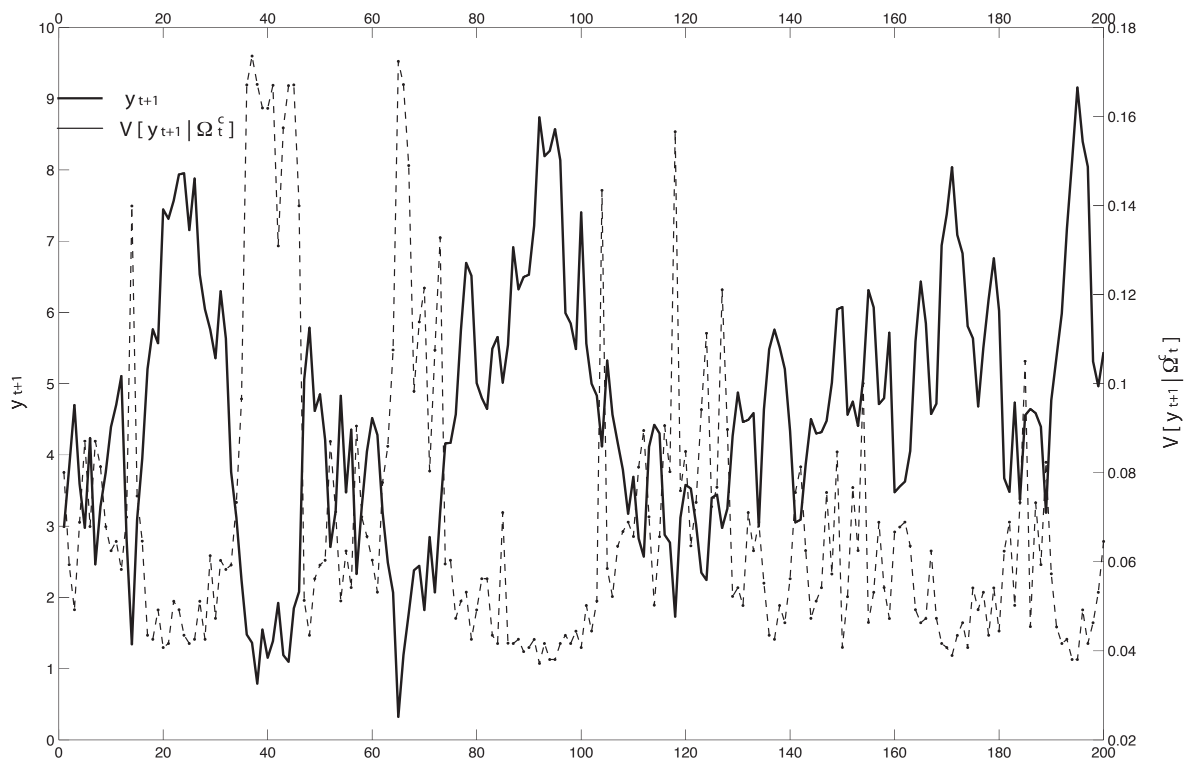
Panel f) in Figure 1 plots the impact of wealth effects on the expected return demanded by households in their risky investments with intermediaries. As discussed above, wealth effects after dividend shocks change local risk aversion of households and affect available funding to intermediaries, further impacting prices and enhancing its wealth implications. But this mechanism has nothing to do with the asymmetric nature of information in the model. Asymmetric information enhances the effects of the aforementioned mechanism because as the mass of intermediaries change, so does the informational content of prices as endogenous signals. In other words, dividend shocks generate both fluctuations in (local) risk aversion and in the *amount* of risk that agents are bearing when they participate in the stock market. The effect of this additional channel is lower aggregate funding in the economy as households cut further back on their allocations to intermediaries, as well as a lower stock demands from intermediaries for any level of aggregate funding. The following section presents a simulation exercise that attempts to isolate the separate contribution of time-varying uncertainty in asset prices.

4.2 Asst Pricing Moments in a Simulated Economy

This section describes a simulation exercise and compares asset pricing implications of alternative assumptions about the underlying structure of the economy. Starting form its long-run level, the exercise simulates 200 periods of random fluctuations in the aggregate state variables y_{t+1} and θ_t, according to its assumed distributions (see table 2 for parameter details).

Figure 2 displays the aggregate series y_{t+1} and its variance conditional on common information ($\mathbb{V}[y_{t+1} \mid \Omega_t^c]$) for a simulated path of the economy. The figure shows how the aggregate series has a strong negative correlation (-0.79) with the conditional variance given public information, as a drop in average dividends lowers the mass of intermediaries and reduces the informational content (signal to noise ratio) of aggregate prices P_t.

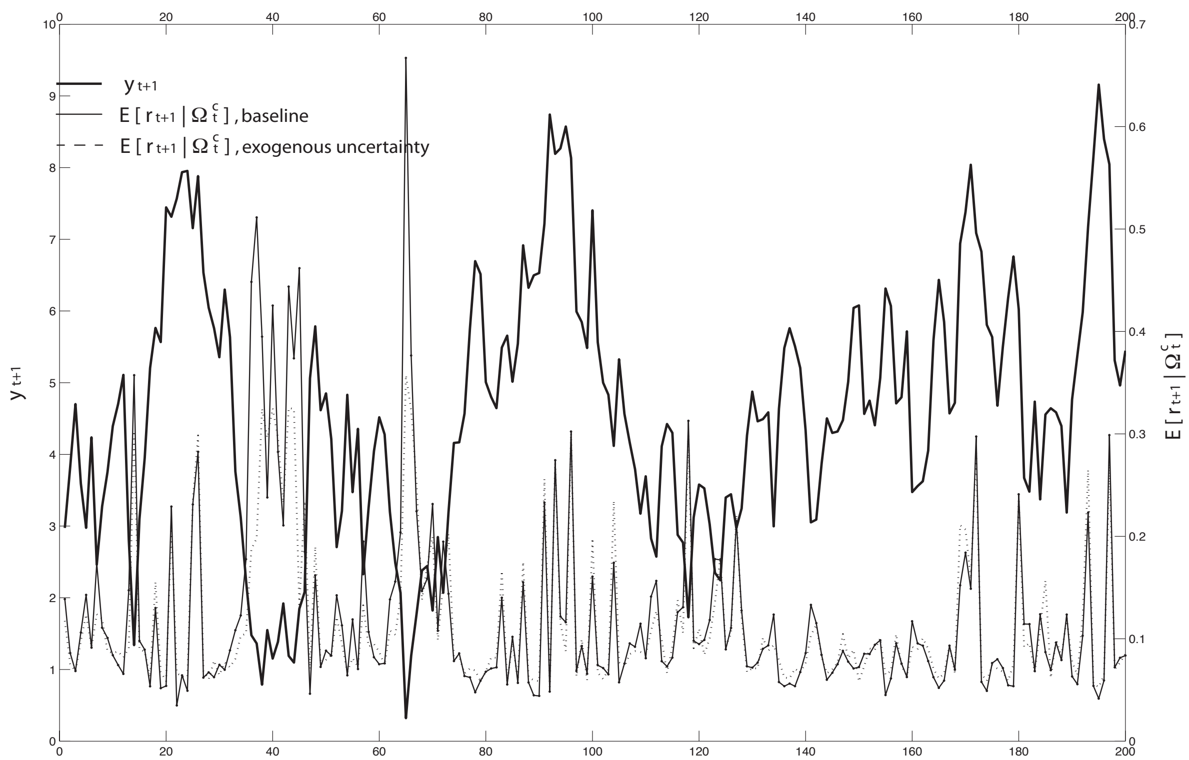
Figure 2: Aggregate dividends and uncertainty



Although the model above is stylized and its quantitative implications should be taken with a grain of salt, a minimum requirement for the asymmetric information mechanism to be relevant is its ability to generate sizable movements in asset prices within the model under consideration. Figure 3 attempts to do just that by comparing our measure of risk premium –the expected return from investing with intermediaries at period t, conditional on public information: $\mathbb{E}[\tilde{r}_{t+1} \mid \Omega_t^c]$– across the baseline and exogenous information economy. For the alternative economy with exogenous uncertainty, the informativeness of prices is set at a level for which both series attain a similar average risk premium.

The additional contribution of the asymmetric information mechanism in generating time-varying uncertainty can be appreciated in the wider movements of the baseline economy (solid line with dots). In particular, while the correlation coefficient between aggregate dividends and risk premium is -0.45 in the baseline economy, it is only -0.30 in the alternative economy with exogenous uncertainty. In terms of the variance decomposition, the economy with exogenous risk has a st.dev. of the risk premium of 7.07%, while in the baseline economy this figure reaches 9.48%, an increase of about 34%.

Figure 3: Risk Premium in Simulated Economies



5 Conclusions

This paper embeds a standard noisy REE model into a OLG macroeconomic framework to study the impact of time-varying wealth on price informativeness, and ultimately, economic uncertainty. As current output contracts and dividends fall, risk-averse households with wealth-sensitive risk tolerance allocate less funding into intermediaries, leading to a contraction in their number and a deterioration in the informational content of prices. The mechanism implies that the level of aggregate dividends (i.e., output) is negatively associated with uncertainty about underlying conditions, and presents a natural explanation for time-varying first and second moments (risk premium and uncertainty) that need not assume preferences with habit formation, nor heteroskedastic dividend processes.

While the mechanism highlighted has an economically relevant effect in the context of the model presented, the main message of the paper is qualitative in nature. Existing models in which uncertainty is specified as an exogenous process implicitly assume away normative implications of the analysis. In contrast, a model in which uncertainty arises endogenously with the limited capacity of informed agents to trade upon information, allows one to reflect on potential policy prescriptions. In particular, the mechanism outlined generates information externalities between agents, as the actions of traders affect the amount of uncertainty faced by other traders and households, both contemporaneously and inter-temporally. Further exploration of the policy implications in environments that exhibit endogenous uncertainty is an important extension of this line of research, but out of the scope of the present paper.

Transplanting a noisy REE framework into a macroeconomic model is not an easy task, and some restrictive assumption must be made at one point or another. Going forward, the agenda should place effort in overcoming the aspects of the model that limit its applicability for studying the quantitative importance of time-varying uncertainty. Specifically, the short-lived nature of assets limit the quantitative effects of the mechanism on risk premia and stock prices. Indeed, while a spike in current levels of uncertainty has a persistent effect on the uncertainty about dividends further into the future, prices of two-period lived assets are by construction prevented from incorporating these effects. In future work, I plan to expand the model by allowing infinitely-lived households and assets, although retaining the OLG structure at the trader level and hopefully prices which exhibit a conditional normal distribution.

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6 Appendix

6.1 Appendix A

Characterization of Beliefs (Proof of Theorem 1 and 2)

Equation (14) describes the transition of the state vector $\Psi_{t+1}^i \equiv \begin{bmatrix} y_{t+1} & \eta_{t+1}^i & \theta_t & \phi_t^i \end{bmatrix}^T$, with

$$C_\psi = \begin{bmatrix} (1-\rho_y) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\rho_\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\Psi} = \begin{bmatrix} \bar{y} \\ 0 \\ \bar{\theta} \\ 0 \end{bmatrix},$$

$$A_\psi = \begin{bmatrix} \rho_y & 0 & 0 & 0 \\ 0 & \rho_\eta & 0 & 0 \\ 0 & 0 & \rho_\theta & 0 \\ 0 & 0 & 0 & \rho_\phi \end{bmatrix}, \text{ and } \quad B_\psi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Similarly, the vector of public signals can be written in terms of Ψ_{t+1}^i as in expression (16), with with

$$C_s = \begin{bmatrix} -(1-\rho_y)/\rho_y & 0 & 0 & 0 \\ -(1-\rho_y)/\rho_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_{s,t} = \begin{bmatrix} \rho_y^{-1} & 0 & 0 & 0 \\ \rho_y^{-1} & \rho_\eta^{-1} & 0 & 0 \\ 1 & 0 & \Delta_t & 0 \\ 1 & 1 & \Delta_t & \Delta_t \end{bmatrix}, \quad \text{and}$$

$$B_s = \begin{bmatrix} -\rho_y^{-1} & 1 & 0 & 0 & 0 & 0 & 0 \\ -\rho_y^{-1} & 1 & 1 & -\rho_\eta^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The filtering problem stated in Theorem 1 amounts to finding posterior beliefs conditional on the vector of public signals, Ω_t^c. Applying a standard Kalman filter gives the posterior expectation stated in equation (17), where K_t^c and O_t^c satisfy the recursive relation described by equation (18) and

$$K_t^c = (A_\psi O_{t-1}^c A_\psi^T + B_\psi \Sigma B_\psi^T) \cdot A_{s,t}^T \cdot (A_{s,t}(A_\psi O_{t-1}^c A_\psi^T + B_\psi \Sigma B_\psi^T) A_{s,t}^T + B_s \Sigma B_s^T)^{-1} \quad (30)$$

Theorem 2 also follows as a result of the Kalman filter, this time using the expectations of Ψ_{t+1}^i conditional on public information Ω_t^c as the prior, and the private signal $s_t^{i,j}$ as the observation leading to the update.

Posterior expectations of trader (i,j) are then given by equation (19), where K_t^p and O_t satisfy the recursive relation described by equation (20) and

$$K_t^p \quad = \quad O_t^c \cdot a_s^T \cdot (a_s O_t^c a_s^T + b_s \Sigma b_s^T)^{-1}, \tag{31}$$

where $a_s = [1 \ \ 1 \ \ 0 \ \ 0]$, and $b_s = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 1]$. This completes the characterization of beliefs.

6.2 Appendix B

Table 1 shows the parameters assumed in the different exercises discussed in section 4.

	Preferences and wealth			Intermediation		Stochastic processes												
	γ	γ^h	W_0	c	$\underline{F}$	y	θ	ρ_y	ρ_θ	ρ_η	ρ_ϕ	σ_y	σ_d	$\sigma_{i,d}$	σ_η	σ_θ	σ_ϕ	σ_s
Figure 1: Impulse-response																		
Persistent shocks	1.5	6	10	0.03	3.6	3	2	0.9	0	0.5	0	1	0.6	0.6	1	0.5	1	2
Transitory shocks	1.5	6	10	0.03	3.6	3	2	0.4	0	0.5	0	1	0.6	0.6	1	0.5	1	2
Figure 2: aggregate dividends and uncertainty																		
Baseline economy	1.5	6	14.5	0.03	5.4	4.5	2	0.9	0.8	0.5	0	1	0.6	0.6	1	0.5	1	2
Figure 3: random path																		
Endogenous uncertainty	1.5	6	14.5	0.03	5.4	4.5	2	0.9	0.8	0.5	0	1	0.6	0.6	1	0.5	1	2
Exogenous uncertainty	1.5	6	14.5	0.03	5.4	4.5	2	0.9	0.8	0.5	0	1	0.6	0.6	1	0.5	1	N.A.*

* The exogenous uncertainty economy assumes the standard deviation of y conditional on prices only is 0.0071.